A Formal Approach to the Design of Multibunch Feedback Systems: LQG Controllers

H. Hindi, J. Fox, S. Prabhakar, L. Sapozhnikov, G. Oxoby, I. Linscott, and D. Teytelman
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309

Abstract

We formulate the multibunch feedback problem as a standard control-systems design problem and solve it using Linear Quadratic Gaussian (LQG) regulator theory. Use of a specific optimality criterion allows quantitative evaluation of different controllers and leads to the design of optimal LQG controllers. Computer simulations are used to show that, as compared to the existing Finite Impulse Response (FIR) control, LQG control can provide the same closed-loop damping for less peak power, thus making more effective use of limited kicker power. Furthermore, LQG control enables us to use more power to provide better damping without the problem of driving instabilities with higher loop gains. The code for the LQG filters described has been written for the Quick prototype installed at ALS.

1 INTRODUCTION

The problem of designing the controller (filtering algorithm) for the longitudinal feedback system is basically a *regulator problem.* The regulator problem has been studied extensively in control theory, and there are many techniques available to solve it. This paper will compare the performances of the existing FIR filter-based control technique to that of the LQG filter-based technique. One of the distinguishing features of LQG design is the use of a specific optimality criterion. This is in contrast to the FIR-based technique, which is based on an adhoc discrete-time approximation of a differentiator [1,2].

In the process of damping synchrotron oscillations, measurements of the beam phase are taken at discrete times and the feedback correction signals are applied at discrete times. Therefore, our analysis will be carried out using discrete-time control formalism. We will also use state-space notation in our description of the plant and controller [3,4]. Given a system described by state-space matrices $\{A, B, C, D\}$, the transfer function is obtained by

$$H(z) = C(zI - A)^{-1}B + D \ . \qquad (1)$$

2 LINEAR QUADRATIC GAUSSIAN REGULATOR THEORY

Figure 1 shows a block diagram of the LQG regulator problem. A precise statement of the problem follows. Given a linear model of the plant P, described by state matrices $\{A, B, C, D\}$ $(D = 0)$,

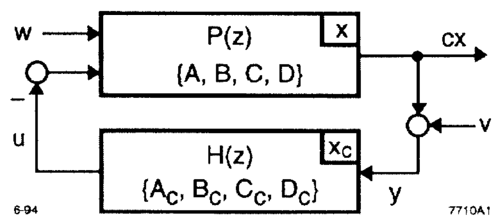

Figure 1. Block diagram of the LQG regulator problem.

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + w(k) \\ y(k) &= C\,x(k) + v(k) \ . \end{aligned} \qquad (2)$$

where x is the plant state (energy and phase), u is the control input (kicker signal), y is the measured regulated output (phase), and w and v are process and measurement noises, respectively, both of which are assumed to be uncorrelated white noises with covariance matrices W and V. We would like to find a controller $H_{\mathrm{lqg}}(z)$, described by $\{A_c, B_c, C_c, D_c\}$, $(D_c = 0)$,

$$\begin{aligned} x_c(k+1) &= A_c\,x_c(k) + B_c\,y(k) \\ u(k) &= C_c\,x_c(k) \ , \end{aligned} \qquad (3)$$

which will minimize the cost functional (or optimality criterion)

$$J_{\mathrm{lqg}} \triangleq \lim_{k\to\infty} \mathbf{E}\left[x(k)^T C^T Q C x(k) + u(k)^T R u(k)\right] \ , \qquad (4)$$

where $\mathbf{E}$ denotes the expectation operator. Notice that the quantity $Cx(k)$ is actually the noiseless plant output. Thus, the cost functional J_{lqg} can be interpreted as the weighted sum of the steady state rms excursion of the regulated plant output $C\,x(k)$ and the rms actuator effort $u(k)$. Here, R and Q are symmetric, positive definite weighting matrices that allow us to trade off between steady state rms output excursions and rms actuator effort.

The solution to the LQG regulator problem is well known [3,4,5]. We state the results here without proof:

$$\{A_c, B_c, C_c, D_c\} = \{A - B\,K_{opt} - L_{opt}\,C, L_{opt}, K_{opt}, 0\} \ , \qquad (5)$$

where L_{opt} is the solution to (Kalman filter, Kalman 1960)

$$L_{opt} = APC^T(V + CPC^T)^{-1} \ , \qquad (6)$$

$$P = APA^T + W - L_{opt}(V + CPC^T)L_{opt}^T \ , \qquad (7)$$

*Work supported by Department of Energy contract DE-AC03-76SF00515.

and K_{opt} is the solution to (Linear Quadratic Regulator, Bellman 1957)

$$K_{opt} = (R + B^T SB)^{-1} B^T SA , \quad (8)$$

$$S = A^T SA + C^T QC - K_{opt}^T (R + B^T SB) K_{opt} . \quad (9)$$

The transfer function of the optimal LQG controller $H_{\mathrm{lqg}}(z)$ can be obtained by applying Eq. (1) to $\{A_c, B_c, C_c, D_c\}$ above. The equations for P and S are matrix quadratic (Riccati) equations. Despite their rather intimidating appearance, they are readily solved using numerical software design tools [7]. At this point, we stress that this controller is "optimal" in the following very precise sense: it minimizes the cost functional J_{lqg}. This does not necessarily make it the "best" controller.

3 COMPARISON WITH A SINGLE-BUNCH BEAM

We now compare the performance of LQG feedback versus FIR feedback on a computer simulation of a single-bunch beam. Initially, these simulations will be without noise; performance in the presence of noise will be discussed in the following section.

The "plant" which we are trying to regulate is the discrete-time linear model of the longitudinal oscillations of a single-bunch beam, with a dc gain of 1 (0 dB), a peak response of 33 dB, a Q of 50, and a synchrotron frequency of 10 KHz, sampled at 20 μs. These parameters model the existing conditions at ALS. Applying Eq. 1 to the state-space model, Eq. (2), yields the following transfer function:

$$P(z) = \frac{0.6854z^{-1} + 0.6794z^{-2}}{1 - 0.6104z^{-1} + 0.9752z^{-2}} . \quad (10)$$

Figure 2(a) shows the frequency response of the beam and Figure 2(b) shows its open-loop impulse response.

The FIR controller is an empirically designed 4-Tap Linear Phase FIR bandpass filter with zero dc response, which approximates a diffentiator. Its gain parameter was adjusted to give the smallest peak gain in the closed-loop response, yielding:

$$H_{fir}(z) = -0.200z^{-1} + 0.000z^{-2} + 0.200z^{-3} + 0.000z^{-4} . \quad (11)$$

An LQG controller (LQG1) can be designed to match the performance of the FIR controller by choosing appropriate values for the design parameters, Q, R, V, and W [3,4]. The values chosen for W and V are arbitrary, since precise measurements of process and measurement noise variances have not been made yet. However, these numbers still lead to a good design, as we shall see. The discretized transfer function calculated by applying Eq. (1) to Eqs. (5) through (13) is:

$$H_{\mathrm{lqg1}}(z) = \frac{-0.3513z^{-1} - 0.0500z^{-2}}{1 + 0.5011z^{-1} + 0.0348z^{-2}} . \quad (12)$$

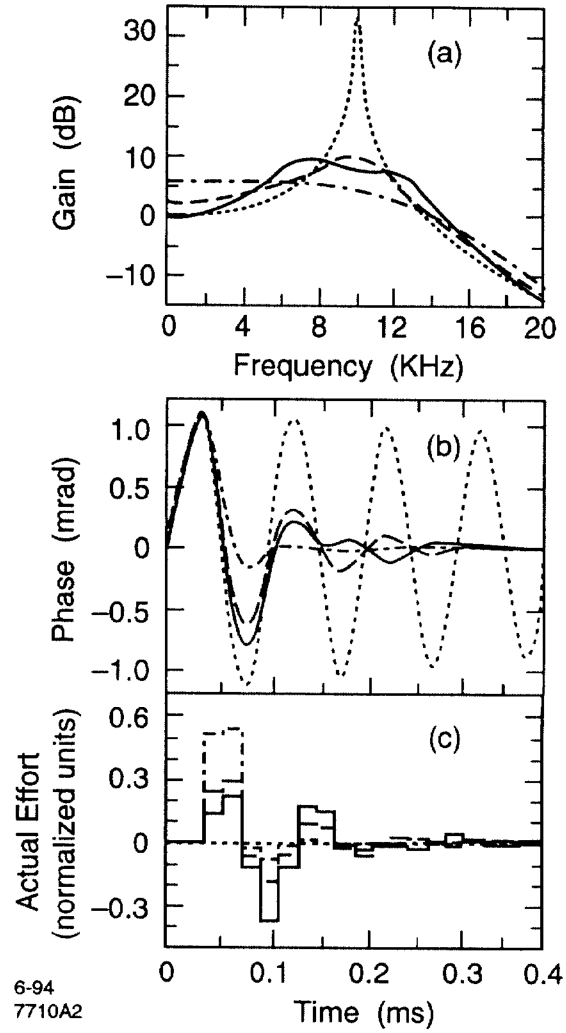

Figure 2. (a) Frequency responses, (b) impulse responses, and (c) actuator efforts. Open-loop (dotted), LQG1 (dashed), LQG2 (dash-dot), and FIR (solid).

It was pointed out in Reference [2] that there is a limit to the damping that can be achieved using the FIR control described above: further increase in the FIR's gain would drive new instabilities. A second LQG controller (LQG2) can be designed that improves the damping *without* causing instabilities. By swapping the values of Q and R, above, while keeping W and V the same, Eqs. (5) to (13) give us a new optimal controller that uses more actuator effort, but achieves better damping:

$$H_{\mathrm{lqg2}}(z) = \frac{-0.6828z^{-1} - 0.4524z^{-2}}{1 + 1.0411z^{-1} + 0.3151z^{-2}} . \quad (13)$$

Notice that both LQG controllers have nonzero dc response [1,2], so they require a simple modification before they can be used in practice: a dc notch filter (for example, $D(z) = (1 - z^{-1})/(1 - 0.9z^{-1})$) must be added on in series. We will return to this point in Section 4.

Figure 2(a) shows the closed-loop frequency responses with the three controllers. The peak of the resonance has been reduced from 33 dB to about 9 dB by both the FIR and LQG1, except that the FIR gives a broader "plateau."[1] The LQG2 flattens the peak past the FIR limit by another 5dB—almost a factor of 2.

Figure 2(b) shows the closed-loop impulse responses. The responses with FIR and LQG1 are remarkably similar, as expected. With LQG2, the system is almost completely damped in a single cycle.

Figure 2(c) shows the actuator efforts required to bring the system to rest. The peak actuator effort of approximately 0.4 units used by FIR is reduced to 0.3 units by LQG1. Thus, LQG1 gives the same performance as FIR, but for less peak power (and lower rms power, of course). For LQG2, however, we see that the improvement in damping comes at a cost: the peak actuator effort has almost doubled to 0.6 units.

Figure 3 shows the closed-loop frequency responses with the FIR, LQG1, and LQG2 in a bunch-by-bunch implementation on a simple simulation of a four-bunch beam [5,6]. The bunches (modeled as masses, springs, and dampers) are coupled by bidirectional, nearest-neighbor coupling at 5% the strength of the restoring force on each individual bunch, but the first bunch is not coupled to the last. The frequency responses were taken by applying an excitation to bunch 1 and measuring its phase. Notice the multiple peaks due to the coupling. The plots show that the performance of the controllers on a weakly coupled beam is almost identical to the single-bunch case.

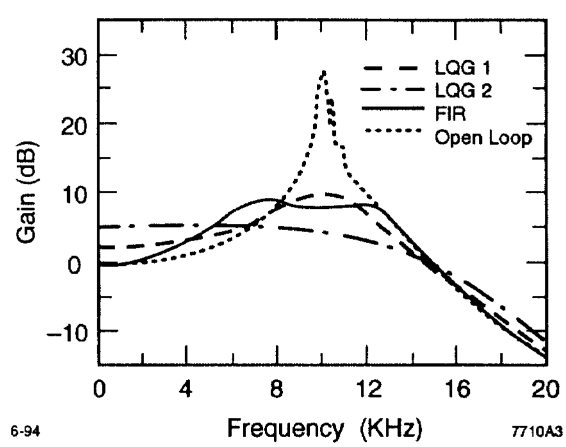

Figure 3. Frequency responses for four-bunch beam.

4 PERFORMANCE WITH NOISE

We now return to the single-bunch senario and quantitatively evaluate the above three controllers in the presence of noise. Once a controller has been specified, it is possible to actually *calculate* the steady-state rms output excursions and the rms actuator effort for the closed-loop system, by solving an appropriate Lyapunov equation [3,4]. As the Q/R ratio is varied from zero to infinity, the family of optimal controllers sweeps out the trade-off curve shown in Figure 4. Points above the curve correspond to suboptimal and hence achievable specifications; points below it are unachievable, since they would have either lower rms excursions or lower rms actuator effort than the corresponding optimal controller.

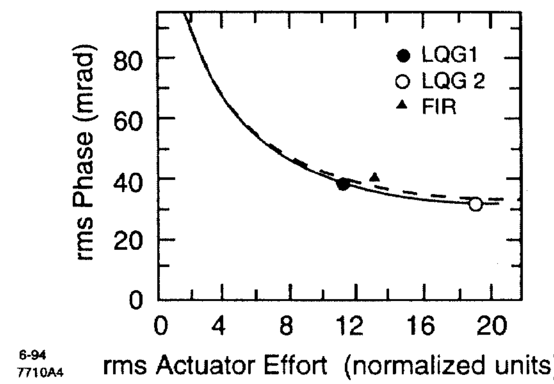

Figure 4. Trade-off curves: LQG (solid), modified LQG (dashed).

For any controller, this curve tells us exactly how close to optimal we are operating, in the LQG sense: since the FIR filter lies close to the "knee" of the curve, it is a reasonable operating point. The trade-off curve for the family of modified LQG controllers, $D(z)H_{\mathrm{lqg}}(z)$, shows that the loss in performance due to this modification is minimal.

In summary, the formulation of the longitudinal feedback problem as an LQG regulator problem allowed us to design better, more efficient controllers. The precise optimality criterion allowed us to evaluate the trade offs between different controllers quantitatively. The ideas presented here will be implemented this summer on the DSP-based bunch-by-bunch longitudinal feedback system at ALS [8].

[1] For larger gains, the edges of this plateau would rise, corresponding to driving new instabilities [2].

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