Photometry Method to Define the Charged Particle Longitudinal Distribution Function in the Bunch[1,2]

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Abstract

In this paper we propose the method of reconstruction of the longitudinal profile of the charged particle bunch by photometry of the film blackening due to optical transition radiation (OTR) generated by those bunches. Three techniques are considered to determine the modules of Fourier spectrum of the bunch particle distribution function envelope. The mathematical processing of the photometry results consists in solution of the Fredholm first-kind integral equation

1. INTRODUCTION

The present work shows the possibility to determine a Fourier transformation image moduls of longitudinal function of charge distribution in an electron bunch using the integral energy characteristic function of the bunch - angular distribution of energy flux density of OTR. The latter is emitted by vacuum-medium boundary and detected in a narrow frequency band of the spectrum optical region during the whole time of the bunch passage through the boundary.

The mentioned characteristic can be determined by results of photometry of the photofilm exposed to OTR effect. For simplicity we consider the normal entrance case. Generalization for the oblique entrance case causes no principal difficulties [1]

2. OTR FROM THE DISPERSIVE MEDIUM BOUNDARY

A linear bunch is moving along the positive direction of the z axis in vacuum towards the planar vacuum-medium boundary (normal to the z axis) at a velocity v. The photofilm's flat surface parallel to the boundary is located at a distance z_0 from it. The point on the film's surface is characterized by a radial ρ and polar φ coordinates of the cylindrical coordinate system connected with the z axis.

At $z_0 >> l$ (l- OTR formation zone length) for the normal entrance case the OTR field is axially symmetric, linearly polarized and for film-detected frequency band can be presented as [1]:

$$E_\rho = Q(\rho,t)\cos\theta;\ E_z = Q(\rho,t)\sin\theta;\ E_\varphi = 0; \qquad (1)$$

where

$$Q(\rho,t) = \int_{\omega_0-\Omega}^{\omega_0+\Omega} A(\rho,\omega)F(\frac{\omega}{v})e^{i\frac{\omega}{v}L}\,d\omega \qquad (2)$$

$$A(\rho,\omega) = \frac{1}{z_0}U(\theta,\omega),\ F(\frac{\omega}{v}) = \int_{-\infty}^{\infty} Z(x)\exp(i\frac{\omega}{v}x)dx \quad (3)$$

$\theta = arctg\,\rho z_0$; $L = \varphi(\rho) - vt$; $\varphi(\rho) = \beta(z_0^2+\rho^2)^{1/2}$;

Here $U(\theta,\omega)$ is OTR frequency-angular characteristic of a single particle [2], $Z(x)$ is bunch particle distribution function, t- time, c- velocity of light in vacuum, $\beta = v/c$.

The film blackening produced as a result of OTR is defined by the expression [3]

$$I(\rho) \approx \int_{-\infty}^{\infty}\left|\vec{E}\right|^2 dt = \cos\theta\int_{\omega-\Omega}^{\omega+\Omega}\left|A(\rho,\omega)\right|^2\left|F(\frac{\omega}{v})\right|^2 d\,\frac{\omega}{v}. \quad (4)$$

being a linear integral transformation [4] of squared Fourier-image modulus of distribution function. The medium dispersion in the observed frequency band is the necessary condition for the presence of dependence of blackening on the type of distribution function.

3. OTR AS COMBINED WITH LENS

If an optical lens is placed between the boundary and photofilm, then the mentioned effect will be observed in the absence of medium dispersion [$U(\theta,\omega) = U(\theta)$ in (3)]. If the lens is round and its axis coincides with the z axis, then the field on the film surface and its blackening will be expressed again through formulae (1) and (4) where now

$$A(\rho,\omega) = \frac{i\omega}{2\pi c}\int_0^a U(\tilde{\theta})\exp\left(i\frac{\omega}{c}gr^2\right)J_0\left(\frac{\omega}{c}\frac{r\rho}{z_0-s}\right)rdr.. \quad (5)$$

$$\varphi(\rho) = \beta\left(z_0 + (n-1)\Delta + \frac{\rho^2}{2(z_0-s)}\right)$$

$$\tilde{\theta} = arctg\,\frac{r}{z_0-s} \qquad (6)$$

$$g = 1/s + 1/(z_0-s) + 1/f.$$

$J_n(x)$ is a first-order Bessel function (n=1,2,3,...), s - the distance from the lens to the boundary, a - the lens radius, f - its focal length, Δ - its thickness, n - frequency-independent refractive index of the lens material.

Expression (5) is obtained in the paraxial approximation ($s >> a$, $z_0 - s >> \rho$) for a thin lens. If the photofilm surface is brought into coincidence with the radiation point image plane (g=0), then (5) turns into Fourier-Bessel transformation of frequency-angular characteristic of a single particle.

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4. OTR DIFFRACTION ON THE SCREEN HOLE

At $f \to \infty$ in (5) and (6) formulae (1) and (4) correspond to the diffraction of OTR field on the round hole in the flat screen in the paraxial approximation. In this case the effect of dependence of film blackening on the type of distribution is preserved. To calculate the field in the screen shadow we'll use the geometric diffraction theory approximation [5]. For the round hole in the screen placed at the distance s from the boundary the field on the surface of the film and its blackening will again be expressed through formulae (1) and (4) where we should put

$$A(\rho,\omega) \approx -U(\tilde{\theta})\left(\frac{if_1(x)}{\sin\frac{\theta}{2}} + \frac{f_2(x)}{\cos\frac{\theta}{2}}\right),\ x = \frac{\omega}{c} a \sin\theta \ldots (7)$$

$$\varphi(\rho) = \sqrt{(z-s)^2 + \rho^2};\ \tilde{\theta} = arctg\frac{a}{s};\ \theta = arctg\frac{\rho}{z_0 - s}$$

Here a is the hole radius, $\tilde{\theta}$ is the angle of OTR incidence on the hole edge $(\tilde{\theta} << \theta)$;

$$f_1(x) = J_1(x),\ f_2 = J_0(x) \qquad (8)$$

In the case of flat slot of $2a$ width, for the plane normal to the screen and containing the z axis we again can use formula (7) putting

$$f_1(x) = -[sinx/(a\omega/c)^{1/2}];\ f_2(x) = cosx/(a\omega/c)^{1/2} \qquad (9)$$

In calculating formula (7) we used along with the geometrical theory of diffraction the method of representation of axially symmetric fields using focal expansion in terms of Bessel functions.

5. RELATIONSHIP BETWEEN THE FILM BLACKENING AND DISTRIBUTION FUNCTION ENVELOPE

In the above considered cases (Sections 2 - 4) the field in the film plane (1) and the blackening caused by it (4) contain high-frequency Fourier spectrum of distribution function stipulated by the bunch microstructure. In order to express the film blackening function through the Fourier spectrum of distribution function envelope stipulated by macroscopic bunch charge distribution, we transform function $Q(\rho,t)$ (2) as

$$Q(\rho,t) = \int_{-\infty}^{\infty} \hat{A}(\rho, L-q)\hat{Z}(q)dq \ldots\ldots\ldots\ldots (10)$$

where

$$\hat{A}(\rho) = \int_{-\infty}^{\infty} A(\rho,\omega)\exp(i\frac{\omega}{v}x)d\frac{\omega}{v};$$

$$\hat{Z}(x) = \int_{-\infty}^{\infty} Z(y)\frac{\sin\frac{\Omega}{v}(y-x)}{y-x}\exp(i\frac{\omega}{v}(y-x)dy$$

Assuming that the Fourier spectrum of envelope $\tilde{Z}(x)$ of distribution function $Z(x)$ lies in the frequency interval 2Ω we can write

$$\tilde{Z}(x) = \int_{-\infty}^{\infty} Z(y)\frac{sin\frac{\Omega}{v}(y-x)}{y-x}dy \qquad (12)$$

Taking into account that because of fast-oscillating nature of the integrand exponent in (11)

$$|\hat{Z}(x)| \le \tilde{Z}(x) \qquad (13)$$

we can claim that

$$Re\,\hat{Z}(x) = B\tilde{Z}(x)cos(\frac{\omega}{v}X(x));$$

$$(14)$$

$$Im\,\hat{Z}(x) = B\tilde{Z}(x)sin(\frac{\omega}{v}X(x))$$

where $X(x)$ and $\tilde{X}(x)$ - are some continuous functions of x, B is const. Further, comparing the first-order derivatives of the real and imaginary parts of function $\tilde{Z}(x)$ defined by (11)and (14) in the approximation of large ω_0 (i.e. discarding in them zero-order terms relative to ω_0) we obtain (at $\omega_0 \to \infty$)

$$X(x) = \tilde{X}(x) = \tilde{X} + const \qquad (15)$$

Substituting the asymptotic value of function $\hat{Z}(x)$ (with account of (14) and (15)) into (10) we arrive at the following expression for function $Q(\rho,t)$

$$Q(\rho,t) \approx B\int_{\omega_0-\Omega}^{\omega_0+\Omega} A(\rho,\omega)F(\frac{\omega-\omega_0}{v})exp\,i(\frac{\omega}{v}L + const)d\frac{\omega}{v} \quad (16)$$

The blackening function (4) takes the form

$$I(\rho) \approx B^2\cos\theta\int_{\omega_0-\Omega}^{\omega_0+\Omega} |A(\rho,\omega)|^2\left|F(\frac{\omega-\omega_0}{v})\right|^2 d\frac{\omega}{v} \qquad (17)$$

which contains a low-frequency part of Fourier spectrum of distribution function with the help of which we can determine its envelope.

6. NUMERICAL CALCULATION

The efficiency of the method confirms the essential difference in the character of the photofilm blackening at different initial bunch distribution functions. Fig.1 presents calculated normalized intensities

$$\bar{I}(\theta) = \frac{I(\theta) - I_{min}(\theta)}{I(\theta) - I_{max}(\theta)}$$

of the film blackening for three different shapes of distribution function: uniform (1), cosinusoidal (2) and Gaussian (3) for the case of flat slot in the screen. Calculations are done for the wave length $\lambda=500\ nm$ and $2\Delta\lambda=10\ nm$. The slot half width s is taken equal to the bunch length: $a=d=100\mu m$.

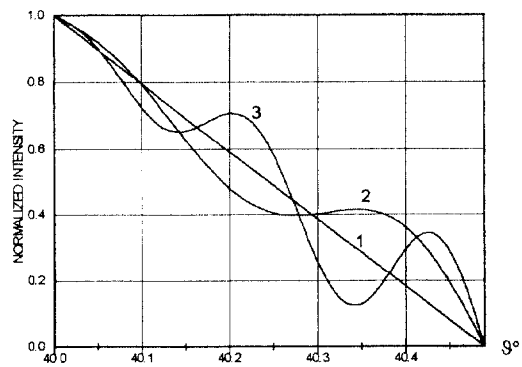

Fig.1

As can be seen from Fig. 1, the difference in the character of blackening displays in different periodicity of their oscillations, in the extent of rapidity of their attenuation as well as in the availability of phase shifts of oscillations relative to each other.

7. CONCLUSION

In this work we considered three different approaches for the determination of modulus of Fourier spectrum envelope of bunch charge distribution function united by common technique: OTR exposed film, its photometry with subsequent mathematical processing of results, which reduces to solution of Fredholm's integral first-kind equation [4].

The problems related to the photofilm sensibility can be solved by detecting the radiation from a train of bunches in the beam: in that case the mathematical processing of photometry results will yield bunch-ensemble-averaged charge distribution.

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