

# Fast Q-measurement for the PS by FFT Analysis

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## Abstract

For speedier and more precise Q-measurements, a new system, based on a Fast Fourier Transform analyser, has been developed. The Q-value is calculated in 2.8 ms from 512 beam position measurements from one pick-up, sampled at the revolution frequency ( $\sim 0.5$  MHz). Amplitude distortion due to rectangular windowing is corrected using amplitude-interpolation based on  $(\sin x)/x$  fit. Frequency interpolation is then applied. For a pure sine-wave, a precision  $\pm 0.0001$  is thus obtained over a 30 dB input range, whilst for an oscillation with a coherence time of 80  $\mu$ s the precision is still  $\pm 0.001$  over a 35 dB input range. To follow rapid changes of machine parameters, e.g. at transition, interleaving of measurements yields a Q-value every 1.8 ms. The Digital Signal Processor memory can store up to 106 ms worth of beam samples for subsequent sliding-FFT analysis.

## 1. INTRODUCTION

The PS accelerates several types of particle : hadrons, leptons and ions. Table 1 shows the total energy  $E$  of these particles and the corresponding revolution frequencies  $f_{rev}$  [1].

Table 1.

Particle type	Kinetic energy		Revolution frequency (kHz)	
	$E_{min}$	$E_{max}$	$f_{rev} (min)$	$f_{rev} (max)$
$p, \bar{p}$	200* MeV	26 GeV	270.2	476.8
$e^+, e^-$	0.6 GeV	3.5 GeV	477.1	477.1
$Pb^{53+}$	95.4 MeV/u	4.25 GeV/u	200.8	469.3

\* Deceleration for injection into LEAR

To calculate the Q-value it is necessary to obtain a coherent betatron oscillation  $f_{\beta}$  of sufficient amplitude, which is achieved by exciting the beam with a magnetic kicker. The use of a single pick-up, however, only allows calculation of the fraction  $q$  or its complement  $1-q$  [2].

The value of Q in the PS lies in the range 6.1 to 6.45, and using the values in Table 1, the betatron frequency range to be measured is

$$20 \text{ kHz} < f_{\beta} < 215 \text{ kHz} \quad (1)$$

The system must be fast enough to allow measurement of rapid Q-changes, e.g. during chromaticity measurements using beam position modulation. This has been achieved using a fast Digital Signal Processor (DSP) system.

## 2. HARDWARE

For each Q-measurement the beam is excited by an air-core kicker receiving a capacitor discharge pulse equal in length to one revolution period.

The acquisition system consists of an electrostatic pick-up which senses the beam position and converts it into an electric signal. The signal is then amplified and filtered before being digitised by a 10 MHz, 12-bit Analogue to Digital Converter (ADC) (see Figure 1).

The sample clock of the ADC is selected according to the type of signal analysis to be performed. Q-measurement is always performed using  $f_{rev}$ , but spectral analysis may also be performed using the accelerating frequency  $f_{RF}$  to observe higher modes of oscillation. A fixed-frequency or tracking anti-aliasing filter is selected depending upon the choice. In the case of Q-calculation the filter does not need to be very steep since the processing system uses amplitude interpolation and can accept some lightly attenuated image frequencies.

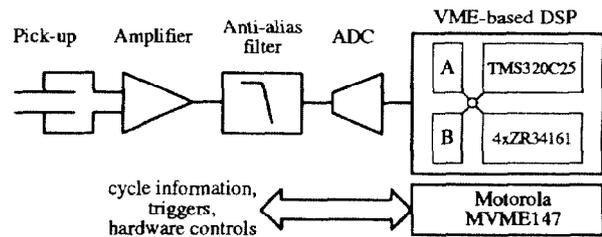


Figure 1. Hardware for the PS Q-measurement system.

Data are transferred into one of the two data memories of the VASP-16 DSP [3] via a custom 16-bit bus. The board is controlled by a Texas TMS320C25 processor, and signal processing is performed by four Zoran ZR34161 vector processors. An arbitration unit governs access to the two data memories, allowing data acquisition in one memory concurrent with processing of the other.

## 3. SIGNAL PROCESSING

512 beam-position samples are acquired by the DSP for Q-calculation, using the machine revolution frequency as derived from the beam pick-up signal. The samples are contained in a rectangular window and are treated using a 512-point magnitude-squared Fast Fourier Transform (FFT), which the four Zoran ZR34161s perform in 0.63 ms [4].

The selection of the spectral line which contains the betatron frequency  $f_{\beta}$  is based on the assumption that, given a sufficient kick, the strongest component of the beam spectrum will be  $f_{rev} \times q$ . Thus a search is performed by the DSP for the tallest spectral line within the frequency range of  $f_{\beta}$ .

Assuming a sine wave as a first-order approximation of the betatron oscillation, the shape of the resulting spectrum will be the well known  $(\sin x)/x$  containing two lines in the main lobe. The fact that the tallest of these two lines does not represent the true amplitude of the signal but is a point

on the  $(\sin x)/x$  curve, introduces the risk that another strong signal, e.g. a coupled betatron oscillation from the other plane, could appear taller than the signal of interest.

To reduce the risk of such errors, amplitude interpolation is employed on the two most prominent signals in the spectrum [5],[6]. In the example shown in Figure 2, the signal on the right is the actual betatron oscillation being measured, while the second peak could be a coupled signal from the other plane, with 90% of the amplitude of the former.

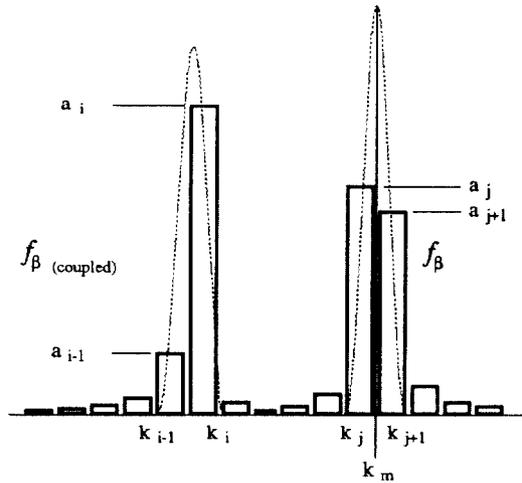


Figure 2. Power spectrum showing a signal  $f_{\beta(\text{coupled})}$  with 10% less amplitude than the signal of interest  $f_{\beta}$ , but which appears to be taller in the spectrum.

For a power spectrum, performed using a rectangular data window, the true amplitude of the signal  $f_{\beta}$  is calculated using [5]

$$A_{f_{\beta}} = \sqrt{a_j} \frac{\pi(k_m - k_j)}{\sin(\pi(k_m - k_j))} \quad (2)$$

where

$$k_m = k_j + \frac{\sqrt{a_{j+1}}}{\sqrt{a_j} + \sqrt{a_{j+1}}} \quad (3)$$

$a_j$  and  $a_{j+1}$  are the amplitudes of the spectral lines  $k_j$  and  $k_{j+1}$  respectively.

For the case of the coupling signal, in which the second line of the main lobe lies to the left of the peak, the true amplitude is found from

$$A_{f_{\beta(\text{coupling})}} = \sqrt{a_i} \frac{\pi(k_i - k_m)}{\sin(\pi(k_i - k_m))} \quad (4)$$

where

$$k_m = k_i - \frac{\sqrt{a_{i-1}}}{\sqrt{a_i} + \sqrt{a_{i-1}}} \quad (5)$$

Consequently the stronger of the two signals is selected as representing the betatron oscillation, and the  $q$  value is interpolated from the FFT using

$$q = \frac{k_m}{N} \quad (6)$$

where  $N$  = Number of points in FFT.

#### 4. RESULTS

The use of a pure sine wave to simulate the betatron oscillation allows a study of the error between known  $q$  values and the results calculated by the system. Figure 3 shows the plot of the absolute error in  $q$ , dependent upon the amplitude of the input signal.

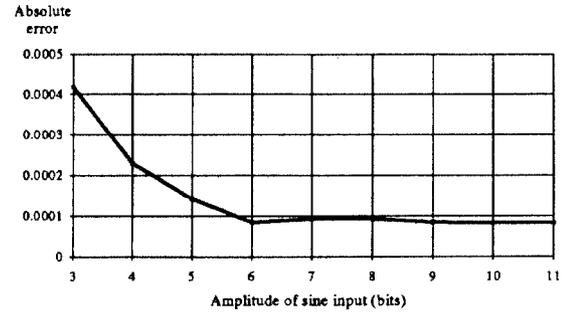


Figure 3. Absolute error in the  $q$  calculation for a pure sine wave input.

The error of the system remains below 0.0001 over a full 30 dB input range, and even with only 3 bits (peak) oscillation, it is less than 0.0005. The Zoran processors use floating point arithmetic during FFT calculation but the amplitudes of the spectral lines are truncated to integer for storage in the DSP, introducing errors into the results of calculations (2)..(5).

Coherence-loss reduces the oscillation amplitude and results in increased measurement errors. The design goal of the system, to obtain an error less than 0.001, has been obtained for a coherence time of 80  $\mu$ s over a 35 dB input range (see Figure 4). Beam excitation voltages and amplifier gains can be selected to keep the signal amplitude in this region.

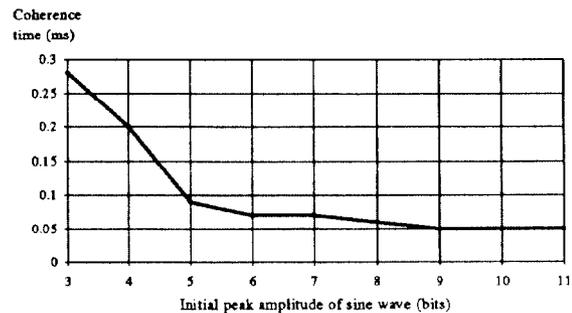


Figure 4. Maximum coherence time which a sine wave may exhibit for the  $q$  error to be less than 0.001.

The execution time of a single  $Q$ -measurement, consisting of an acquisition, data pre-processing [4], FFT and interpolation of the  $q$  value, is performed by the DSP in 2.8 ms. Amplitude interpolation by the TMS320C25 takes a further 0.4 ms. The division of the DSP data memory into two data banks and the access-arbitration unit allows a

significant increase in the power of the system through interleaving of measurements. Acquisitions are performed concurrently with the processing of previous samples, increasing the throughput of the system to one  $q$  measurement every 1.8 ms. For normal machine operations a limit has been defined allowing up to 300  $q$  measurements in each machine cycle. The results are stored locally in the DSP until the end of the cycle when they are transferred to the VME host and entered into the PS control system [8].

Figure 5 shows the results of the FFT Q-measurement for an LHC-type beam in the PS. The magnetic cycle exhibits a 1.2s front porch followed by acceleration, through transition, to a flat top at 26 GeV.

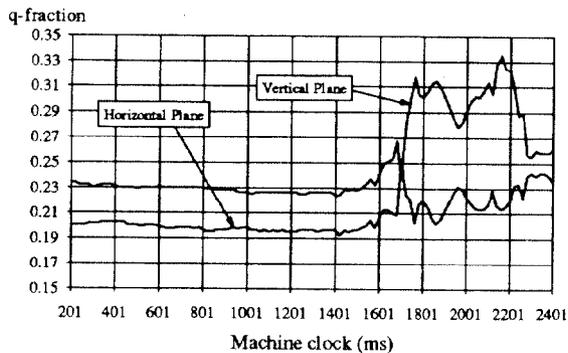


Figure 5. PS FFT Q-measurement of an LHC-type beam.

Further increases in measurement detail can be obtained from a "Sliding FFT" option which allows a picture of the beam spectrum to be constructed on a turn-by-turn basis. A single acquisition of 53248 samples is performed using  $f_{rev}$  or  $f_{RF}$ , and an FFT is moved over the data by a user-defined step to generate the display shown in Figure 6. This measurement allows very detailed analysis during a selected portion of the machine cycle. In the example shown, an instability is seen to grow significantly around 5 ms after injection.

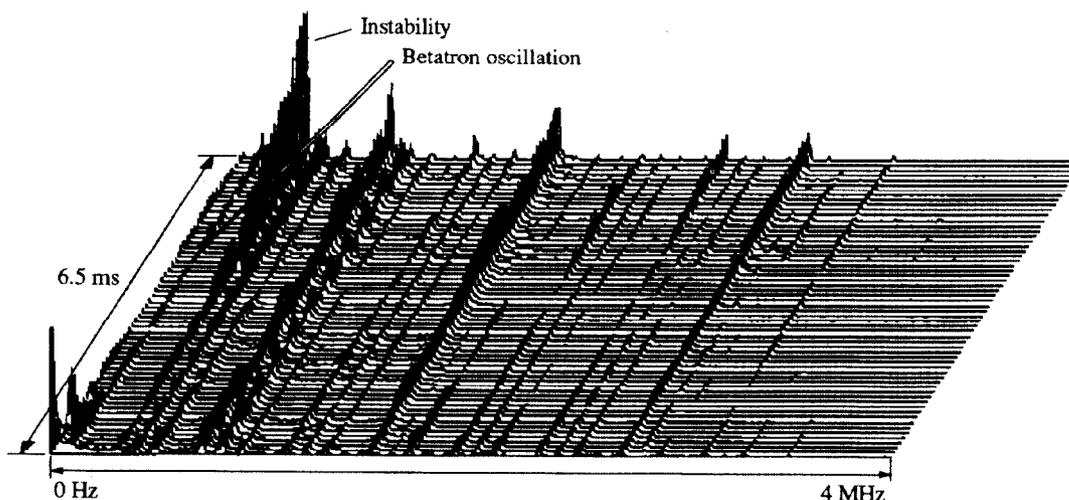


Figure 6. Sliding FFT analysis showing the spectrum of a beam during 6.5 ms following injection. The beam intensity was  $1.6 \times 10^{13}$  particles, in 20 bunches, sampled at  $f_{RF} = 8$  MHz. The bandwidth of the pick-up signal was limited to 2.5 MHz by a low-pass filter.

## 5. CONCLUSIONS

A fast Digital Signal Processor has been successfully used to perform Q-measurement and spectral analysis in the PS machine. The flexibility of the system allows users to perform a series of Q-measurements covering each machine cycle, or to select particular points of interest for detailed spectral analysis.

Amplitude interpolation has reduced the risk of Q-measurement errors in the event of high coupling between the horizontal and vertical planes as in the case of LHC-type beams. Subsequent use of frequency interpolation results in a  $q$  error below 0.0001 over a 35 dB input range.

## 6. ACKNOWLEDGEMENTS

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## 7. REFERENCES

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