On Analysis of On-Line Adjustment Procedures

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Abstract

Real-time operation of adjustment procedures set some specific limitations on possible algorithms. In the framework of linear model, an analysis of requirements to algorithms due to instability of correctors, their operation range and precision of mathematical model, is done. The coefficient of transport system adjustability, allowing to compare different algorithms, is defined. On this basis, the necessary condition of adjustability of an beam transport system is derived. Examples of analysis of beam direction adjustment in the real transport system are given.

1 INTRODUCTION

As any control system, an accelerator control system is aimed to
1. monitor accelerator and beam parameters;
2. adjust these parameters.

In our opinion, recently the major efforts have been directed onto monitor system developments, where modern computer technologies such as distributed data bases, graphics, etc. were intensively used, and much less attention was paid to the problems of online adjustment of beam parameters.

With keeping this in mind, we intend here to consider some aspects of the analysis of algorithms for the online adjustment of beam parameters. The goal of our consideration is to develop tools to answer the question: can the given algorithm keep beam parameters in specified limits under the condition that the parameters of both the input beam and magnets used for adjustment are changed spontaneously in time.

2 ADJUSTABILITY COEFFICIENT

Let us consider some beam transport system (BTS), whose current state is described by array y, depending on the input beam parameters z, and corrector parameters x. Then the adjustment procedure may be treated as an online solution of a system of, generaly speaking, nonlinear equations:

$$y(x, z) = y^*, \tag{1}$$

where array y^* characterizes the required state of BTS.

Furthermore, we assume that linearization of equations (1) is valid and will consider the following system of linear equations:

$$Ax + z = y^*, \quad A = \{\frac{\partial y_i}{\partial x_i}\}. \tag{2}$$

To be sure in the capacity of the algorithm to adjust beam parameters, we should analyze two very closely related, but nevertheless different, points:
1. numerical stability of the equations;
2. requirements to the algorithm due to real time operation and other technical limitations.

The first point is rather traditional and the urgency of such an analysis is transparent; neither the matrix A, nor the right-hand side of equations (2) are known exactly. To estimate how this might affect the solution of the linear inverse problem, the condition numbers[1] defined as

$$condA = ||A|| \cdot ||A^{-1}||, \tag{3}$$

or more adequate here statistical analogues of condition numbers[2], can be used. At the same time, adjustment algorithms have some peculiarities. First of all, to some extent we are not interested in the obtained values of x; the only important thing is how close is the state of BTS $y(x)$ to the required value y^*. And secondly, as a rule, solution of system (2) is obtained by iterating with some matrix B^{-1}, close in a sense to the matrix A^{-1}. If

$$||1 - AB^{-1}|| \leq$$
$$\leq cond(A) \cdot \frac{||A^{-1} - B^{-1}||}{||A^{-1}||} \leq \frac{\varepsilon \cdot cond^2(A)}{1 - \varepsilon \cdot cond(A)} < 1, \tag{4}$$

where $\varepsilon = \frac{||A-B||}{||A||}$, the iterations converge to the required state.

The obtained relation guarantees only numerical convergency of the algorithm, but while deriving it we have neglected some very important, from the practical point of view, details. First of all, we are not interested in the values of the corrector parameters x, while in practice the range of permissible values is limited. Secondly, we implicitly suppose that the corrector parameters do not change spontaneously. And, last but not least, an adjustment algorithm should operate in real time. So, besides numerical stability we have to analyze technical capacity of the algorithm to adjust the beam.

To this end, let us define the following quantities:

- the corrector stabilization level, i.e. the magnitude of maximum spontaneous deviations of corrector parameters x from the prescribed ones in the operation time of one algorithm loop:

$$\delta_{st} = \max_{t \in [t_0, t_0 + t_k]} ||x(t) - x(t_0)||,$$

where $x(t)$ are corrector parameters at the moment t, t_k — the operation time of one algorithm loop;

- the input beam stabilization level, i.e. the magnitude of the spontaneous deviations of z from prescribed ones in the operation time of one algorithm loop:

$$\gamma_{st} = \max_{t \in [t_0, t_0 + t_k]} ||z(t) - z^0||;$$

- the required beam state stabilization level, i.e. the magnitude of permissible deviations from the required state y^* during the whole operating period:

$$\varepsilon_{st} = \max_{t \in [0, t_{end}]} ||y(t) - y^*||;$$

- the width of the corrector operation range:

$$\delta_{max} = \max_{x, x' \in D_x} ||x - x'||,$$

where D_x means the corrector operation range;

- the width of the input beam parameters range:

$$\gamma_{max} = \max_{z, z' \in D_z} ||z - z'||,$$

where D_z is a set of permissible values of the input beam parameters;

- the width of operation regimes:

$$\varepsilon_{max} = \max_{y, y' \in D_y} ||y - y'||,$$

where D_y is a set of possible operation regimes.

To ensure that in the operation time of one loop of the algorithm the instability of the input beam and corrector parameters will not cause unpermissible deviations of the output beam, we have to demand that:

$$||\Delta y|| \leq ||A \Delta x|| + ||\Delta z|| \leq ||A|| \cdot ||\Delta x|| + ||\Delta z|| \leq$$

$$\leq ||A|| \cdot \delta_{st} + \gamma_{st} \leq \varepsilon_{st}$$

or

$$||A|| \leq \frac{\varepsilon_{st} - \gamma_{st}}{\delta_{st}} \tag{5}$$

On the other hand, we should provide for correction of any deviation of the input beam, as well as possibility to change operation regime:

$$||\Delta x|| = ||A^{-1}(\Delta y - \Delta z)|| \leq ||A^{-1}|| \cdot (||\Delta y|| + ||\Delta z||) \leq$$

$$\leq ||A^{-1}|| \cdot (\varepsilon_{max} + \gamma_{max}) \leq \delta_{max}.$$

As a result, we obtain:

$$||A^{-1}|| \leq \frac{\delta_{max}}{\varepsilon_{max} + \gamma_{max}}. \tag{6}$$

It should be noted that the quantities Δz appear in the derivation of both relation (5) and (6). However, they have different meaning. In the first case these are the deviations in a short period of time — for example, small oscillations around some mean values. In the second one these are the deviations after the whole operation period, including both above-mentioned short period oscillations and long range (may be very smooth in time, but large in magnitude) input beam deviations.

It is convenient to combine relations (5),(6) in a single adjustability criterion:

$$||A|| \cdot ||A^{-1}|| \equiv cond(A) \leq coad, \tag{7}$$

where

$$coad = \frac{(\varepsilon_{st} - \gamma_{st}) \cdot \delta_{max}}{(\varepsilon_{max} + \gamma_{max}) \cdot \delta_{st}}. \tag{8}$$

In the following, we will call this quantity the BTS adjustability coefficient.

Thus, we have obtained the necessary[1] condition of adjustability of BTS by means of the given algorithm. It should be emphasized that, although the obtained relation includes a condition number, it is not relevant for the accuracy of the solution of (2). This relation means a technical restriction on the instability of parameters, the range of operation regimes and the algorithm operation time. Its nonfulfillment means that one cannot guarantee that the required BTS state will be held at the required stabilization level, although system (2) is well conditioned and the matrix A^{-1} is known with high accuracy.

Moreover, adjustability coefficient is independent of the matrix A and can take arbitrary values, while the condition number is always greater, or equal to unity. So, if the adjustability coefficient is less than unity, there exists no algorithm described by system (2) permiting to adjust the given BTS at given tolerances.

The condition (7) was obtined using absolut values of the tolerances and instabilities, but it can be easy rewritten in statistical terms [3].

To demonstrate the application of the approach described, let us consider as an example the analysis of two schemes of beam location adjustment. These schemes (see figs. 1 and 2) have been studied experimentally and the results were reported in [4]. The goal was to pass a beam, using three bending magnets ϕ_i, through three given space points determined by coordinate detectors R_i.

In the frames of the first order Brown formalism[5], the beam displacement from the reference trajectory may be written as[6]

$$\vec{\xi} = T\vec{\xi}_0 + \vec{S} - H\delta\vec{B}, \tag{9}$$

where

$$H = \begin{pmatrix} T_{12}(1)\alpha_1 & \dots & T_{12}(n)\alpha_n \\ T_{22}(1)\alpha_1 & \dots & T_{22}(n)\alpha_n \end{pmatrix},$$

T is the total transfer matrix, $\vec{\xi}_0 = (\xi_0, \xi_0')$, $\vec{\xi} = (\xi, \xi')$ — beam phase coordinates at BTS input and output respectively, ξ and ξ' — the space and angular coordinates, respectivly, $\vec{S}$ — the term related to particle momentum deviation, $\delta\vec{B}$ — magnet field deviations, $T(i)$ — the matrix, describing beam transportation from the center of the

[1] not sufficient, because (7) does not entail the validity of inequalities (5),(6)

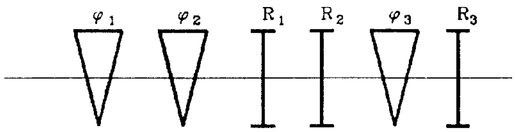

Figure 1: Beam location adjustment. Scheme 1.

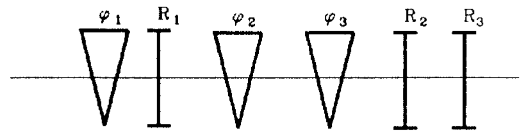

Figure 2: Beam location adjustment. Scheme 2.

i-th magnet to BTS output, α_i — bending angles in the i-th magnet.

Then the problem of beam location adjustment may be formulated in terms of system (2), where $A = \{T_{12}(k, \bar{i})\}$, and $T(k, \bar{i})$ is the matrix, related to beam transfer between the center of k-th magnet and the i-th detector, $y = \{\xi_i\}$ are beam space coordinates at detectors, $z = T(i)\vec{\xi_0} + \vec{S}$.

The layout of detectors and corrector magnets for the first setup is shown in fig. 1. In this case, we have

$$A = \begin{pmatrix} T_{12}(1,\bar{1}) & T_{12}(2,\bar{1}) & 0 \\ T_{12}(1,\bar{2}) & T_{12}(2,\bar{2}) & 0 \\ T_{12}(1,\bar{3}) & T_{12}(2,\bar{3}) & T_{12}(3,\bar{3}) \end{pmatrix}, \qquad (10)$$

$$det(A) = T_{12}(3,\bar{3}) \cdot T_{12}(1,2) \cdot T_{12}(\bar{1},\bar{2}).$$

So, the weakest restriction imposed on this adjustment scheme is that there should be no "point to point" transformations between the first and second magnets, the first and second detectors, the third magnet and third detector.

In the described case, experimental measurements gave the following matrix:

$$A = \begin{pmatrix} 0.011 & 0.73 & 0. \\ 0.013 & 0.46 & 0. \\ 0.021 & -0.02 & 0.37 \end{pmatrix}$$

To estimate the numerical stability of the system, let us calculate the condition number in the $\sqrt{tr}$ norm. In accordance with (3), one has $cond_{\sqrt{tr}} = 183.3$. So, the system is fairly stable. The smallness of the condition number allows, as follows from (7), to hold the given BTS to very close tolerance. Although Ref. [4] does not provide all necessary information, but under rather realistic assumption that $\delta_{max}/\delta_{st} \sim 10^4, \gamma_{max} \sim 10$ mm and neglecting ε_{max}, we obtain that beam may be adjusted with $\varepsilon_{st} = \gamma_{st} + 0.18$mm.

In spite of all that the experiment yielded a negative result — iterations have not converged. In the author's opinion, the reason of divergency is smallness of the distance between the first and second magnets which leads , as follows from (10), to the smallness of the matrix determinant. But, as is clear from the calculations of condition number the matrix is far from being singular. The real reason of divergency becomes clear if we take into account the matrix measurement accuracy:

$$\Delta A = \begin{pmatrix} 0.001 & 0.03 & 0. \\ 0.002 & 0.02 & 0. \\ 0.002 & 0.02 & 0.02 \end{pmatrix}.$$

Direct calculations yield $\varepsilon = ||\Delta A||/||A|| \simeq 5 \cdot 10^{-2}$, and from

$$\frac{\Delta||x||}{||x||} \leq \frac{\delta \cdot cond(A)}{1 - \varepsilon \cdot cond(A)} + \frac{\varepsilon \cdot [cond(A)]^2}{1 - \varepsilon \cdot cond(A)},$$

we obtain that the relative error is $\frac{||\Delta x||}{||x||} \simeq 5$, i. e. not a single digit in the solution we have obtained can be relied upon.

The second scheme of interest is represented in fig. 2. The matrix of the related system is

$$A = \begin{pmatrix} T_{12}(1,\bar{1}) & 0 & 0 \\ T_{12}(1,\bar{2}) & T_{12}(2,\bar{2}) & T_{12}(3,\bar{2}) \\ T_{12}(1,\bar{3}) & T_{12}(2,\bar{3}) & T_{12}(3,\bar{3}) \end{pmatrix}.$$

The measurements revealed the following values for the matrix elements and their errors:

$$\begin{pmatrix} 0.46 & 0. & 0. \\ -0.02 & 0.37 & 0.02 \\ -0.05 & -0.3 & 0.1 \end{pmatrix} \pm \begin{pmatrix} 0.002 & 0. & 0. \\ 0.002 & 0.02 & 0.02 \\ 0.002 & 0.02 & 0.02 \end{pmatrix}$$

In this case, the condition number of the matrix is $cond_{\sqrt{tr}} = 1.9$, and, as a result, the relative error of the solution is no more than 0.33. Indeed, as the experiment shows, the iteration process converges and the required beam adjustment is achieved.

3 CONCLUSION

Thus, the approach proposed for analysing of online adjustment procedures is a simple but rather powerful tool allowing at the developmental stage: to compare different adjustment schemes; to establish bottlenecks; to estimate the required precision of measurements.

4 REFERENCES

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