New Method for Displaying the RF Cavity Impedance on a Smith Chart

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Abstract

A new technique for the Smith Chart display of RF cavity impedance is described. The system was developed for the compact synchrotron HELIOS 2 currently being built by Oxford Instruments. There are three major improvements compared with the standard four probe method: firstly this technique uses two probes on the transmission line instead of four, secondly AGC (Automatic Gain Control) on the forward signal (signal going from the source to the cavity) requires only 10 dB dynamic range of amplitude detection, the precision being limited by the linearity of the detector. (With the four probe method, the linearity over the wide dynamic range of amplitude detection (more than 60dB) severely limits the precision). Finally in contrast to the standard four probe method, this technique does not require a complicated calibration procedure. The Smith Chart is displayed via a C program on an IBM compatible PC with a 80286 processor, under Windows, saving the need for a dedicated Network Analyser.

1 INTRODUCTION

A standard method to display the cavity impedance on a Smith Chart uses four probes on the transmission line, $\lambda/8$ apart (λ being the wavelength of the cavity frequency). Detecting these voltages by squaring them makes possible the display of the reflection coefficient in polar coordinates on an X-Y oscilloscope. See [1] for further details. The four probe method was not used because of the need to operate over a wide range of cavity voltages - typically a 40dB range. The detector dynamic range would have to be even larger than this in order to cope with the signal levels associated with different voltage levels expected at the four probe positions as the cavity impedance changes.

The method we propose here demonstrates two ways of calculating the reflection coefficient S_{11} and shows that in combination they will minimize the dynamic range of amplitude detection required.

2 METHOD

According the transmission line theory, the equations of the voltage and the current along a transmission line at θ are:

$$\begin{cases} V(\theta) &= V^+e^{-j\theta} + V^-e^{j\theta} \\ Z_cI(\theta) &= V^+e^{-j\theta} - V^-e^{j\theta} \end{cases}$$

where $\theta = 2\pi l/\lambda$, l being the position of the probe on the transmission line, Z_c is the characteristic impedance (50Ω), $V^+ = \sqrt{Z_c}a$ the incident voltage wave, and $V^- = \sqrt{Z_c}b$ the reflected voltage wave, the reflection coefficient being $S_{11} = b/a = V^-/V^+$.

Taking these equations at $\theta = 0$ and $\theta = -\pi/2$:

$$\begin{cases} V_0 &= V(\theta=0) &= V^+ + V^- \\ Z_cI_0 &= Z_cI(\theta=0) &= V^+ - V^- \\ V_{-\pi/2} &= V(\theta=-\frac{\pi}{2}) &= j(V^+ - V^-) \end{cases}$$

Hence we can express the current at $\theta = 0$ in terms of the voltage at $\theta = -\pi/2$:

$$Z_cI_0 = -jV_{-\pi/2}$$

Thus the cavity impedance Z_L can be determined from the two probe voltages:

$$Z_L = \frac{V_0}{I_0} = Z_c\frac{V_0}{-jV_{-\pi/2}}$$

Therefore we have two methods to construct the reflection coefficient S_{11}:

1. calculate the normalised impedance Z:

$$Z = \frac{Z_L}{Z_c} = \frac{V_0}{Z_cI_0} = \frac{|V_0|}{|-jV_{-\pi/2}|}e^{j\alpha} = |Z|e^{j\alpha}$$

where α is the impedance angle measured by: $\alpha = arg(V_0) - arg(V_{-\pi/2})$. From $|Z|$ and α, S_{11} can be calculated using its definition:

$$S_{11} = \frac{Z-1}{Z+1} = \frac{|Z|^2 - 1 + j2|Z|\sin\alpha}{|Z|^2 + 1 + 2|Z|\cos\alpha} \quad (1)$$

2. construct the two voltages V^- and V^+ from V_0 and $-jV_{-\pi/2}$, i.e.:

- $V^- = \frac{1}{2}(V_0 + jV_{-\pi/2})$
- $V^+ = \frac{1}{2}(V_0 - jV_{-\pi/2})$

therefore the reflection coefficient can be calculated directly:

$$S_{11} = \frac{V^-}{V^+} = \frac{|V^-|}{|V^+|}e^{j\psi} \quad (2)$$

where $\psi = arg(V^-) - arg(V^+)$ is the reflection coefficient angle.

Equation 1 and 2 are two ways to express S_{11}, the first one from the two voltages V_0, $V_{-\pi/2}$ and α, the second from V^-, V^+ and ψ.

These two expressions can be combined to minimize the required voltage amplitude detection ranges.

3 VOLTAGE AMPLITUDES

The variation of $|V^-|$ is linear with $|S_{11}|$:

$$V^- = S_{11}V^+$$

$$|V^-| = |S_{11}||V^+| \qquad (3)$$

This equation is represented on figure 2. As this figure shows, when the cavity impedance Z_L is close to 50Ω ($|S_{11}| \approx 0$), then the amplitude of the reflected wave V^- is very small, therefore difficult to detect.

Because we have implemented an Automatic Gain Control with $|V^+|$ on the input to the probe signal processing board, the amplitude variation of V^+ is not taken into account in the following study.

The amplitude variations of V_0 and $-jV_{-\pi/2}$ are represented on figures 3 and 4. These figures have been drawn according to the following equations:

$$\begin{aligned} V_0 &= V^+ + V^- = (1+S_{11})V^+ \\ -jV_{-\pi/2} &= V^+ - V^- = (1-S_{11})V^+ \end{aligned}$$

which amplitude variations in terms of $|S_{11}|$ and ψ are:

$$|V_0| = \sqrt{1+2|S_{11}|\cos\psi + |S_{11}|^2}|V^+| \qquad (4)$$

$$|-jV_{-\pi/2}| = \sqrt{1-2|S_{11}|\cos\psi + |S_{11}|^2}|V^+| \qquad (5)$$

When $|V^-|$ is very small, i.e. $|S_{11}| \approx 0$, $|V_0|$ and $|V_{-\pi/2}|$ are both in the same voltage amplitude range around $|V^+|$ and easy to detect. So one can use the impedance calculated from the voltages V_0 and $V_{-\pi/2}$ when $|S_{11}| \approx 0$ or the reflection coefficient calculated from the voltages V^- and V^+ when $|S_{11}| \approx 1$.

The minimum dynamic range required using the above technique can be found. From equation 3 the dynamic range of $|V^-|$ is $|S_{11}|$. The maximum amplitude variation of $|V_0|$ compare to $|V_{-\pi/2}|$ takes place for $\psi = 0^o$ or $\psi = \pm 180^o$. Equations 4 and 5 fix the maximum variation of $|Z|$ to $(1-|S_{11}|)/(1+|S_{11}|)$. Therefore the minimum dynamic range is given by the following equation:

$$|Z| = \frac{1-|S_{11}|}{1+|S_{11}|} = |S_{11}|$$

which gives as intercept point $|S_{11}| = \sqrt{2}-1$. The critical linearity of amplitude detection takes place in fact in a 7.65dB dynamic range.

4 CIRCUIT SCHEMATIC

V_0 and $V_{-\pi/2}$ are the two probe voltages, a quarter of the cavity wavelength apart on the transmission line between the RF transmitter and the cavity. To simplify signal processing in the following schematic, the cacity frequency can be downconverted before processing and detection.

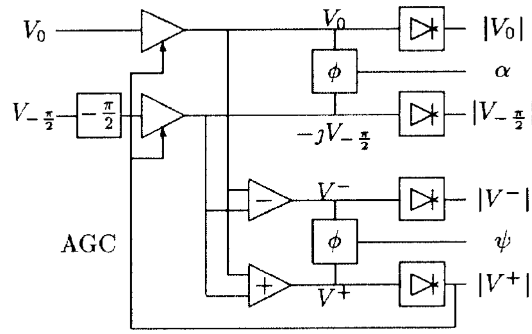

Figure 1: Probe signal processing board

The 90^o phase shifter was implemented using the technique described by T.E.Daughters in [2]. In order to provide a minimum difference of AGC gain amplification between the two channels (tracking error), a dual variable gain amplifier was used. The amplitude detectors consist of half wave rectifier ([3] p.120) followed by low-pass filters. These detectors allow a detection range wider than 20dB with a linearity better than 1% over 15dB. The phase detector used gave a monotonic response only between 0^o and 180^o. It is symmetrical about 0^o, therefore the sign of the angle is ambiguous over a 360^o range ([3] p.429). Considering that the impedance angle α is in the interval $-90^o, +90^o$, α can be found without ambiguity, and the ambiguity of the reflection coefficient angle ψ can be solved considering that $\text{sign}(\psi) = \text{sign}(\alpha)$.

The 6 voltages were loaded into a PC via a 12 bit ADC (Analog to Digital Converter).

5 SOFTWARE ALGORITHM

The algorithm starts by calculating the reflection coefficient modulus:

$$|S_{11}| = \frac{|V^-|}{|V^+|}$$

Depending on this value, one of the two following calculation methods are used:

- if $|S_{11}| > \sqrt{2}-1$:
  With the S_{11} angle ψ, the reflection coefficient can be displayed using equation 2.
- else ($|S_{11}| < \sqrt{2}-1$) :
  The ratio of $|V_0|$ by $|-jV_{-\pi/2}|$ gives $|Z|$ and with the impedance angle α the reflection coefficient can be displayed using equation 1.

This very basic algorithm has been implemented in a straightforward fashion on a IBM compatible PC using C.

The reference plane of the reflection parameter S_{11} in this study is defined by the placement of the probe V_0 on the transmission line. This is not a suitable reference plane to display the cavity impedance on the Smith chart. The cavity's short circuit can be established by de-tuning the cavity. The reflection coefficient displayed from the

V_0's reference plane can be anywhere on the $\Re(Z) = 0$ imaginary circle of the Smith chart. To set up the correct reference plane, the cavity is well de-tuned and the Smith chart's short circuit is set to coincide with this.

6 ERROR ANALYSIS

The accuracy of this method is mainly defined by the linearity of the amplitude and phase detectors. A good approximation of the error of the impedance display on the Smith chart in terms of % of its radius is given by the following formula: $\epsilon = \sqrt{1.1025A^2 + 0.74P^2}$ where A the amplitude error is expressed in %, and P the phase error expressed in degrees.

For example, a 1% error of amplitude detection linearity and 5 degrees from the phase detector gives a 4.4% maximum error on the radius of the Smith chart.

7 CONCLUSION

This method compared with the former method has the following advantages:

- it requires two probes placed on the transmission line instead of four.
- as it is possible to implement a gain compression loop (AGC) for the transmitter power, the amplitude detector dynamic range need only be 10dB.
- the two algorithms require the ratio of two voltages, therefore in contrast with the four probes method no calibration procedure is needed to normalize the variation of the transmitter drive levels.

The drawback of this method is the requirement of a computer. However this enables other facilities not normally found on a network analyser to be easly programmed, such as:

- maintenance of the data points of the last 5 seconds on the screen, continuously.
- freezing of the data on the screen if the electron beam is lost.
- logging data, e.g. movement during a ramp.
- high quality graphics hardcopy.

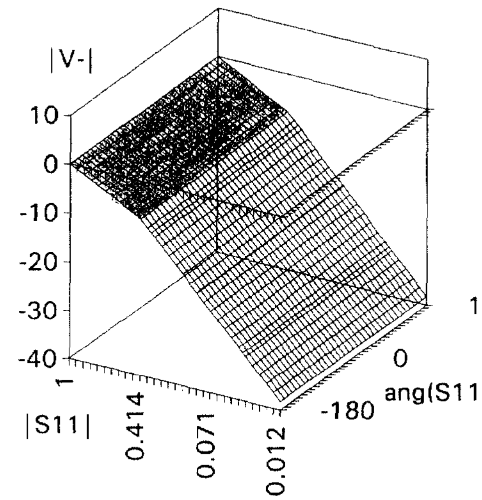

Figure 2: Modulus of V^-

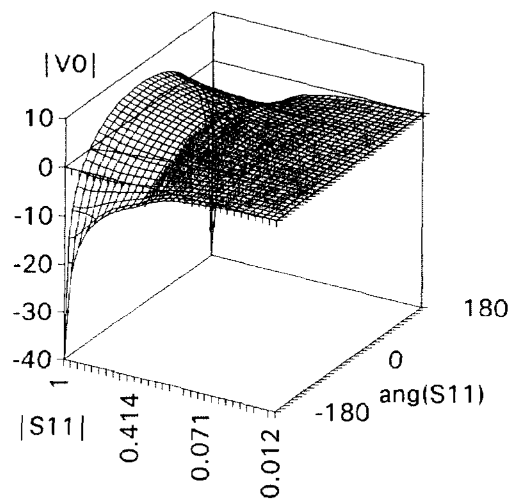

Figure 3: Modulus of V_0

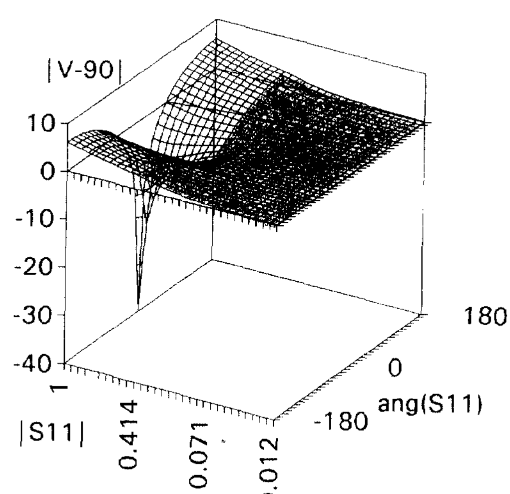

Figure 4: Modulus of $V_{-\pi/2}$

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[2] T.E. Daughters, *Lumped Elements Improve Phase Shifter Design*, MICROWAVES&RF, January 1992.

[3] Horowitz and Hill, *The Art of Electronics*, Cambridge University Press.