

# Analytical Calculation of a Coupler for the Linear Collider Accelerating Section.

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## Abstract

Calculation formula for approximate determination of the coupler inner cavity diameter and coupling slots dimensions are obtained by means of representation of the coupler as a resonator-prototype. Electromagnetic field in the vicinity of the slot which connects the coupler with the feeding waveguide is considered in one mode approximation. The results of calculation and measurements for the coupler with two feeding waveguides are presented. Such a coupler is supposed to be used for the DESY Linear Collider.

## 1 INTRODUCTION

In the process of constructing a linear accelerator structure the final determination of dimensions of the slot coupling the input rectangular guide with the coupler cavity is always made experimentally. To decrease the number of necessary hardware elements the preliminary evaluation of the slot dimensions is made either by means of analytical formula or on the basis of a three - dimensional code. For example the dimensions of a coupler for a linac accelerating section can be calculated using data of [1] or by means of analytical formulae based on Bethe theory [2] or on the basis of lumped parameter equivalent circuit analysis[3]. In case of the symmetrical double input coupler for the Linear Collider [4] MAFIA code is used to study the influence of main dimensions variation on the matching. In this paper calculation formulae are presented. They are obtained on the basis of the coupler representation as a resonator-prototype [5].

## 2 BASIC THEORY

The main dimensions of a coupler are its inner cell diameter  $2b_c$  and dimensions of slots coupling the cylindrical cell with feeding rectangular waveguides  $h_s$ . As for the height of the cell  $D_c$  its value is usually chosen equal to that of the disk loaded guide cell. The diameter of the input drift tube  $2a_1$  is chosen a priori. The schematic draft of a coupler with two feeding rectangular guides is shown in Fig.1. Such a construction is supposed to provide good field amplitude and phase symmetry near the beamline.

For evaluation of  $2b_c$  and  $h_s$  let's write down the expression for relative input impedance of the coupling slot which can be represented as an inductance connected in

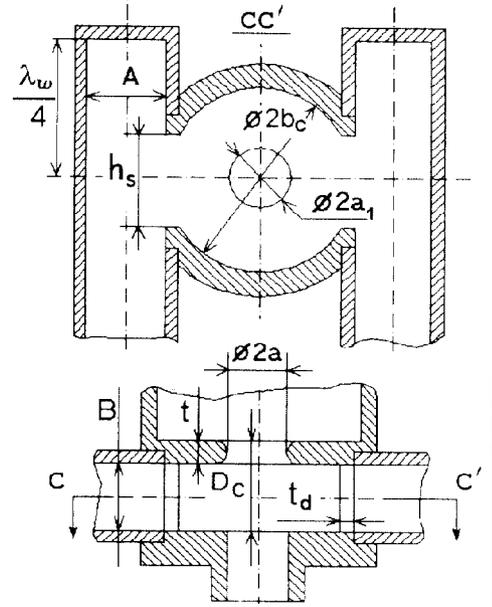


Figure 1: Coupler with two feeding rectangular guides

parallel to a transmission line [5]

$$\sqrt{Z_{in}} = \frac{Z_0 \xi_{dig} \xi_{rw} \pi V_m}{\lambda_0} \quad (1)$$

where  $\xi_{rw}$  and  $\xi_{dig} = H/\sqrt{p}$  are magnetic field strength parameters taken near the slot inside rectangular waveguide and disk loaded guide correspondingly.  $U_m$  is magnetic polarizability of the coupling slot. In case a slot with dimensions  $B \times h_s$  is made in the narrow wall of the waveguide it is given by

$$U_m = \frac{\lambda_w^2 B}{16\pi} \tan^2 \left[ \frac{\pi (h_s - t_d)}{\lambda_w} \right] \quad (2)$$

Using (1) and (2) we can obtain for the matched system ( $Z_{in} = 1$ )

$$h_s - t_d = \frac{\lambda_w}{\pi} \arctan \left( \frac{4}{\lambda_w} \sqrt{\frac{\lambda_0}{Z_0 B \xi_{dig} \xi_{rw}}} \right) \quad (3)$$

The parameter  $\xi_{rw}$  for the construction shown in Fig.1 (it's taken on the axial line of the coupling cell, the short-circuiting plunger in the rectangular waveguide being accounted for) can be written as

$$\xi_{rw} = \sqrt{\frac{\lambda\lambda_w}{A^3 B Z_0}} \quad (4)$$

The parameter  $\xi_{dig}$  can be roughly determined in terms of the electric field parameter  $\zeta_e = E\lambda_0/\sqrt{P} = \lambda_0\xi_e$  which is known for the disk loaded guide at operating mode.

For estimations one can use the relation for  $E_{01}^o$  mode at  $r = b_c$ , i.e.  $\xi_m = \frac{J_1(\nu_{01})}{Z_0} \frac{\zeta_e}{\lambda_0}$ . Keeping in mind there is standing wave regime in the coupler and by using measured field distribution for the cross-section under consideration we can obtain

$$\xi_{dig} = \frac{2J_1(\nu_{01})}{Z_0} \frac{\zeta_e}{\lambda_0} \cos \frac{\pi z}{2D} \quad (5)$$

The coordinate  $z$  is taken from the butt-end wall of the disk loaded guide. Taking into account that the power is fed through two coupling slots we can write down the mean value of  $\xi_{dig}$  over the slot length  $B$

$$\xi_{dig} = \frac{2J_1(\nu_{01})}{Z_0} \frac{\zeta_e}{\lambda} \frac{\sin \frac{\pi B}{2D}}{\frac{\pi B}{2D}} \sqrt{2} \quad (6)$$

So, the coupling slot width can be determined by formula (3) combined with (4) and (6). Because the field component  $H_\varphi(z)$  at  $r = b_c$  is known only approximately we have to set the dimension  $h_s$  slightly (by 10 – 15%) lower than obtained by calculations and afterwards to correct it experimentally.

While determining the dimension  $2b_c$  one should keep in mind that the coupler cell diameter differs from that of the disk loaded guide with the same dimensions  $2a, D, t$ . It can be explained by geometrical modifications. Thus there is only one disk with a hole. Second disk is replaced by the butt-end wall with cut off tube having diameter  $2a_1$ . In addition two slots are cut on the coupler cylindrical surface. To evaluate the diameter  $2b_c$  change due to these reasons we can use the expression

$$\Delta 2b_c = \frac{\Delta f_c}{\partial f / \partial 2b_c} \quad (7)$$

where

$$\Delta f_c = \Delta f_b + \Delta f_{c.o} + 2 \Delta f_s \quad (8)$$

Here  $\Delta f_b$  is the coupler frequency shift due to replacement of one disk by the butt-end wall,  $\Delta f_{c.o}$  is the frequency shift due to cut-off tube influence,  $2 \Delta f_s$  is the frequency shift due to coupling slots influence.  $\Delta f_b$  could be determined from comparison  $2b$  value obtained from the dispersion equation for the disk loaded guide with  $2b$  value obtained for similar cell but with no irises in disks, i.e. by using the formula which defines the frequency of  $E_{010}^o$  mode for a cylindrical resonator:

$$f_{010} = \frac{c\nu_{01}}{\pi 2b_c}$$

In our case the coupler has only one short circuited disk. Other frequency shifts can be evaluated according to formula, obtained in one mode approximation

$$\Delta f_{c.o} = f_{2/3} 0,36 \left(\frac{a_1}{b_c}\right)^2 \frac{a_1}{B} \quad (9)$$

$$2 \Delta f_s = -\frac{f_{2/3}}{4\nu_{01}^2} \left(\frac{\lambda_w}{\lambda_0}\right)^2 \tan^2 \left[\frac{\pi(h_s - t_d)}{\lambda_w}\right] \quad (10)$$

$$\frac{\partial f}{\partial (2b_c)} = \frac{f\pi}{\lambda\nu_{01}} \quad (11)$$

### 3 CALCULATION RESULTS AND COMPARISON WITH EXPERIMENTAL RESULTS

As an example the results of calculations for DESY Linear Collider input and output couplers are presented in Table 1. The calculations were made with

Table 1:

Parameters	Input Coupler	Output Coupler
$2a, mm$	31,02	21,77
$a/\lambda_{2/3}$	0,1551	0,10885
$\xi_{rw} \frac{1}{\Omega m^{1/2} m}$	1,88	1,88
$\xi_{dig} \frac{1}{\Omega m^{1/2} m}$	6,50	12,5
$h_s - t_d, mm$	29,9	23,2
$2b'_c, mm$	81,23	78,39
$f_{E_{010}^o}, MHz$	2829,3	2927,6
$2 \Delta f_b, MHz$	-173	-70
$\Delta f_b, MHz$	-86,5	-35
$2 \Delta f_s, MHz$	-161	-83
$2 \Delta f_{c.o}, MHz$	86	93
$\Delta f_c, MHz$	-161,5	-25
$\Delta 2b_c, mm$	-4,1	-0,65
$2b_c, mm$	77,1	77,75

the use of formula given above. The coupler parameters study was carried out for input and output couplers. For this purpose disk loaded guide sections consisting of 11 cells with the ratio  $a/\lambda = 0,1551$  and  $a/\lambda = 0,10885$ , the structure period equal to 33,33mm and cell shape as in [6] were manufactured. To obtain the traveling wave

regime in the disk loaded guide the movable absorbing load technique was used.

For the case when the short circuited plungers were placed at the distance  $z = \frac{\lambda_w}{4}$  from the centers of the coupling slots the matched regime was obtained at  $2b_c = 76\text{mm}$ ,  $h_s - t_d = 29,2\text{mm}$ ,  $z=24\text{mm}$  for the input coupler and at  $2b_c = 77,25\text{mm}$ ,  $h_s - t_d = 22,5\text{mm}$ ,  $z=26\text{mm}$  for the output coupler [7].

## 4 CONCLUSIONS

Apparently the discrepancy between theoretical and experimental values can be explained by the approximate field representation in the vicinity of the coupling slot at the both sides of it, i.e. in the rectangular guide and in the coupler cell. Nevertheless the technique described above can be used for preliminary approximate calculations.

## 5 ACKNOWLEDGMENTS

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