Error Sensitivity of a Double Side Coupled Muffin-Tin

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Abstract

The knowledge of the sensitivity is of interest for a proposed side coupled muffin-tin structure as compared to a single periodic structure. The aim is to make a confluent structure with a high error tolerance which does not need tuning. In order to estimate this tolerance the structure is replaced by a chain of lumped element resonators with losses. The currents in the equivalent circuit can easily be found as the solution of an inhomogeneous linear equation. The shunt impedance is deduced from the currents and its sensitivity is found by variation of the parameters of the system.

1 INTRODUCTION

The proposed Muffin-Tins [1, 2] would be hard to tune due to their physical size of typically 1 mm. Therefore it is necessary to have a structure that does not need tuning.

This paper reports sensitivity studies for accelerating structures with one, two or three different resonators per period length. The geometries are found with MAFIA, but their sensitivity to errors cannot be found using MAFIA because the expected errors are so small that the discretization of the geometrical errors would require a not realizable fine mesh. Also is the effect of the errors depending on the number N of period lengths considered. This also prohibits the use of MAFIA.

On the computer used, a MAFIA run takes approximately 5 minutes for one period length. The lumped circuit model takes for 100 period lengths approximately 4/1000 second. Therefore it is possible to vary the frequency of the resonators (the geometry in MAFIA) several thousand times to get a solid base for statistical statements in only a few hours.

The approach used here is calculating the currents in a linear passive network. It is not used that these networks are almost periodic. In [3, 5, 6] the currents were calculated using a pertubation method starting from the currents of the loss free, perfect periodic structure, [3] considers also a double periodic chain. This approach was tried for the single periodic chain by the author. For small errors the results are the same as with the direct solution of the system, but this calculation is much more computer time intensive than the direct solution. (Actually the number of arithmetic operations required for the pertubation approach scales as N^2, as compared to the $\sim N$ operations of the direct solution).

In [4] the direct approach was used also. They calculate the currents in a single periodic chain as it is presented here.

None of the above cited references calculate many different perturbations of a structure. This may be due to the non availability of high speed computers at the time they were written.

2 LINEAR SYSTEMS

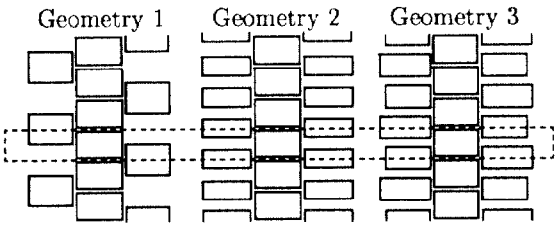

Figure 1: Top view of the double-sided muffin-tin structures as presented in [2]

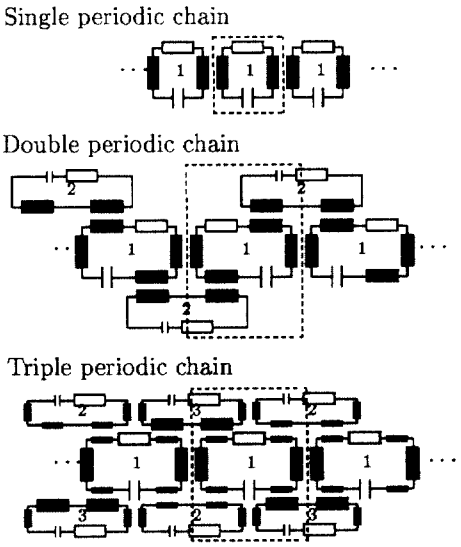

Figure 2: The coupled circuits, representing chains of coupled resonators

The geometries (Fig. 1) of the proposed Muffin-Tins are such that one two or three different resonator geometries are present per period length. These resonators are coupled with up to six neighbors. The lumped circuit model

has therefore to take into account coupling between almost every mesh. The coupling between a mesh of type i and one of type j is denoted k_{ij}. In the double periodic chain only k_{22} vanishes, in the triple periodic chain only $k_{22} = k_{33} = 0$. Fig. 2 shows the equivalent circuits of the three types of investigated structures, coupling arrows are omitted. The dashed boxes include the model of one period length. The first and last of the cells are special: In the mesh type 1 of the first cell is the driving voltage. The meshes type 2 and 3 of the first and last cells also have special termination impedances to simulate the end cells. Fig. 3 shows a triple periodic chain with four cells and terminations. The termination impedances in meshes of type 2 and 3 are calculated iteratively from the currents in the travelling wave case. The travelling wave is calculated with a termination impedance in the mesh type 1 of the last cell which equals the impedance of the network as it is seen by the voltage source.

$$Z_{\text{term},2} = \frac{i_3}{i_2} \mathrm{j}\omega M_{23} \; ; \; Z_{\text{term},3} = \frac{i_2}{i_3} \mathrm{j}\omega M_{23}$$

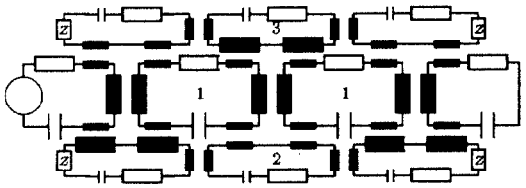

Figure 3: Triple periodic chain with 4 cells, termination impedances and voltage source

Kirchhoff's mesh equations for the N, $2N$, or $3N$ meshes of these networks lead to linear systems with matrix bandwidths of 3, 7 or 11. Due to lack of space the mesh equations are not shown here.

3 SENSITIVITY STUDY

The sensitivity studies were performed by varying the parameter resonance frequency of the cells with a Gaussian distributed random variable, the parameters of the lumped circuit model are deduced from these and plugged into the linear system, this is solved and the shunt impedance is deduced from it's solution.

The shunt impedance is almost totally insensitive to errors in the coupling, so only frequency errors are considered.

To get comparable results for the different structures operating in different modes, all structures are chosen so that the total structure length $L = N \times l = 7$cm is for all structures the same. For an operating frequency of 120 GHz this results in $N = 112$ for the $\pi/2$ structure, in $N = 84$ for the $2\pi/3$ structure and $N = 56$ for the π structures.

The confluent structures are designed to operate in standing wave. Since the sensitivity depends on the operation in standing wave or travelling wave, all structures are simulated for standing wave operation. The frequency errors in the cells have the effect that the frequency of the chosen operating mode changes slightly. The shunt impedances are calculated at this resulting resonance frequency.

Every point in the figs. 4, 5 is deduced from 1000 different random error sets. The figures show the mean and standard deviation of the resulting shunt impedances. The crossing lines indicate the allowable frequency errors for a decrease in shunt impedance down to 90% of the ideal shunt impedance. All figures have as ordinate the logarithm of the standard deviation of the relative frequency errors.

3.1 *Sensitivity of single periodic structures*

The reason for this investigation is to decide whether the confluent structures of [2] are less sensitive to errors than the single periodic structure of [1]. Therefore the sensitivity of such structures has to be evaluated first.

Three types of single periodic structures were calculated. All structures are designed to have their operating mode at 120 GHz. The parameters are listed in table 1. The results are shown in Fig. 4. The tolerable frequency errors are $\sigma f/120\text{GHz} = 10^{-2.75} = 1.8 \times 10^{-3}$ for the $\pi/2$ structure, $10^{-2.8} = 1.6 \times 10^{-3}$ for the $2\pi/3$ structure and $10^{-3.35} = 0.45 \times 10^{-3}$ for the $162\pi/180$ structure.

3.2 *Sensitivity of confluent structures*

Three confluent structures were presented in [2], the parameters of these structures were found by changing the parameters of the lumped circuit until the same Brillouin-diagram appeared. These values are shown in table 2.

The sensitivity of the double and triple periodice chains simulating the geometries 1-3 of [2] operating at the confluence point at $\beta l = \pi$ and $f = 120$ GHz is shown in Fig. 5. The tolerable frequency errors are $\sigma f/120\text{GHz} = 10^{-2.7} = 2 \times 10^{-3}$ for geometry 1, $10^{-2.95} = 1.1 \times 10^{-3}$ for geometry 2 and $10^{-2.85} = 1.4 \times 10^{-3}$ for geometry 3.

4 CONCLUSIONS

This sensitivity study calculated the effects of frequency errors in the resonators, it was not considered that geometrical errors have different effects on the resonance frequency depending on the cell where the error occurs and what side of cavity is effected.

But within this restriction, the conclusion is, the confluent structures geom. 1 to 3 of [2] are not less sensitive as the $2\pi/3$ structure of [1].

As was already known, in a single periodic chain the π-mode is the most sensitive, but what is new is that the $2\pi/3$-mode is almost as stable as the $\pi/2$-mode.

5 ACKNOWLEDGEMENT

The author would like to thank the APS project at Argonne National Laboratory, especially the RF group for supporting this work.

6 REFERENCES

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Table 1: Parameters for the single periodic chains

	$\pi/2$	$2\pi/3$	$162\pi/180$
N	112	84	60
f_1 [GHz]	120	118.63	118.28
k_{11} [%]	-5.5	-4.5	-3.0
Q	1670	2150	2860
r/Q [kΩ/m]	150	141	200
$\frac{\sigma f}{120GHz}\Big\vert_{\frac{<r_0>}{r_0(\sigma f=0)}=0.9}$ [%]	0.18	0.16	0.045

Table 2: Parameters for the confluent chains

	double periodic Geom. 1	triple periodic Geom. 2	triple periodic Geom. 3
N	56	56	56
f_1 [GHz]	118.794	117.880	117.881
f_2 [GHz]	120	118.793	119.919
f_3 [GHz]		118.793	115.5
k_{11} [%]	-2.0	-3.5	-3.5
k_{12} [%]	7.6	5.4	5.0
k_{13} [%]		5.4	5.0
k_{23} [%]		-2.0	-1.0
Q_1	2800	2590	2550
Q_2	2800	2730	2450
Q_3		2730	2550
r_1/Q_1[kΩ/m]	92	94	94
r_2/Q_2[kΩ/m]	60	45	68
r_3/Q_3[kΩ/m]		45	60
$\frac{\sigma f}{120GHz}\Big\vert_{\frac{<r_0>}{r_0(\sigma f=0)}=0.9}$ [%]	0.2	0.11	0.14

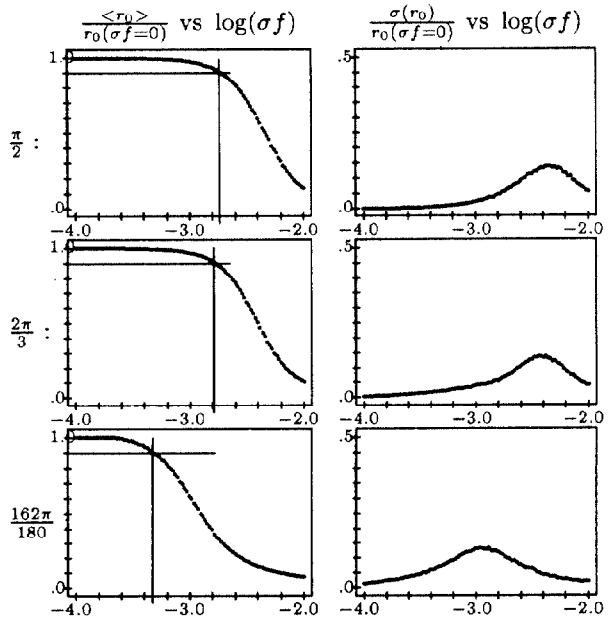

Figure 4: Mean and standarddeviation of the shunt impedances of single periodic chains in $\pi/2, 2\pi/3, 162\pi/180$-mode vs. standard deviation of the resonance frequencies

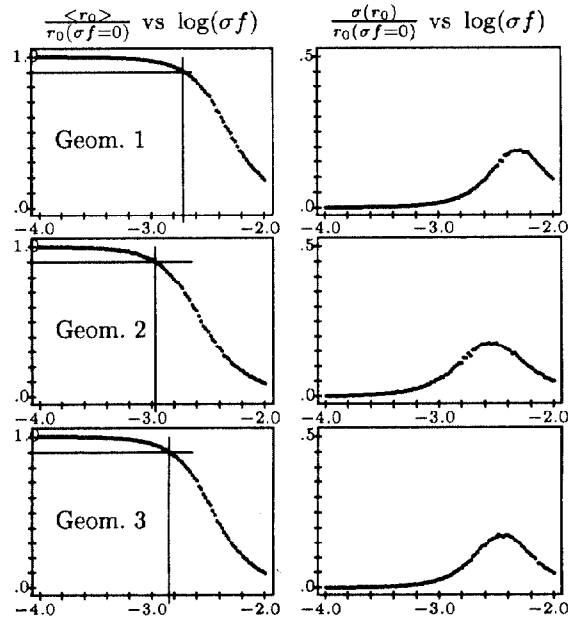

Figure 5: Double and triple periodic chains at confluent π-mode: $< r_0 >$ & $\sigma(r_0)$ vs $\sigma\delta$