

# Computer-Aided Studies of the Three-Cavity System for Heavily Beam-Loaded Accelerators

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## Abstract

In extremely high-luminosity storage rings, a longitudinal coupled-bunch instability will arise even from the accelerating mode of cavities. In order to solve this problem, a three-cavity system, comprising an accelerating cavity and an energy-storage cavity with a coupling cavity in between, is a possible candidate, since it drastically reduces the detuning frequency owing to the large stored energy.

In this study the three-cavity system was investigated using computer codes. The results are compared with an analysis based on a coupled-resonator model. The geometrical shape of the system was optimized regarding the mode separation in the storage cavity and the effect of the coupling hole. The optimized design showed that even the fastest possible growth rate is comparable to the radiation damping rate, if the Q values of dangerous parasitic modes are reduced to less than 100, for example, by installing a damper in the coupling cavity. The feasibility of the three-cavity system for heavily beam-loaded accelerators was realistically indicated.

## 1. INTRODUCTION

In high-luminosity electron and/or positron storage rings, such as B-factories, the stored current should be extremely high. In order to compensate the reactive component of heavy beam loading, a large detuning is induced, which can amount to several times as much as the revolution frequency. It excites several modes of the coupled-bunch instability arising from the accelerating mode, the growth rates of which are too fast to cure with a bunch-by-bunch feedback system.

In order to avoid this instability, the detuning frequency should be decreased. If we increase the stored energy, namely, if we decrease  $R/Q$  keeping high shunt impedance, the detuning frequency is decreased. One may consider that the use of superconducting cavities is a possible solution in this direction. In some cases, such as the positron ring (LER) of KEKB, a collider for B-physics under construction at KEK, however, the detuning frequency exceeds a revolution frequency, even if the superconducting cavities are used.

As pointed out by Shintake,<sup>1</sup> this problem can be solved by utilizing a high-Q large-volume energy-storage cavity coupled to an accelerating cavity. In order to put this scheme into practical use, Yamazaki and Kageyama proposed a three-cavity system,<sup>2</sup> in which a coupling cavity is used between an accelerating cavity and an energy-storage cavity to couple them, operated in a  $\pi/2$  mode in the sense of a bi-periodic structure. In order to suppress the instabilities arising from two parasitic modes (0 and  $\pi$  modes), a damper is installed in the coupling cavity where no field is excited in the  $\pi/2$  mode.

Furthermore, the growth of each coupled-bunch mode is cancelled by its damping, since the 0 and  $\pi$  mode frequencies are situated symmetrically around the  $\pi/2$  mode frequency.

The discussion regarding the three-cavity system was based on the coupled-resonator model. It should be noted here that many other parasitic modes exist, the frequencies of which are close to the operating mode, since the storage cavity will be operated in a higher-order mode, such as, for example, the TE015 mode. These nearby modes, which were not taken into account in the coupled-resonator model analysis, may cause serious problems. For example, mode mixing with a low-Q parasitic mode due to a coupling hole reduces the Q value of the operating mode. Another potential problem is that cancellation of the 0 and  $\pi$  modes may not actually occur.

In this paper we discuss the characteristics of the three-cavity system utilizing computer codes. We demonstrate that the optimized design of the system satisfies the requirements for the operating mode and that we can manage the presence of many nearby modes in the storage cavity.

## 2. STORAGE CAVITY

As a storage cavity we chose a pill-box cavity operating in the TE015 mode, the same mode as the energy-storage cavity for SLED,<sup>3</sup> for following reasons. For a frequency of 508.6 MHz and the conductivity of copper, a Q value of more than  $2.2 \times 10^5$  can be obtained in the case of the TE01p mode with  $p \geq 5$ , which is sufficiently high for our purpose. In order to maximize the intra-cavity coupling, it is preferable to locate a coupling hole at the place where the surface magnetic field is high. Then, the TE01p mode with an odd p is more appropriate than that with an even p, since we can locate the coupling hole symmetrically with respect to the mid-plane between the two end plates. It is advantageous to keep this symmetry, regarding mode separation, as discussed below.

There exist many parasitic modes in the storage cavity, the frequencies of which are close to that of the TE015 mode. If a low-Q parasitic mode is mixed with the operating mode, the Q value of the operating mode will be reduced. We first attempted to sufficiently remove the degeneracy of the TE015 and TM115 modes, investigating the effect of a groove at the end plates, which is used in SLED.<sup>3</sup> Figure 1 shows a schematic view of the storage cavity with a coupling hole and grooves. We chose a groove length of 80 mm, which gives rise to a sufficient separation of 16 MHz.

Next, we consider the mode mixing with a neighboring mode. A perturbation theory shows that the  $TE_{mnp}$  ( $p$ =odd) and  $TM_{mnp}$  ( $m > 0$ ,  $p$ =odd) modes can easily be mixed with the operating mode through the perturbing effect of a coupling hole. The ratio of the radius to length was chosen so as to

maximize the separation between the TE<sub>015</sub> and any of these modes. Other modes are less dangerous, since the perturbing effect of half of the hole is cancelled by another half owing to the symmetry with respect to the midplane.

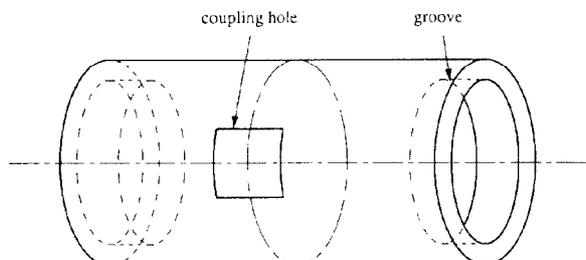


Figure 1 Schematic view of the storage cavity.

### 3. DESIGN OF THE THREE-CAVITY SYSTEM

In order to optimize the shape of the three-cavity system, we studied its characteristics with MAFIA.<sup>4</sup> The accelerating, coupling and storage cavities are referred to as the a-, c- and s-cavities, respectively, from now on. We simply assumed pill-box cavities operating in the TM<sub>010</sub> mode for both the a- and c- cavities, since it does not affect our analysis significantly which type of damped structure will be used as an a- cavity. Figure 2 shows a schematic view of the system. According to Yamazaki and Kageyama,<sup>2</sup> the requirements for this system can be summarized by three points: (1) The field pattern of the  $\pi/2$  mode should be such that no field is excited in the c- cavity. At the same time, (2) the ratio of the stored energy in the s- to the a-cavity ( $U_s/U_a$ ) should be about 10 in order to decrease the detuning frequency by an order of magnitude. (3) The 0 and  $\pi$  modes should be situated symmetrically, at least approximately, around the operating mode frequency.

#### 3.1 Tuning of Cavities

The resonant frequency of each cavity was adjusted in such a way that we tune one cavity by changing its radius while detuning neighboring cavities by setting a boundary condition in the mid-plane including the cavity axis so as not to excite the resonant mode. When the frequency of each cavity was thus adjusted to an identical value,  $U_c$  was very small compared to  $U_a$  or  $U_s$  in the  $\pi/2$  mode. This is in agreement with the requirement (1) from the coupled-resonator model, indicating that this method is appropriate for the tuning.

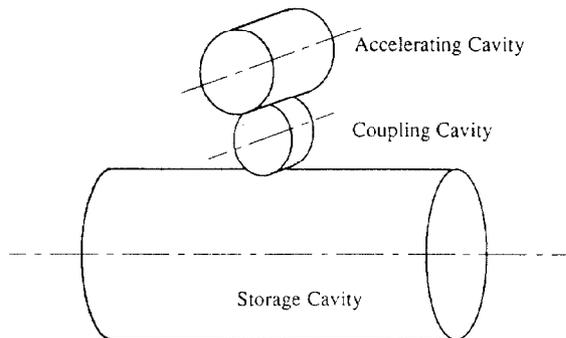


Figure 2. Schematic view of the three-cavity system.

#### 3.2 Coupling Factors and Stored Energy

We calculated the coupling factor ( $k_a$ ) between the a- and c-cavities and ( $k_s$ ) between the s- and c-cavities. After each cavity was tuned to minimize  $U_c$ , the ratio of  $U_s$  to  $U_a$  was compared with that calculated with the coupled-resonator model. The ratio calculated with MAFIA was different from that of the coupled-resonator model by a factor of two or three. Further investigation convinced us that the quantitative disagreement is the effect of neighboring modes in the s- cavity (detailed discussion is given in Ref. 5). Thus, we continued the optimization by adjusting the sizes of coupling holes so as to practically obtain the required ratio of  $U_s$  to  $U_a$ .

#### 3.3 The 0 and $\pi$ Modes

The coupled-resonator model predicted that the  $\pi/2$  mode frequency is situated approximately at the center of the 0 and  $\pi$  mode frequencies. MAFIA calculations showed, however, that this is not necessarily the case. A preliminary calculation showed that  $f(\pi)-f(\pi/2)=4.7\text{MHz}$  and  $f(\pi/2)-f(0)=1.8\text{MHz}$ . The reason of the frequency asymmetry is considered to be again due to the effect of neighboring modes in the s-cavity. We finely adjusted the ratio of the radius to length of the s-cavity so that the frequencies of two nearest-neighboring modes with high R/Q are situated symmetrically around the  $\pi/2$  mode.

## 4. DISCUSSIONS

Properties of the optimized design is listed in Table 1. The field pattern of the  $\pi/2$  mode (Figure 3) seems to be a nearly pure TE<sub>015</sub> mode in the s-cavity, a nearly pure TM<sub>010</sub> mode in the a-cavity and almost no field excited in the c-cavity. Most of the energy is stored in the s-cavity ( $U_s$  is about ten times  $U_a$ ) in the form of the high-Q TE<sub>015</sub> mode. This energy distribution gives the required properties of the system: the value of R/Q is reduced by an order of magnitude smaller than that of conventional normal-conducting damped cavities without a storage cavity, while the total Q value is increased much higher. As a result, the detuning frequency is reduced by an order of magnitude. In the case of LER of KEKB, the fastest growth time due to the accelerating mode is 47 ms, which is longer than the radiation damping time.

Next, we demonstrate that parasitic modes do not cause any serious coupled-bunch instabilities. There are many parasitic modes, the frequencies of which are close to the operating mode, as shown in Table 1. Among them, the TE<sub>mnp</sub> ( $p=\text{odd}$ ) or the TM<sub>mnp</sub> ( $m>0, p=\text{odd}$ ) modes can couple to the a-cavity through the c-cavity. In these modes, however, a strong field is excited in the c-cavity, as shown in Figure 4. Therefore, these modes can be damped by a damper installed in the c-cavity, while the operating  $\pi/2$  mode is not affected, since almost no field is excited in the c-cavity in this mode. The growth rate in the case of KEKB was calculated by taking all of these modes into account. As shown in Figure 5, if the Q values are damped down to 50 with the damper, the fastest growth time is about 40 ms, which is about the same as the radiation damping time. Even if the damper can damp them to only 100, the growth time is not very disastrous and it can probably be cured by a feedback system.

## 5. CONCLUSIONS

We studied the three-cavity system for heavily beam-loaded accelerators by using computer codes. The dimensions of the system were optimized in order to meet the requirements for both the operating and parasitic modes. The operating mode has the desired field pattern for our purpose. The growth rates of coupled-bunch instabilities due to the operating mode and neighboring parasitic modes are comparable to the radiation damping rate. It was thus more realistically indicated than the coupled-resonator model analysis that this system is feasible for heavily beam-loaded accelerators.

Consequently, the three-cavity system is one of the most promising candidates for the KEK B-factory (KEKB). The accelerating cavity should have a damped structure in order to suppress the coupled-bunch instability arising from the higher-order modes.<sup>6,7</sup>

Table 1 Properties of the optimized design

operating mode (TE015, $\pi/2$ )		
frequency	(MHz)	508
$Q_{total}$		$1.8 \times 10^5$
$R/Q_{total}$	( $\Omega$ )	13.9
$k_a, k_s$		5.6 %, 1.0 %
parasitic modes; TEM <sub>np</sub> (p=odd), TM <sub>mnp</sub> (m>0, p=odd)		
frequency(MHz)	mode in s-cav.	$R/Q_{total}$ ( $\Omega$ )
474.0	(TE411)	0.12
479.5	(TE121)	0.04
491.2	(TM115)	0.02
499.1	(TM213)	0.75
501.7	(TE413/TM213)	105. (0 mode)
512.8	(TE413)	71.8 ( $\pi$ mode)
518.3	(TE315)	1.04
521.0	(TE413)	16.4
parasitic modes; other modes		
472.3 (TM212)	473.4 (TE116)	475.0 (TE314)
484.3 (TM016)	492.0 (TE412)	497.2 (TE122)
501.0 (TM022)	516.4/537.3 (TE216/TM214)	
522.4 (TM023)	537.7 (TM116)	

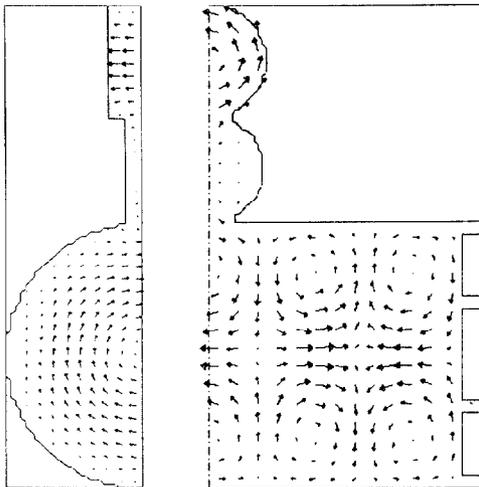


Figure 3. Field pattern of the  $\pi/2$  mode calculated by the MAFIA code; (left) electric field, and (right) magnetic field.

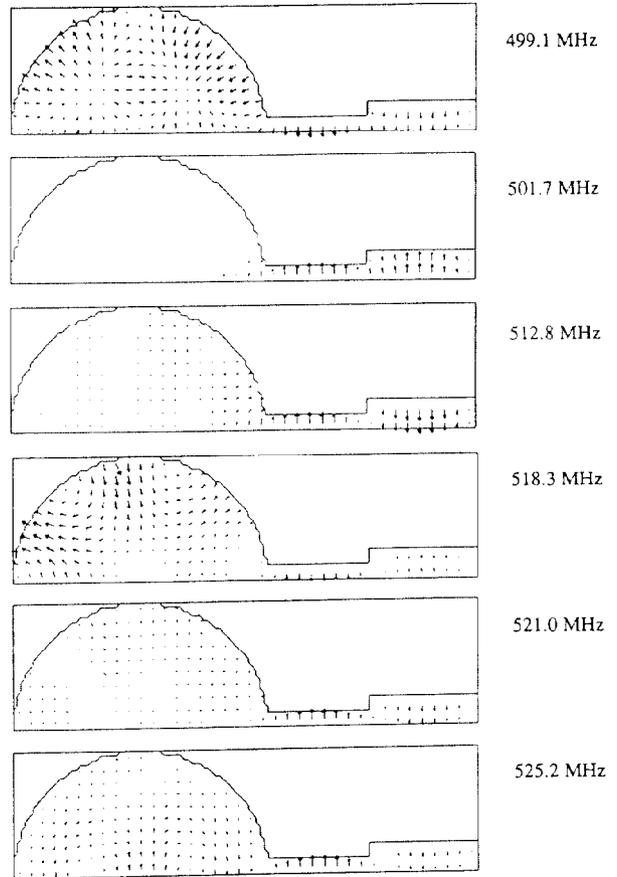


Figure 4. Field patterns of the parasitic modes, the frequencies of which are close the operating modes.

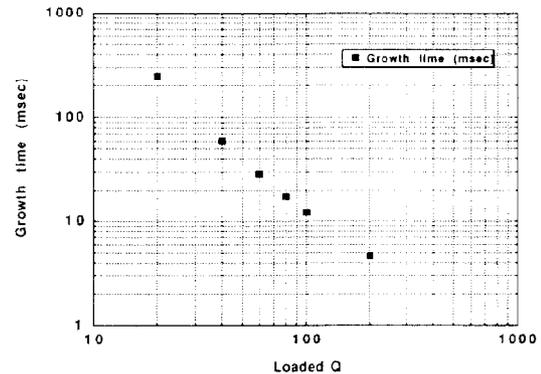


Figure 5. Fastest growth time of the coupled-bunch instability caused by parasitic modes as a function of the damped  $Q$  value.

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