# Investigation of planar mm-wave RF structures for nonrelativistic electron acceleration, focussing and bunching

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*Abstract*

Four different RF structures for acceleration, focussing and bunching of nonrelativistic electrons at 120 GHz have been considered. The structures are planar and adapted to fabrication by lithography. They were calculated numerically with MAFIA and subsequently approximated by analytical expressions. The equations of motion are derived and a relation between the focussing forces in the three dimensions is given. Finally, a method for acceleration, focussing and bunching of a continuous beam is proposed .

## 1. INTRODUCTION

Radio-Frequency-Quadrupoles (RFQ) have been developed for accelerating and bunching, while simultaneously focussing, low-energy heavy particles such as protons or heavy ions. They are periodic structures which operate at low frequencies, typically between several tens to several hundreds of MHz. Their period length is much shorter than the RF wavelength and determined by the particle's velocity ß. In addition, owing to the symmetry of the structure (vanes), the magnetic field around the axis is negligible. Under these conditions it suffices to replace the wave equations for the electric field by the Laplace equation and to assume quasi-static electric fields in the rest frame of the synchronous wave.

For electrons and in case of very high RF frequencies the situation is completely different. At first, electrons become rapidly relativistic and, thus, the period length of the structure is no longer short as compared to the RF-wavelength. Secondly, the small RF-wavelengths require structure geometries which are appropriate to micro-mechanic fabricational techniques as for instance proposed in [1]. Such structures have to be planar. Under these circumstances we can not neglect the magnetic field and have to solve the full wave equation. This was done numerically with MAFIA. On the other hand, since the structure is made of many periods, we can restrict the calculation to the synchronous wave only. Assuming a synchronous wave with negligible vertical magnetic field, as resulting from MAFIA calculations, we derive analytically the equations of motion for the electrons and a relation between the focussing forces in the three dimensions. Adding an external axial magnetic field it is possible to focus the beam transversely and at the same time provide longitudinal bunching. Finally, typical bunch parameters are given in a numerical example.

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## 2. ANALYTICAL DERIVATION OF THE EQUATIONS OF MOTION

In the planar RF structures we have investigated, we can assume $B_y = 0$. The solution $E_z$ of the wave equation consists of an infinite series of space harmonics of the type

$$E_z = E_0 \cos k_x x \cdot \cos k_y y \cdot e^{j(\omega t - k_z z)} \tag{1}$$

with

$$k_z^2 - k^2 = k^2\left(1/\beta^2 - 1\right) = -k_x^2 - k_y^2. \tag{2}$$

for the synchronous wave.

The lefthand side of equ. (2) is always positive and therefore either $k_x^2$ or $k_y^2$ or both have to be negative. Thus, three types of synchronous space harmonics are possible

$$\begin{aligned} E_z &= E_0 \cos k_x x \cdot \cosh k_y y \cdot e^{j(\omega t - k_z z)} \quad (I) \\ E_z &= E_0 \cosh k_x x \cdot \cos k_y y \cdot e^{j(\omega t - k_z z)} \quad (II) \\ E_z &= E_0 \cosh k_x x \cdot \cosh k_y y \cdot e^{j(\omega t - k_z z)} \quad (III) \end{aligned} \tag{3}$$

The other field components follow from Maxwell's equations considering $B_y = 0$.

Then, near the axis, $k_x x$, $k_y y \ll 1$, the Lorentz forces acting on an electron which travels with ßc parallel to the z-axis are

$$\begin{aligned} F_x &= eE_0 \frac{\beta}{k} k_x^2 x \sin\Phi \begin{Bmatrix} +1 \\ -1 \\ -1 \end{Bmatrix}, \quad F_z = -eE_o \cos\Phi \\ F_y &= -eE_0 \frac{\beta}{k} y \sin\Phi \begin{Bmatrix} k_z^2 + k_x^2 - k^2 \\ k_z^2 - k_x^2 - k^2 \\ k_z^2 - k_x^2 - k^2 \end{Bmatrix}, \quad \Phi = k_z z - \omega t. \end{aligned} \tag{4}$$

Let us now define the phase $\Phi_0$ as the one of a synchronous particle with the velocity $\beta_0 c$. For a non-synchronous particle we obtain then

$$\dot{\Phi} = k_z \beta c - \omega = \omega\left(\beta / \beta_0 - 1\right). \tag{5}$$

Assuming ß close to $\beta_0$ we can approximate the equation of longitudinal motion of a non-synchronous particle

$$\begin{aligned} mc\frac{d}{dt}(\gamma\beta) &\approx mc\frac{d}{dt}\left(\gamma_0\beta_0\right) + mc\frac{1}{\omega}\frac{d}{dt}\left(\gamma_0^3\beta_0\dot{\Phi}\right) \\ &= -eE_0\cos\Phi. \end{aligned} \tag{6}$$

The first term at the left side equals the force $-eE_0\cos\Phi_0$ acting on the synchronous particle. The right side can be developed for a small phase difference $\delta\Phi = \Phi - \Phi_0 \ll 1$ between synchronous and non-synchronous particles and we obtain from (6)

$$\delta\ddot{\Phi} = \frac{ek}{m\beta_0\gamma_0^3} E_0 \sin\Phi_0 \,\delta\Phi. \tag{7}$$

if $\beta_0$ varies adiabatically in time. The equations for the transverse motion follow from (4), again assuming an adiabatical change of $\beta_0$,

$$\ddot{x} = \frac{e}{m\gamma_0} E_0 \frac{\beta_0}{k} k_x^2 \begin{Bmatrix} 1 \\ -1 \\ -1 \end{Bmatrix} \sin\Phi_0 \cdot x$$

$$\ddot{y} = -\frac{e}{m\gamma_0} E_0 \frac{\beta_0}{k} \begin{Bmatrix} k_z^2 + k_x^2 - k^2 \\ k_z^2 - k_x^2 - k^2 \\ k_z^2 - k_x^2 - k^2 \end{Bmatrix} \sin\Phi_0 \cdot y. \tag{8}$$

As can be seen from (8), the sums of the coefficients of x and y are equal for all three cases I to III, namely

$$-\frac{e}{m\gamma_0}\frac{\beta_0}{k} E_0\left(k_z^2 - k^2\right)\sin\Phi_0 = -\frac{ek}{m\beta_0\gamma_0^3} E_0 \sin\Phi_0, \tag{9}$$

which is the negative coefficient of the longitudinal equation (7). Since the coefficients are proportional to the optical strengths of the RF-structures, we conclude that the sum of the transverse and longitudinal optical strengths vanishes for all structures. This result is also valid if $B_y \neq 0$ as can be shown easily.

As a consequence it is impossible to form longitudinal buckets and at the same time have focussing and defocussing of equal strength in the transverse planes. On the other hand, if we allow for debunching in longitudinal direction we can have focussing in both transverse planes (cases I, II) which may even be of equal strength (case III). Thus, by switching periodically between + and - $\Phi_0$ along the structure we get AG-focussing in all three planes called alternating phase focussing (APF). For relativistic particles, $\gamma_0 \gg 1$, it is easily seen from (7), (8) that the longitudinal focussing vanishes while the transverse forces are of opposite signs and equal strength.

## 3. Numerical Calculations of Different Structures

Four different structures (Fig. 1) have been calculated with MAFIA.

The dimensions of the structures are given in Fig. 1 for a 120 GHz $\pi$-mode which is synchronous with a particle of velocity $\beta = 0.4$. Calculated are further field integrals parallel to the z-axis (Table 1), such as

$$W^E = \int_0^L E e^{jk_z z} dz, \quad W^B = c\beta \times \int_0^L B e^{jk_z z} dz. \tag{10}$$

We realize that the contribution coming from the magnetic field ($W_y^B$) is small as compared to the contribution of the electric field ($W_y^E$). The only exception is structure b.

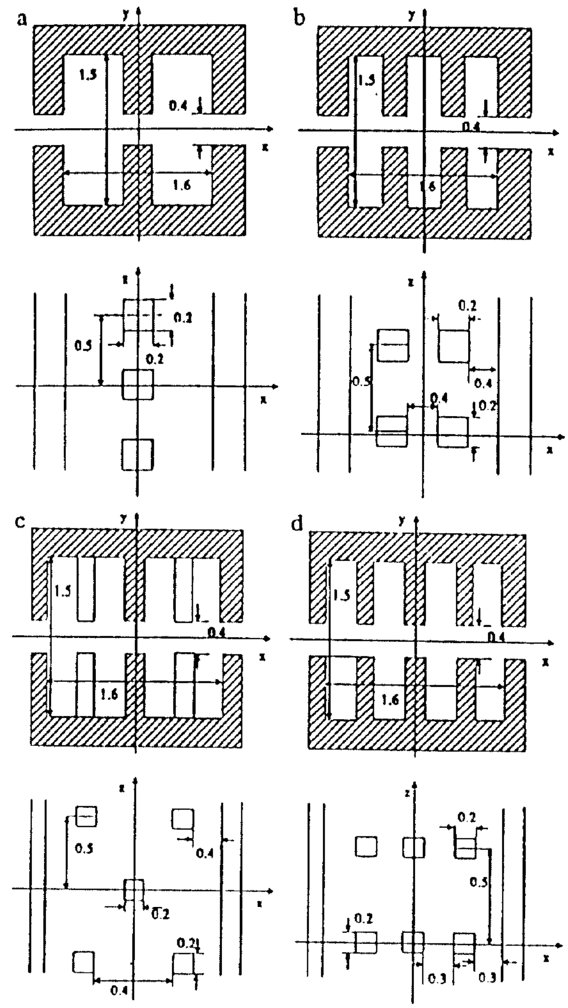


Figure 1. Cross sections and top views of four different RF-structures.

Table 1
Field integrals (equ. 10) parallel to the z-axis for the structures given in Fig. 1

| wake-potential in V \ Structure | a | b | c | d |
|---|---|---|---|---|
| $W_z^E$ (x = y = 0) | 6.4 | 6.1 | 9.3 | 4.2 |
| $W_z^E$ (x = 0.1 mm, y = 0) | 5.7 | 6.6 | 9.0 | 3.3 |
| $W_z^E$ (x = 0, y = 0.1 mm) | 8.2 | 6.6 | 11.3 | 5.9 |
| $W_x^E$ (x = 0.1 mm, y = 0) | j1.8 | -j1.4 | j1.1 | j2.4 |
| $W_y^E$ (x = 0, y = 0.1 mm) | -j5.9 | -j1.7 | -j6.6 | -j5.3 |
| $W_y^B$ (x = 0, y = 0.1 mm) | 0.53 | 0.47 | 0.2 | 0.35 |

The magnetic x-component, not given here, is even much smaller and justifies our assumption of vanishing $B_y$. By comparing the MAFIA results with the three types of space-harmonics, equation (4), we note that structure a carries waves of type I with opposite and different optical strength in transverse directions. Structure b refers to type III. The focussing forces in x- and y-directions have equal sign and can even be of equal magnitude if $k_x^2 = 1/2(k_z^2-k^2)$. Finally, structures c and d are mixtures of the types I and III.

## 4. ESTIMATES OF BUNCH DIMENSIONS

As it was shown, longitudinal bunching and symmetric conditions in x- and y-direction are only possible if the fields are defocussing transversely. In such a case we get from (8) for a synchronous particle at $\Phi_0 = -\pi/2$

$$\ddot{x} = \nu^2 x, \quad \ddot{y} = \nu^2 y, \quad \nu^2 = \frac{1}{2}\frac{ekE_0}{m\beta_0\gamma_0^3} \tag{11}$$

which yields exponentially growing solutions. The motion, however, can be easily stabilized by adding an external constant magnetic field $B_{z0}$ in axial direction. Then, the longitudinal equation of motion (7) does not change and (11) becomes

$$\ddot{x} = \nu^2 x - \alpha\dot{y}, \qquad \ddot{y} = \nu^2 y + \alpha\dot{x}$$

which can be written as

$$\ddot{\zeta} = \nu^2\zeta + j\alpha\dot{\zeta}, \quad \zeta = x + jy, \quad \alpha = \frac{eB_{z0}}{m\gamma_0}. \tag{12}$$

The solution of (12) is

$$\zeta = C_1 e^{j\Omega_1 t} + C_2 e^{j\Omega_2 t}, \quad \Omega_{1/2} = \frac{\alpha}{2}\left[1 \pm \sqrt{1-4(\nu/\alpha)^2}\right]. \tag{13}$$

Therefore, the solution is finite and stable for

$$2\frac{\nu}{|a|} < 1 \quad \text{or} \quad |B_{z0}| > \sqrt{\frac{2mkE_0}{e\beta_0\gamma_0}}. \tag{14}$$

The constants $C_{1/2}$ in (13) are obtained from the initial values $\zeta_0, \dot{\zeta}_0$

$$C_1 = -\frac{\Omega_2\zeta_0 + j\dot{\zeta}_0}{\Omega_1 - \Omega_2}, \quad C_2 = \frac{\Omega_1\zeta_0 + j\dot{\zeta}_0}{\Omega_1 - \Omega_2}. \tag{15}$$

Bunch radius and divergence are then given by

$$r_b = |\zeta| \le |C_1| + |C_2|$$

$$r_b' = |\dot{\zeta}|/c\beta_0 \le \frac{1}{\beta_0 c}\left[|\Omega_1 C_1| + |\Omega_2 C_2|\right]. \tag{16}$$

Under these circumstances the longitudinal force leads to a bunching of the beam at phase values $\Phi_0 = -\pi/2 \pm 2\pi n$ until the RF bunching force equals the space charge force

$$E_z^{RF} + E_z^{SC} = 0. \tag{17}$$

We estimate the space charge force by assuming a cylindrical bunch shape of length $l_b$ and radius $r_b$ with homogeneous charge distribution of total charge $Q = 2\pi I_0/\omega$. In case of bunches which are much shorter than their radius $l_b \ll r_b$ we can neglect the transverse derivatives of $E^{SC}$ in $\nabla \cdot E = \rho/\varepsilon 0$ and obtain

$$\frac{\partial E_z^{SC}}{\partial z} \approx \frac{\rho}{\varepsilon_0} = \frac{2I_0}{\omega\varepsilon_0 l_b r_b^2}. \tag{18}$$

Now, we differentiate (17) with respect to z and substitute (18) together with the field (3) at x = y = 0 and $\Phi_0$

$$\frac{\partial E_z^{RF}}{\partial z} + \frac{\partial E_z^{SC}}{\partial z} \approx -k_z E_0 + \frac{\rho}{\varepsilon_0} = 0$$

yielding a bunch length of

$$l_b = 2\beta_0 c I_0 / \varepsilon_0 \omega^2 r_b^2 E_0. \tag{19}$$

As an example we consider a structure operating at 120 GHz with an accelerating field of $E_0$ = 2 MV/m. The electron velocity is taken as $\beta_0 = 0.4$ corresponding to $\gamma_0 = 1.091$. The change in $\beta$ and $\gamma$ for an electron passing through one structure period is about 2 % and 0.4 %, respectively, which justifies our assumption of adiabatic changes. The external magnetic field, according to (14), must be

$$|B_{z0}| > 0.36 \text{ T}.$$

Choosing $B_{z0}$ = 1 T we obtain for (13)

$$\Omega_1 = 1.54 \cdot 10^{11} s^{-1}, \quad \Omega_2 = 5.6 \cdot 10^9 s^{-1}.$$

In order to calculate the beam radius and divergence we assume initial values

$$|\zeta_0| = 0.1 \text{ mm}, \qquad \alpha_0 = |\dot{\zeta}_0|/\beta_0 c = 10 \text{ mrad}$$

and obtain the amplitudes C in (15)

$$|C_1| = 9\ \mu\text{m}, \quad |C_2| = 104\ \mu\text{m}$$

and finally the bunch radius and divergence, equ. (16)

$$r_b \le 0.11 \text{ mm}, \quad r_b' \le 16 \text{ mrad}.$$

As we can see both beam radius and divergence are essentially maintained. The bunch length, in case of $I_0$ = 10 mA average current, follows from (19) as

$$l_b = 20\ \mu\text{m}$$

which is smaller than $r_b$ and our assumption is at least partially correct.

## 5. ACKNOWLEDGEMENT

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## 6. REFERENCES

[1] H. Henke, Y.W. Kang and R.L. Kustom, "A mm-wave RF structure for relativistic electron acceleration", Argonne National Laboratory, report ANL/APS/MWM-1, 1993