Analytic Study of Transverse Shunt Resistance and Even-Odd Mode Coupling of a Rod Type RFQ

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Abstract

To minimize the ohmic power losses, it is necessary to maximize the transverse shunt resistance, R_{shunt}. The cell of a rod-type RFQ is modelled by a parallel two-rod transmission line supported above a parallel ground conductor by two legs. Due to coupling between neighbouring supports, the loading impedance is modified depending on the leg spacing. The shunt resistance is improved by reducing the cell length and increasing the leg spacing, and maximized when the legs are equally spaced. However, this is also the condition for strong excitation of the unwanted 'even-mode' in which a potential difference exists between the ends of the rods mid-plane and the grounding conductor or tank. Once the legs of the support are longitudinally separated, some even-mode excitation of the structure is inevitable because some current must be injected into the ground conductor; the even-mode excitation rises as leg separation increases. Further, when the desired odd-mode voltage is symmetric about the cell centre, the even-mode voltage is anti-symmetric This paper is a very much abridged version of two internal design notes[3],[4].

1 INTRODUCTION

We divide the RFQ into series connected unit cells. R_{shunt} is the cell-averaged inter-rod-voltage squared, divided by cell power loss. Our basic purpose will be to optimize R_{shunt} as a function of cell length l. The support structure is, itself, an issue. How does one compare cases with different separation s between the two legs of the support? How does one assess the relative electrical coupling between the supports of neighbouring cells? Our strategy has been to model the support by a parallel plate transmission line, and for given transverse dimensions and resonance frequency of the structure let the model itself determine the leg separation s as a function of the cell length l. This is discussed in sections 2 and 4.

In the naive model[1], a plane passing through the centre line between the two rods (and perpendicular to the plane containing them) is a ground plane. This aspect of the model does not agree with MAFIA code calculations reported by Andreev[2] and has prompted us to consider a model in which the structure is excited in a combination of 'odd' and 'even' transmission line modes. This is discussed in section 3.

2 NAIVE MODEL

The unit cell (see Fig.1) consists of a length l of two-rod transmission line, characteristic impedance Z_0, bridged at

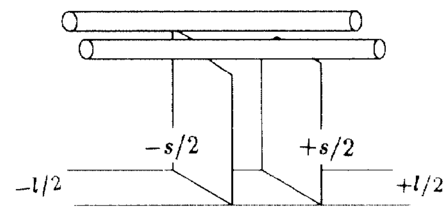

Figure 1: Sketch of RFQ unit cell model

its centre by a support of lumped impedance Z_L. The line terminations are taken to be open-circuit so that the cell may function equally well as an end-cell or one with left and right neighbours. The condition for resonance is $\tan(\beta l/2) = +jZ_0/2Z_L$ where $\beta = \omega/c$ is the phase constant of the line.

2.1 *Transverse shunt resistance*

The power dissipated is the sum of ohmic losses in the rods and in the support. The rod current $I(z)$ and voltage $V(z)$ distributions, as a function of position z, come from solving the telegraphists equations, assuming a time dependence $\exp(+j\omega t)$. The current changes step-wise across the support at location $z = 0$ by an amount $V(0)/Z_L$. Hutcheon[1] modelled the support as a section of parallel plate transmission line. The current flow is confined to the interior (i.e. facing) surfaces of the two plates. For a given width and separation of the plates, their height h is determined from the resonance condition and so cannot be held constant while cell-length is varied.

The transverse shunt resistance per unit length is

$$R_{\text{shunt}} = Z_0^2[1+\mathrm{sinc}\beta l]/\{R_{\text{surf}}^{\text{rods}}[1-\mathrm{sinc}\beta l] + P_{\text{plates}}\}\ , \quad (1)$$

$$P_{\text{plates}} = R_{\text{surf}}^{\text{plates}}[\sin(\beta l/2)/\cos\beta h]^2[1+\mathrm{sinc}\beta h\cos\beta h](4h/l)\ .$$

Here $R_{\text{surf}}^{\text{rods}}$ and $R_{\text{surf}}^{\text{plates}}$ are the resistances per unit length (for given diameter of rods, and width of plates). $R_{\text{shunt}}(l)$ has minima at $\beta l = 0$, π and is a rather skewed bell-shaped curve between. Hence there is a maximum shunt resistance. However, it may occur that the optimum cell length is impractically small. Further, there are three deficiencies of the model which make this result rather unrealistic. (i) We assumed negligible separation of the support legs. (ii) We have not held the support height constant. (iii) We have ignored coupling between supports of adjacent cells.

3 EVEN AND ODD MODES OF LINE

In the odd-mode excitation, there are positive and negative charges on the rods, equal and opposite currents flowing in the rods, and no free charges on the ground conductor.

In the even-mode excitation, there are equal positive (say) charges on both rods and a free negative charge on the ground conductor; there are equal currents flowing in the same sense in both rods and an oppositely directed current flowing on the ground conductor. In the even-mode excitation, the mid-plane between the rods is not a ground plane, and there exists a potential difference between the centre line and the grounded vacuum tank.

3.1 *Symbol definitions*

We define odd-mode voltage V_x to be the electric field integral for a path perpendicular to the two rods and in the plane containing the rods. The V_x path starts on one rod and terminates at the other. We shall call the odd-mode current I.

We define even-mode voltage V_y to be the field integral for a path perpendicular to the ground conductor and in the mid-plane between the rods. The V_y path starts on the ground conductor and terminates at the plane containing the V_x path. We shall denote the even mode current J.

3.2 *Telegraphists equations*

It can be shown that V_x couples only to I, and that V_y couples only to J, and the relations between them are:

$$\partial V_x/\partial z = -L_x \partial I/\partial t \ , \quad \partial I/\partial z = -C_x \partial V_x/\partial t \quad (2)$$
$$\partial V_y/\partial z = -L_y \partial J/\partial t \ , \quad \partial J/\partial z = -C_y \partial V_y/\partial t \ . (3)$$

L_x and C_x are the mutual inductance and capacitance per unit length between the two rods. L_y and C_y are the distributed reactances between the ground conductor and the plane containing the rods.

3.3 *Currents in legs of support*

The unit-cell support is composed of two legs. The left leg is connected between the left rod and the ground conductor. The right leg is connected between the right rod and the ground conductor. Left and right refer to viewing along the rods in the direction of positive z. We take the cell to extend between $z = \pm l/2$, and the legs of the support to be symmetrically placed at $z = \pm s/2$. Suppose, that a current K flows off the left rod, down one leg, and then flows up the opposite leg of the support and on to the right rod. We suppose the whole support has impedance Z_L, and that each leg contributes $Z_L/2$. We assume that this impedance is independent of the separation of the two legs and is independent of the presence or absence of neighbouring unit cells. Each leg is considered to be connected between a single rod and the centre-line of the ground conductor. The current down the left leg is $K_{\text{left}} = (2/Z_L)(V_y + V_x/2)\big|_{z=-s/2}$. The current down the right leg is $-K_{\text{right}} = (2/Z_L)(V_y - V_x/2)\big|_{z=+s/2}$. The negative sign in front of K_{right} derives from our choice that the current flows up the right leg.

3.4 *Fundamental symmetries of V_x and V_y*

If K_{left} and K_{right} are not equal, then the currents flowing across each end of the unit cell are unequal, and the cell ceases to be a repeatable unit. Hence the currents K_{left} and K_{right} are equal, in which case

$$(V_x/2 + V_y)\big|_{z=-s/2} = (V_x/2 - V_y)\big|_{z=+s/2} \ . \quad (4)$$

From this it follows that if V_x is symmetric about the centre of the unit cell, then V_y must be anti-symmetric.

3.5 *Odd-mode with 'open'-terminations*

We take the boundary condition for the odd-mode of zero current (I) and maximum voltage (V_x) at the ends of the unit cell. The distribution in the interior of the cell is then found by simultaneously integrating equations (2) from the cell ends towards the cell centre. The voltage distribution $V_x(z)$ is symmetric by choice. Integrating across the current discontinuities, that is rightwards across $z = -s/2$ or leftwards across $z = +s/2$ gives two expressions for the current distribution $I(z)$ in the region $-s/2 < z < +s/2$. Their equality implies the condition:

$$\tan[\beta(l-s)/2] + \tan[\beta s/2] = j\frac{Z_0^x}{Z_L}\left[\frac{1}{2} + \frac{V_y(-s/2) - V_y(+s/2)}{V_x(-s/2) + V_x(+s/2)}\right] \ . \quad (5)$$

Equation (5) gives the support impedance necessary for resonance. Here $Z_0^x = \sqrt{L_x/C_x}$.

3.6 *Even-mode with 'open'-terminations*

To evaluate the resonance condition (5) we need an explicit expression for the even-mode voltage V_y and mode current J . Because we have stipulated $I(\pm l/2) = 0$, it follows that $J(\pm l/2) = 0$ also. Thus the cell-ends are 'open'-terminations for the even-mode.

We already know the voltage distribution $V_y(z)$ to be anti-symmetric. To find the current J we integrate (3) outwards from the centre ($z = 0$) across the discontinuities at $-s/2$ or $+s/2$ and towards the cell ends. Now, $J(\pm l/2) = 0$, and from this equality it follows that

$$[V_y(-s/2) - V_y(+s/2)]/[V_x(-s/2) + V_x(+s/2)] =$$
$$- Z_0^y/\{2Z_0^y + jZ_L \tan[\beta(l-s)/2] - jZ_L/\tan(\beta s/2)\} \ . \quad (6)$$

The expression on the left hand side characterizes the relative importance of the even and odd mode contributions. For reasons of beam dynamics, it is desirable to suppress the even-mode; and to accomplish this we must make the right hand side as small as possible. The most direct method is to reduce the leg separation to zero ($s \to 0$), in which case the even-mode excitation is zero irrespective of the magnitudes of Z_L and $Z_0^y = \sqrt{L_y/C_y}$.

3.7 *Unit cell resonance condition*

We may combine equations (5) and (6) to determine the resonance condition :

$$\tan[\beta(l-s)/2] + \tan[\beta s/2] = j\frac{Z_0^x}{2Z_L}\left[1 - \frac{2Z_0^y}{2Z_0^y + jZ_L\tan[\beta(l-s)/2] - jZ_L/\tan(\beta s/2)}\right] \ .$$

It is worth noting that Z_L pure imaginary is a solution of this equation.

4 FULL MODEL

We introduce a very crude model for the coupling of the supports of nearest neighbour cells. However, it does include the features: (i) coupling increases as cell size is reduced, and (ii) the desired load impedance (for resonance) can be obtained without varying the height of the support. We model the support as parallel plates, but allow current flows on both faces of the plates; transmission lines are formed (i) between the interior faces of the plates of a single support, and (ii) between the exterior faces of the plates of adjacent supports. This has the effect of replacing the single loading impedance Z_L by two parallel impedances whose complementary values depend on the leg separation s. Thus, by varying s we may obtain (almost) any desired load impedance, while keeping the support height h constant. This model becomes quite accurate in two extreme cases: (i) very long cells and (ii) very short unit cells.

4.1 Current and voltage distribution

The unit cell has support legs placed at $\pm s/2$. We shall assume that the even-mode excitation is small and can be ignored. For simplicity, we suppose that current flow along the ground conductor is piece-wise constant. Let us suppose a current $K_{\rm int}$ flows on the inner faces of the legs, and a current $K_{\rm ext}$ flows on the outer faces of the legs, and let K denote their sum. Let the impedance of the current path between the interior faces of the legs of a single support be Z_L, and the impedance of the path between the exterior faces of nearest neighbour legs of two adjacent supports be Z_M. We take the cell ends to have voltage V_x maxima, and odd-mode current I zero. The step changes in current are $\Delta I(-s/2) = K/2$ and $\Delta I(+s/2) = K/2$ where $K = V_x(\pm s/2)[1/Z_L + 1/Z_M]$.

4.2 Resonance condition

The resonance condition yields the requirement:

$$\tan[\beta(l-s)/2] + \tan[\beta s/2] = j(Z_0/2)[1/Z_L + 1/Z_M] \, . \quad (7)$$

Now, for parallel plate supports Z_L and Z_M of height h, width w, and respective plate separations s and $l-s$,

$$\frac{1}{Z_L} + \frac{1}{Z_M} = \frac{w}{j\eta\tan(\beta h)} \times \frac{l}{s(l-s)} \, . \quad (8)$$

For fixed resonance angular frequency ω, and fixed support height h, equations (7, 8) may be combined to give a transcendental equation for the leg separation s.

In general, as the cell length l is reduced, so the total loading impedance $Z_{\rm Tot} = (Z_L \times Z_M)/(Z_L + Z_M)$ must increase to maintain resonance. Now, $Z_{\rm Tot}$ is maximized when $Z_L = Z_M$. Hence it follows that as l decreases, so the ratio s/l increases towards the limit value of $s/l = 1/2$. The limiting cell length satisfies the condition:

$$\tan(\beta l/4) \times \tan(\beta h) = (Z_0/\eta)(w/l) \, . \quad (9)$$

Beyond this value, l cannot be further reduced except by modifying the support dimensions h and w . Equation (9) is also the condition for minimizing the impedance Z_M and thereby maximizing the current $K_{\rm ext}$ flowing across the ends of the cell; which in turn implies the strongest possible excitation of the undesired even-mode. Here $\eta = \sqrt{\mu/\epsilon}$.

4.3 Transverse shunt resistance

To calculate the total power loss, we should sum the even and odd mode contributions. However, we shall consider only the odd-mode, and so shall give an approximate result for the transverse shunt resistance $R_{\rm shunt} \approx$

$$\frac{Z_0^2\left\{(1-s/l)[1+{\rm sinc}\beta(l-s)] + (s/l)[1+{\rm sinc}\beta s](V_1/V_0)^2]\right\}}{P_{\rm rods}(l,s) \; + \; P_{\rm plates}(l,s)} \quad (10)$$

where the rod and support contributions are

$$P_{\rm rods} \approx R_{\rm surf}^{\rm rods}\left\{(1-s/l)[1-{\rm sinc}\beta(l-s)] + (s/l)[1-{\rm sinc}\beta s](V_1/V_0)^2\right\}$$

$$P_{\rm plates}(l,s) \approx 4R_{\rm surf}^{\rm plates}\left[\frac{\sin[\beta(l-s)/2] + (V_1/V_0)\sin[\beta s/2]}{\cos(\beta h)}\right]^2 \times$$
$$\left\{\left(\frac{s}{l}\right)\left(1-\frac{s}{l}\right) + \left(\frac{h}{l}\right)[1+{\rm sinc}(\beta h)\cos(\beta h)]\left[1+\frac{2s(s-l)}{l^2}\right]\right\}$$

and the voltage ratio is $V_1/V_0 = \cos[\beta(l-s)/2]/\cos[\beta s/2]$.

4.4 Properties of shunt resistance

Usually there will be a region of βl values where the shunt resistance rises as $1/l^2$. At the same time, the ratio s/l rises so that one goes from the case of (i) long cells, supports far apart and legs very closely spaced; toward the case of (ii) short cells and leg spacing roughly one half the support spacing. Eventually, the limiting condition (9) will be achieved; when $s = l/2$ we find $V_0 = V_1$ and the shunt resistance reaches the maximum value

$$\hat{R}_{\rm shunt} \approx \frac{[1+{\rm sinc}(\beta l/2)]Z_0^2}{R_{\rm surf}^{\rm rods}[1-{\rm sinc}(\beta l/2)] + P_{\rm plates}} \, , \quad (11)$$

$$P_{\rm plates} = 4R_{\rm surf}^{\rm plates}\left[\frac{\sin(\beta l/4)}{\cos(\beta h)}\right]^2 [1 + 2(h/l)(1+{\rm sinc}\beta h\cos\beta h)] \, .$$

5 CONCLUSION

The transverse shunt resistance of a rod-type RFQ can be increased by reducing the cell length. This implies increasing the spacing of the two legs comprising the unit cell support. However, this is also the condition for increasing the current flows along the ground conductor; and thereby causing strong excitation of the unwanted even-mode.

6 REFERENCES

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