SPECTRAL–ANGULAR DISTRIBUTION OF BREMSSTRAHLUNG BY SETS OF FAST CHARGED PARTICLES MOVING IN A THICH TARGET

A.V.Koshelkin

Theoretical Physics Department, Moscow Engineering Physics Institute
Moscow 115409, Russia

1.INTRODUCTION

We consider the radiation emission by a set of ultrarelativistic charged particles which undergo multiple elastic collisions with atoms of a amorphous scattering medium. The radiation emission by a bunch of identical particles as well as by a high energy electron-positron pair and an ultrarelativistic hydrogen- like atom in the medium is investigated theoretically in detail.

2.STATEMENT OF THE PROBLEM.

We consider the system of charged ultrare lativistic ($E_\mu \gg m_\mu$) classically fast ($E_\mu \gg \omega$ is a radiation frequency) particles which do not interact with each other (E_μ, m_μ, e_μ are the energy, the mass and the charge of the particle, $\hbar = c = 1$). These particles enter a homogeneous, semi-infinite, amorphous scattering medium with a permeability $\varepsilon(\omega)$. In the initial period, $t = 0$, particles are located in the points $\vec{r}_{01}$, $\vec{r}_{02}$ ..., $\vec{r}_{0N}$, and are the velocity $\vec{v}_{01}, \vec{v}_{02}, \ldots, \vec{v}_{0N}$, which are equal to $v_o = [1 - (m_\mu/E_\mu)^2]^{1/2}$. They are directed at angles $| \theta_\mu | \ll 1$ ($\mu = 1, \ldots, N$ - is the number of the particles) to the $\vec{e}_z$ vector (vector of the inward normal to the boundary of the medium). Let the characteristic longitudinal size of the beam L be such that $Lv_0 \ll T$ (the time when the particles move in the medium).

3.SOLUTION OF THE PROBLEM.

The spectral-angular distribution of the energy emitted by these particles is

$$\frac{d\mathcal{E}_\omega}{d\omega d\Omega_{\vec{n}}} = \frac{\omega^2}{2\pi^2} Re\Big\{ \varepsilon^{1/2}(\omega) \sum_{\mu,\nu=1}^{N} e_\mu e_\nu \int_0^T dt \int_0^{T-t} d\tau \times \exp\big[-i\omega\tau + i\vec{k}(\vec{r}_{0\mu} - \vec{r}_{0\nu}) + i\vec{k}(\vec{r}_\mu(t+\tau) - \vec{r}_\nu(t))\big] \times [\vec{n} \times \vec{v}_\mu(t+\tau)][\vec{n} \times \vec{v}_\nu(t)] \Big\}, \quad (1)$$

where $\vec{k}$ is the wave vector of the radiation field, $d\Omega_{\vec{n}}$ is an element of solid angle in the direction $\vec{n} = \vec{k}/k; k = \omega\varepsilon^{1/2}(\omega), \vec{r}_\mu(t+\tau) + \vec{r}_{0\mu}; \vec{r}_\mu(t) + \vec{r}_{0\mu}; \vec{v}_\mu(t+\tau); \vec{v}_\nu(t)$ are radius vectors and the velocities of the particles at the time $t + \tau$ and t respectively, τ is the time scale of the radiation formation (the coherence time), and the t is the time at which the radiation is emitted.

To calculate the observed spectral distribution of the emission energy of the particles in the medium, $dE_\omega/d\omega$, we must average expression (1) over all possible trajectories of the particles in the scattering matter [1]. To solve this problem it is necessary to find [2] two-time distribution function of the particles in a scattering medium. In the case of ultrarelativistic classically fast particles the solution of the problem is determined by the Fourier component of this function $F_{\vec{k}}(\vec{v}_\mu; \vec{v}_\nu, t, \tau)$:

$$\frac{dE_\omega}{d\omega d\Omega_{\vec{n}}} = \frac{\omega^2}{2\pi^2} Re\Big\{ \varepsilon^{1/2}(\omega)$$

$$\sum_{\mu,\nu=1}^{N} e_\mu e_\nu \int d\vec{v}_\mu \int d\vec{v}_\nu \int_0^T dt \int_0^{T-t} d\tau \tag{2}$$

$$\times \exp[-\imath\omega\tau + \imath\vec{k}(\vec{r}_{0\mu} - \vec{r}_{0\nu})]$$

$$\cdot [\vec{n}\times\vec{v}_\mu][\vec{n}\times\vec{v}_\nu]\Big\} F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau),$$

The function $F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)$ satisfies to the following equations and the initial condition [2]:

$$\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)}{\partial\tau} - \imath\vec{k}\cdot\vec{v}_\mu(\vec{\eta})\cdot F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau) = \Xi_\mu{}^2 \cdot \frac{q_0}{4}\cdot\frac{\partial^2 F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)}{\partial\vec{\eta}^2} \tag{3}$$

$$\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial t} - \imath\vec{k}\cdot(\vec{v}_\mu(\vec{\eta}) - \vec{v}_\nu(\vec{\zeta}))\cdot F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0) = \frac{q_0}{4}\cdot\Big\{\Xi_\mu\cdot\frac{\partial F_{\vec{v}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial\vec{\eta}} + {} + \Xi_\nu\cdot\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial\vec{\zeta}}\Big\}^2 \tag{4}$$

$$F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,0,0) = \delta(\vec{\eta}-\vec{\theta}_\mu)\cdot\delta(\vec{\zeta}-\vec{\theta}_\nu)$$

$$\Xi_\mu = U_\mu(\vec{g})\cdot E_0\cdot[E_\mu\cdot U(\vec{g})]^{-1}$$

where q_0 is the average square of multiple scattering angle for a positron per unit of path [3], $U_\mu(\vec{g}); E_\mu; U(\vec{g}); E_0$ correspond to the Fourier component of the potential of interaction with the atom of a medium and the energy for the particles which number is μ and for the positron respecti vely; $\vec{\eta}$ and $\vec{\zeta}$ angle vectors which are connected with $\vec{v}_\mu(\vec{\eta})$ and $\vec{v}_\mu(\vec{\zeta})$ by the ordinary formulae of the theory of multiple scattering in a amorphous medium [3].

4.RESULTS.

The theory of radiation emission by the set of ultrarelativistic charged particles which are scattered multiple elastic in a amorphous medium is developed. The spectral-angular distribution of the emission energy by these particles is obtained.

The emission of of a bunch of identical particles ($E_\mu \equiv E; m_\mu \equiv m; e_\mu \equiv e; q_\mu \equiv q_0\cdot\Xi^2 \equiv q$) is investigated in detail. We show that the spectral distribu tion of the emission energy has at least one extremum. If the bunch of the particles is a pulse one the extremum is the maximum , moreover, the maximum is unique. The value of the emission energy at the maximum, the maxi mum position as well as the width of the maxi mum depend essentially on both the initial condition of the particles and the scattering properties of the medium. Moreover if the width of the initial beam D is such that the condition $qD(E/m)^{-3} \ll (m/E)(qT)^{-1/2}$ holds, the maximum of bremsstrahlung energy spectrum is a plateau with a width on the order $D(qT)^{-1/2}$ .The ratio of $(dE_\omega/d\omega)_{max}$ to the background level ($(dE_\omega/d\omega)_{BH} = 2e^2qTN/3\pi(m/E)^2$) is approximately equal to N ,the number of emitting particles. As the parameter $qD(m/E)^{-3}$ increases $qD(E/m)^{-3} \leq (m/E)(qT)^{-1/2} \ll 1$ the plateau convert into " strict " maximum . As before , we have $(dE_\omega/d\omega)_{max}/(dE_\omega/d\omega)_{BH} = N$. If we have $qD(E/m)^{-3} \gg 1$, then the quantities $(dE_\omega/d\omega)_{max}$ and $(dE_\omega/d\omega)_{BH}$ become approximately equal to each other.

The bremsstrahlung by a highly anisotropic point source of ultrarelativistic particles is investigated. We show that in this case the spectral disrtibution of the emission energy has a maximum. The last is unique. The shape of the maximum as well as the magnitude of the emission energy depend strongly both on the value

$\Theta_0 \ll 1$ of cone vertex angle which the includes the initial velocities of the particles and on the scattering properties of a scattering medium.

We have studied in detail the emission by a pulse beam of fast charged particles in a dispersive scattering medium. If the latter is an electrical neutral plasma the presence of polarization in the medium leads under certain conditions to a radical change in the bremsstrahlung spectrum of such a system of the particles. In particular, for sufficiently high Langmuir frequencies, $\omega_0 \gg q(m/E)^{-3}$, and for $D\omega_0 \gg (Nm^2/qTE^2)^{1/2}$, the maximum in the spectral distribution of the bremsstrahlung radiation is suppressed by the interaction of radiation field with plasma medium, and $(dE_\omega/d\omega)$ becomes a monotonously increasing function of ω in contrast to the case $\varepsilon = 1$. However, in the case when the permeability of the medium satisfies the inequality $\varepsilon(\omega)^{1/2} > v_0{}^{-1}$ (the situation when Cherenkov effect takes place) in the very long-wavelength region of the spectrum, as for in the $\varepsilon(\omega)^{1/2} < v_0{}^{-1}$, the radiation turns out to be essentially bremmstrahlung (as before, $(dE_\omega/d\omega) \propto \omega^{1/2}$ as $\omega \to 0$). If, however, the frequency $\omega \gg q(1 - v_0[(\varepsilon(\omega)]^{1/2})^{-2}(\varepsilon(\omega))^{1/2}$, the bremsstrahlung strongly suppressed and mechanism for the formation of the radiation is the interaction of the system of fast particles with the coherently emitting medium.

The emission by an ultrarelativistic electron-positron pair ($-\Xi_{positron} = \Xi_{electron} = -1$; E_0, m_0, e_0 are energy , the mass, the charge of a positron, respectively; q_0 is the average square of the angle of multiple scattering of a positron per unit path) in a scattering medium is researched in detail. We have shown that the interference of waves emitted by the electron and by the positron leads to the suppression of the intensity of the emission energy at long wave frequency range

$$\frac{dE_\omega}{d\omega} = \frac{4}{3} \cdot \frac{e_0{}^2 T^3}{\pi} \cdot (q_0\omega)^{3/2}; \quad (\varepsilon(\omega) = 1)$$

At shortwave frequency region the interference effects is negligible an the value of the emission energy is proportional to the number of the irradiating particles. We show that under the certain conditions the emission spectrum of the electron-positron pair has overbending point which is situated on the frequencies in order of $q_0 E_0{}^2 m_0{}^{-2}$. It is shown that the angular distribution of the emission energy of the electron-positron pair has a maximum which is on the emission angles $\theta \simeq (q_0 T)^{1/2}$.

We consider the radiation emission by a hydrogen-like atom in a "strong" scattering medium. We show that under the certain conditions $(q_n < q; Z_n{}^2 , (q/q_n)^{1/2})$, where q_n and Z_n are the average square of multiple scattering angle per unit path and the charge of the core for hydrogen-like atom) the spectral distribution of the emission energy by this atom has step-like view. The width and the height of the "step" depend sufficiently on the charge and on the mass of the core of the emitting hydrogen- like atom.

REFERENCES.

1. A.B.Migdal, Phys.Rev., 103, 1811 (1956).
2. A.V.Koshelkin. Zh. Eksp.Teor.Fiz.(Sov. JETP) 100, 1724-1738 (1991).
3. N.P.Kalashnikov, V.S.Remizovich, M.I.Ryazanov, Collisions of Fast Charged Particles in Solids, Gordon and Breach Science Publishers, New York, 1985.