ELECTRON–POSITRON PAIR PRODUCTION BY A SET OF ULTRARELATIVISTIC CHARGED PARTICLES IN A SCATTERING TARGET

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1.INTRODUCTION

Production of an electron-positron pair by a set of fast charged particles which do not interact with each other but which do undergo multiple elastic scattering by randomly positioned atoms of the medium with dielectric permeability is studied. On basic of a systematic kinetic analysis of process of emisson of photons in a medium the cross-section of the pair production of these particles is calculated.

2.STATEMENT OF THE PROBLEM.

We consider the system of charged ultrare lativistic ($E_\mu \gg m_\mu$) classically fast ($E_\mu \gg \omega$ is a radiation frequency) particles which do not interact with each other (E_μ, m_μ, e_μ are the energy, the mass and the charge of the particle, $h = c = 1$). These particles enter a homogeneous, semi-infinite, amorphous scattering medium with a permeability $\varepsilon(\omega)$. In the initial period, $t = 0$, particles are located in the points $\vec{r}_{01}, \vec{r}_{02} \ldots, \vec{r}_{0N}$, and are the velocity $\vec{v}_{01}, \vec{v}_{02}, \ldots, \vec{v}_{0N}$, which are equal to $v_o = [1 - (m_\mu/E_\mu)^2]^{1/2}$. They are directed at angles $| \theta_\mu | \ll 1$ ($\mu = 1, \ldots, N$ - is the number of the particles) to the $\vec{e}_z$ vector (vector of the inward normal to the boundary of the medium). Let the characteristic longitudinal size of the beam L be such that $Lv_0 \ll T$ (the time when the particles move in the medium).

3.SOLUTION OF THE PROBLEM.

In the case of ultrarelativistic charged particles the cross-section of pair production may be calculated by the method of "equivalent photons" [1]. The latter consists in the substitution of virtual photons which is in the process of pair production by the real photons emitted by the bunch of initial particles. If interaction of the emitted photons with atoms of a scattering medium is negligible the cross-section of pair production $d\Sigma$ which is summed over all conditions of an electron and a positron is given by the expression:

$$d\Sigma = \sigma_{el-pos} \cdot g(\omega) \cdot d\omega \qquad (1)$$

In formulae (1) σ_{el-pos} is the well-known cross-section of pair production by an individual photon with energy ω [1], $g(\omega)$ is the number of photons per unit range of frequency. In the case when the energy of a photon ω more greater then a mass of an electron m_e the cross-section σ_{el-pos} is [1]:

$$d\sigma_{el-pos} = \frac{28}{9} z^2 \alpha r_e{}^2 (\ln \frac{2\omega}{m_e} - \frac{109}{42}), \qquad (2)$$

where r_e is the classical radius of an electron which is equal to e^2/m_e (e is the charge of an electron), α is the constant of fine structure.

The the number of photons per unit range of frequency $\mathcal{G}(\omega)$ emitted by the set these particles is

$$\mathcal{G}(\omega) = \frac{\omega}{2\pi^2} Re\Big\{ \varepsilon^{1/2}(\omega) \sum_{\mu,\nu=1}^{N} e_\mu e_\nu \int d\Omega_{\vec{n}} \int_0^T dt \int_0^{T-t} d\tau \times \exp\big[-\imath\omega\tau + \imath\vec{k}(\vec{r}_{0\mu} - \vec{r}_{0\nu}) + \imath\vec{k}(\vec{r}_\mu(t+\tau) - \vec{r}_\nu(t))\big] \times [\vec{n}\times\vec{v}_\mu(t+\tau)][\vec{n}\times\vec{v}_\nu(t)]\Big\}, \tag{3}$$

where $\vec{k}$ is the wave vector of the radiation field, $d\Omega_{\vec{n}}$ is an element of solid angle in the direction $\vec{n} = \vec{k}/k; k = \omega\varepsilon^{1/2}(\omega), \vec{r}_\mu(t+\tau) + \vec{r}_{0\mu}; \vec{r}_\mu(t)+\vec{r}_{0\mu}; \vec{v}_\mu(t+\tau); \vec{v}_\nu(t)$ are radius vectors and the velocities of the particles at the time $t+\tau$ and t respectively, τ is the time scale of the radiation formation (the coherence time), and the t is the time at which the radiation is emitted.

To calculate the observed density of photons emitted by the particles in the medium, $g(\omega)$, we must average expression (1) over all possible trajectories of the particles in the scattering matter [2]. To solve this problem it is necessary to find [3] two-time distribution function of the particles in a scattering medium. In the case of ultrarelativistic classically fast particles the solution of the problem is determined by the Fourier component of this function $F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)$:

$$g(\omega) = \frac{\omega}{2\pi^2} Re\Big\{ \varepsilon^{1/2}(\omega) \sum_{\mu,\nu=1}^{N} e_\mu e_\nu \int d\Omega_{\vec{n}} \int d\vec{v}_\mu \int d\vec{v}_\nu \int_0^T dt \int_0^{T-t} d\tau \times \exp\big[-\imath\omega\tau + \imath\vec{k}(\vec{r}_{0\mu} - \vec{r}_{0\nu})\big] \cdot [\vec{n}\times\vec{v}_\mu][\vec{n}\times\vec{v}_\nu]\Big\} F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau), \tag{4}$$

The function $F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)$ satisfies to the following equations and the initial condition [3]:

$$\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)}{\partial\tau} - \imath\vec{k}\cdot\vec{v}_\mu(\vec{\eta})\cdot F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau) = \Xi_\mu{}^2 \cdot \frac{q}{4} \cdot \frac{\partial^2 F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,\tau)}{\partial\vec{\eta}^2} \tag{5}$$

$$\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial t} - \imath\vec{k}\cdot(\vec{v}_\mu(\vec{\eta}) - \vec{v}_\nu(\vec{\zeta}))\cdot F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0) = \frac{q}{4}\cdot\Big\{\Xi_\mu\cdot\frac{\partial F_{\vec{v}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial\vec{\eta}} + \Xi_\nu\cdot\frac{\partial F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,t,0)}{\partial\vec{\zeta}}\Big\}^2 \tag{6}$$

$$F_{\vec{k}}(\vec{v}_\mu;\vec{v}_\nu,0,0) = \delta(\vec{\eta}-\vec{\theta}_\mu)\cdot\delta(\vec{\zeta}-\vec{\theta}_\nu)$$

$$\Xi_\mu = U_\mu(\vec{g})\cdot E\cdot[E_\mu\cdot U(\vec{g})]^{-1}$$

where q is the average square of multiple scattering angle for a positron per unit of path [4], $U_\mu(\vec{g}); E_\mu; U(\vec{g}); E$ correspond to the Fourier component of the potential of interaction with the atom of a medium and the energy for the particles which number is μ and for the positron respecti vely; $\vec{\eta}$ and $\vec{\zeta}$ angle vectors which are connected with $\vec{v}_\mu(\vec{\eta})$ and $\vec{v}_\mu(\vec{\zeta})$ by the ordinary formulae of the theory of multiple scattering in a amorphous medium [4].

4.RESULTS.

The theory of electron-positron pair production by the set of ultrarelativistic charged particles which are scattered multiple elastic in a amorphous medium is developed. The cross-section of the production of electron-positron pairs by these particles is obtained.The cross-section depends essentially on both the characteristics of the initial bunch of ultrarelativistic particles and the scattering properties of the medium.

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