AMPLITUDE DEPENDENT TUNE SPREAD AND FIELD ERRORS OF SUPERCONDUCTING LOW-β QUADRUPOLES

R. Ostojic and T. M. Taylor
LHC Division, CERN, Geneva, Switzerland

Abstract

The nonlinear effects in the low-β insertions are studied on the basis of the amplitude dependent tune spread. Several methods of estimating the tune spread in superconducting low-β quadrupoles are compared. The main feature of these magnets is that the random errors dominate the multipole spectrum. The proposed methods allow an analysis of the final focus layout in the early stages of the insertion design, and point to the critical magnets and the dominant multipole errors.

1 INTRODUCTION

The large β-functions in the low-β insertions which are associated with small values of β^*, chosen for optimising the luminosity, are generally considered a limiting factor of hadron collider performance. High-β conditions inside the final focus quadrupoles are the source of chromatic and amplitude dependent effects. While both are related to the precise layout of the low-β section, the field quality of the low-β quadrupoles drives non-linear resonances, which together with the beam-beam collisions, limit the lifetime of the colliding beams. In order to estimate the relevance of the large amplitude motion, the contribution of each low-β quadrupole should be considered by taking into account the details of the local optics, in particular of the variation of the β-function and of the central orbit inside the quadrupoles, and of the possible asymmetries of the layout around the collision point. It is also important to study the layout in the early stages of the design, and to determine those features of the low-β quadrupoles which contribute mostly to exciting the non-linearities. An important indicator in this respect is the amplitude dependent tune spread.

In this report we propose a method of estimating the amplitude dependent tune spread of a low-β triplet of superconducting quadrupoles, in which the random errors typically dominate the multipole spectrum. These quadrupoles cannot be treated as thin lenses both because of their length and gradient, and because of the displaced beam trajectories arising from finite crossing angles. Furthermore, the optical functions vary considerably over short lengths, comparable to the extension of the quadrupole end field, where the systematic multipole errors are compensated on the average. With modest computational effort, dominant features of the triplet layout can be determined, and tolerance limits on the multipole errors can be derived.

2 AMPLITUDE DEPENDANT TUNE SPREAD IN PERTURBATION THEORY

The long-term particle motion in non-linear magnetic fields is an area of extensive studies. The long term dynamic aperture is correlated to the tune spread of the beam, with the value of around $\Delta Q = 0.015$, chosen for the LHC [1], considered as the upper limit for a hadron collider. The dominant part of the tune spread budget is attributed to the head-on and parasitic beam-beam collisions. While only a small fraction is related to the single particle motion (typically $\Delta Q = 0.005$), mostly due to large amplitude oscillations in the low-β insertions, the amplitude dependent tune spread beyond this level clearly limits the dynamic aperture.

We base our approach of estimating the amplitude dependent tune spread on perturbation theory, where the non-linear terms of the Hamiltonian are treated as perturbations to the well known linear motion [2]. To first order, the tune shift is obtained by averaging the phase independent part of the perturbation around the ring. As the perturbation is directly related to the multipole spectrum of the guide field, customarily represented as :

$$B_y + iB_x = B_0 \sum_{n=1} (b_n + ia_n)(\frac{x+iy}{r_0})^{n-1}$$

the contribution of the k-th multipole error to the tune shift may be written in terms of the optics and multipole field errors of each magnet in a given section of the low-β insertion:

$$\delta Q_x = \int \frac{ds}{2\pi\rho} \beta_x \sum_{n=1}^{10} (C_n^{x,b} b_n + C_n^{x,a} a_n) \qquad (1)$$

The most important coefficients for the bending plane are :

$$\begin{aligned} C_4^{x,b} &= (1-\delta)(1.5\Delta^2 + \frac{J_1}{4}) \\ C_5^{x,b} &= (1-\delta)\Delta_x J_1 \\ C_6^{x,b} &= (1-\delta)(2.5\Delta^2 J_1 + \frac{J_2}{6}) \end{aligned}$$

where:

$$\begin{aligned} \Delta_x &= x_c + D_x\delta \\ \Delta_y &= y_c + D_y\delta \\ \Delta^2 &= \Delta_x^2 - \Delta_y^2 \\ J_1 &= 3(J_x\beta_x - 2J_y\beta_y) \\ J_2 &= \frac{15}{2}(J_x^2\beta_x^2 - 6J_x\beta_x J_y\beta_y + 3J_y^2\beta_y^2) \end{aligned}$$

Here, $J_{x,y}$ are the x, y - action co-ordinates of the motion, R - the average radius of the ring, x_c - the central orbit displacement, and D_x - the dispersion. All quantities under the integral depend on the longitudinal co-ordinate s.

The coefficients C_n^x for the bending plane, as well as the equivalent coefficients for the y-plane, C_n^y, are sums of products of $J_x\beta_x$ and $J_y\beta_y$ of progressively higher order and increasing number of terms. It should be noted that for $n \leq 3$, the coefficients C_n depend only on the momentum error, δ, and central orbit excursion, i.e. the tune shift is linear in β. For $n \geq 4$, the order of $J\beta$ in C_n increases by one for every even multipole, so that the tune shift contribution of the 20-pole field error is of order 5 in β and beam emittance. The series is truncated with this term as it is considered to be the last known with a reasonable accuracy. In this approximation the non-zero C_n^a coefficients are of odd order (C_1^a is identically zero), and depend linearly on the central orbit and dispersion in the vertical plane.

3 TUNE SPREAD IN A LOW-β TRIPLET

The total tune spread is in general obtained by superposing tune shifts of different origins. In low-β triplets, the beam dynamics is dominated by the single-particle motion, and the tune spread can be obtained from the amplitude dependent tune shift, since the beam-beam and chromatic effects are reduced by the large beam size and small dispersion usually imposed in this section of the ring. Therefore, equation 1 is a good starting point for estimating the tune spread.

The multipole errors of accelerator magnets are expressed as a combination of systematic and random parts. In typical large aperture superconducting quadrupoles, the systematic errors are usually small by design, but nevertheless cannot be ignored. The "return" and "lead" ends of the magnet are not identical, and are constructed to give small integral field errors. However, the end errors vary considerably over a short distance, and oscillate with peaks of opposite sign comparable to several random sigmas, whereas it is the random errors which dominate in the body of the magnet. Therefore, an estimate of the tune spread has to consider both sources of error, and treat their interference appropriately.

In a situation when the field errors in the low-β triplet are predominantly systematic, the amplitude dependent tune spread can be obtained by examining the tune "footprint" of the particle distribution. In the case of a Gaussian distribution, particles are launched with initial amplitudes between 0σ and 3σ in each plane individually, and along several contours of constant total action $J_x + J_y$, where the ratio of horizontal and vertical actions is smoothly varied. The resulting tune spread in each plane is defined on the basis of the maximum tune shift difference over the distribution:

$$\Delta Q_{x,y} = \max\{\mathrm{abs}(\delta Q_{x,y}(J_{x_i}, J_{y_i}) - \delta Q_{x,y}(J_{x_j}, J_{y_j}))\}$$

The situation is more complicated in the case of predominantly random field errors. Equation 1 suggests that under the assumption of negligible systematic errors, the rms tune spread can be obtained as a weighted sum of random multipole errors, where the weights depend on the (J_x, J_y) values in the wings of the particle distribution. For practical calculations, we consider four points in the (J_x, J_y) plane: $(0,0), (3,0), \ldots (3,3)$ (expressed in terms of the beam emittance). For each of these points, the average tune shift and its rms value are calculated. The "rms" tune spread is then defined on the basis of the largest separation of any of the four points from the average tune shift $\overline{\delta Q_{x,y}}$:

$$\Delta Q_{x,y} = 2\max\{\pm < \delta Q_{x,y} > \pm 2\sigma_{\delta Q_{x,y}} \pm \overline{\delta Q_{x,y}}\} \quad (2)$$

Due to the interplay of the systematic and random errors, the actual rms tune spread tends to be smaller than the value calculated from equation 2; however, it cannot be calculated analytically. In order to find a relation between the limiting case and a more realistic situation including systematic errors, we consider the following cases:

- "Gaussian beam" tune spread: generate a number of combinations of systematic and random multipole errors for all magnets of the low-β triplet. For each seed, find the tune spread over a Gaussian distribution of particle amplitudes. The tune spread is defined as the maximum spread over the multipole error seeds.
- "4σ" tune spread: consider two extreme (J_x, J_y) points, (4,0) and (0,4). For these two amplitudes, calculate the tune spread on the basis of a large number of random seeds for the multipole errors.

Finally, we also consider the unlikely but limiting "maximum error" case, where each multipole is assigned either a value of the sum or of the difference of the systematic error and two random σ's, whichever gives a greater contribution to the tune spread.

3.1 Implementation of Procedures

The above procedures are performed in a program which takes as input the detailed description of a section (or full) machine in the MAD style format. A particular segment of the accelerator, e.g. a single low-β triplet, is selected, and additional transformations which may influence the multipole errors are applied (e.g. rotation of a magnet about its vertical axis). Each magnet class is described by the systematic and random error tables for the body, and systematic errors for the lead and return ends. In addition, correlations between multipoles may be specified for determining the error tolerances.

The optical functions and the central orbit are tracked inside each magnet, and the contributions of all multipole errors recorded. In this way, critical magnets and multipole terms can be identified, and the effect of varying the layout and optical conditions can be determined.

3.2 Comparison of the Tune Spread Estimates

The results of tune spread calculations with the procedures described above are shown in Fig. 1 in case of the LHC high luminosity insertions [1]. The "Gaussian beam" tune spread is considered a baseline method, but is itself time consuming and inappropriate for examining a large number of different situations. A very precise upper limit is obtained by using the "4σ" method, which is presented in Fig. 1 by its "four point" footprint $((J_x, J_y) = (0,0), (0,4), (4,0), (4,4))$. The non-zero average tune shift in these two cases is due to the fact that all random errors are considered, including the strong b_3 term which contributes to the average tune shift but not to the tune spread. The tune spread obtained by the "rms" and the "maximum error" methods, are on the contrary centred at the tune shift produced by the small systematic errors. The "rms" tune spread is by a factor of 1.6 larger than the "4σ" value and requires the smallest computational effort. However, we consider the "4σ" method as the most appropriate as it consistently gives good results within short computational time. The "maximum error" case is larger by a factor of 1.7.

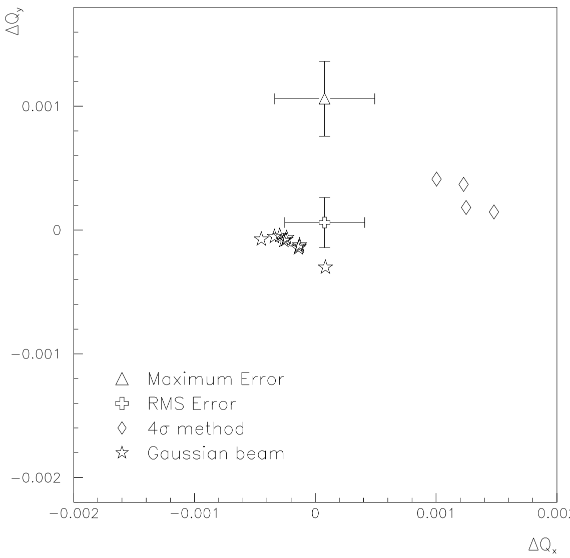

Figure 1: Comparison of the tune spread estimates for the LHC low-β triplet. For clarity, the "maximum error" tune spread has been shifted by 0.001 in ΔQ_y.

4 ANALYSIS OF LOW-β TRIPLET LAYOUTS

The major advantage of a simple method of estimating the amplitude dependent tune spread is that a number of configurations can be studied in the early stages of the insertion design without resorting to time consuming tracking studies. In the case of the LHC low-β quadrupoles [3], the application of this method gave the following results:

- The tune spread of the low-β insertion is dominated by the quadrupoles sitting in regions of peak β-values, which contribute 90% of the total tune spread. The off-centred central orbit ($\leq 4\sigma$) increases the tune spread by a factor of 2.5. In the LHC, changes in β^* influence the tune spread twice as much as changes of the crossing angle.
- The multipoles which contribute most to the tune spread are the octupole and decapole random, and dodecapole systematic and random errors. The errors of quadrupole ends, in particular the b_6 component of the lead end, contribute to about half of the total tune spread. These errors are systematic, and can be compensated by passive or active methods.
- By choosing the side on which to put magnet connections, the low-β triplet can be made such that the two LHC beams have the same tune spread. The value of the tune spread is reduced from $\Delta Q = 5\ 10^{-4}$ to $3\ 10^{-4}$ by the correct choice of the position of the connection side.
- Assuming the "maximum error" case and four low-β insertions tuned at a β^* of $0.5\,\mathrm{m}$, the LHC tune spread is by a factor 1.6 below the tentative limit of 0.005.

The last result indicates that the dynamic aperture of the LHC at top energy should not be limited by the random multipoles. This is confirmed by recent tracking studies [4], which give a dynamic aperture of 10 σ in physics conditions, on the edge of the "good field region" of the $70\,\mathrm{mm}$ aperture low-β quadrupoles.

5 CONCLUSIONS

We have compared several methods of estimating the amplitude dependent tune spread in superconducting low-β quadrupoles. The approach addresses the main feature of the multipole spectrum of these magnets, i.e. the dominance of the random errors. Furthermore, the details of the local optics, in particular the rapid variation of the β-function and central orbit deviation, and the asymmetries of the insertion optics are taken into account. The method allows to determine the critical quadrupoles and multipole errors, to investigate the role of magnet orientation and connections, and to set limits on multipole error tolerances.

6 REFERENCES

[1] The LHC Study Group, *The Large Hadron Collider, Conceptual Design*, CERN/AC/95-05.

[2] S. Heifets, "Advanced Non-linear Theory", Physics of Particle Accelerators, AIP Conf. Proc. 184, New York 1989.

[3] R. Ostojic T.M. Taylor and G.A. Kirby, "Design and Construction of a One-Metre Model of the 70 mm Aperture Quadrupole for the LHC Low-β Insertions", Proc. 13th Int. Conf. on Magnet Technology (MT13), Victoria, Canada, September 1993, LHC Note 245.

[4] A. Faus-Golfe, A. Verdier, "Dynamic Aperture Limitations of the LHC in Physics Conditions due to Low-β Insertions", Proc. of this Conference.