ASSESSMENT OF THE ACHIEVABLE EMITTANCE RATIO IN DIAMOND

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Abstract

The performance of DIAMOND as a light source is determined by the vertical beam size, that is by the vertical emittance. The ε_y is determined by two factors: the vertical dispersion generated in the bending magnets by errors, and the coupling of the oscillation in the vertical and horizontal plane. In this paper we review the theory concerning the evaluation of the vertical emittance and the coupling. We give an estimate of the vertical emittance for DIAMOND and an assessment of the requirements of the correction system.

1 INTRODUCTION

In a perfect machine, the vertical emittance is determined by the synchrotron radiation opening angle, with a value that is several orders of magnitude inferior to the horizontal natural emittance.

However, in a real machine the vertical emittance is determined by two factors:

1.The finite vertical dispersion generated in the dipoles by horizontal dipolar field imperfections.

2.Coupling of the vertical and horizontal modes of oscillation, generated by skew quad error and vertical orbit displacement in the sextupoles.

In principle, these two processes are independent and uncorrelated, and the final ε_y would be given by the sum of the two contributions. We defined the *emittance ratio* χ as:

$$\chi = \frac{\varepsilon_y}{\varepsilon_x} \qquad \varepsilon_x = \frac{\varepsilon_{x0}}{1+\chi} \qquad \varepsilon_y = \frac{\chi\,\varepsilon_{x0}}{1+\chi} \tag{1}$$

It can be show that the performance of a light source is heavily influenced by the emittance ratio, and for small values of χ, the brilliance B_n is:

$$B_n = \frac{F_n}{V_{\text{phase space}}} \propto \frac{1}{\chi\,\varepsilon_{x0}^2} \tag{2}$$

In order to have a high brilliance, the emittance ratio χ, therefore the vertical emittance ε_y, should be as low as possible.

2 VERTICAL DISPERSION

The existence of imperfections on the magnetic lattice will induce a finite vertical dispersion along the lattice and change the values of the horizontal dispersion.

The emittance ratio would be given by:

$$\chi = \frac{\varepsilon_y}{\varepsilon_x} = \frac{J_x}{J_y}\frac{\langle \mathcal{H}_y \rangle}{\langle \mathcal{H}_x \rangle} \tag{3}$$

where $\langle \mathcal{H}_{x,y} \rangle$ is defined as:

$$\langle \mathcal{H} \rangle = \int_{bending} \left(\beta D'^2 + 2\alpha D D' + \gamma D^2\right) ds \tag{4}$$

It should be noted that both the vertical and the horizontal emittances are affected and modified by the effect of the errors. The correction system would compensate those errors, and the final relative change would be much smaller in the horizontal plane.

It is difficult to give an analytical formula for the emittance ratio in a machine with correction system. In [1] it is given a possible approach. However, an easier solution would be through simulation.

For a bare machine with dipolar errors, it is possible to give an approximate value of the vertical emittance, and the results would give us an estimate of the requirements for the correction system. Following the results from [2], the first step is to evaluate the vertical dispersion. The equation for it can be written, in normalised co-ordinates, as:

$$\frac{d^2\eta}{d\phi^2} + Q^2\eta = Q^2\beta^{3/2}F(\phi) \tag{5}$$

where we have drop the *y* subindex to simplify the notation. *F*(ϕ) is the driving function associated to the magnetic imperfections and η is the normalised dispersion:

$$\eta(\phi) = \frac{D(s)}{\sqrt{\beta}} \tag{6}$$

The previous equation has the following solution:

$$D(s) = \frac{\sqrt{\beta(s)}}{2\sin(\pi Q)} \int_s^{s+L} \sqrt{\beta(\sigma)}F(\sigma)\cos[\mu(\sigma) - \mu(s) - \pi Q]\,d\sigma \tag{7}$$

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The rms value at a point s_0 would be given by:

$$\left\langle D(s_0)^2 \right\rangle = \frac{\beta(s_0)}{4\sin^2(\pi Q)} \times \left[\int_s^{s+L} \sqrt{\beta(\sigma)} F(\sigma) \cos[\mu(\sigma) - \mu(s) - \pi Q] d\sigma \right]^2 \quad (8)$$

If we make the usual assumptions:

- The different errors are uncorrelated.
- The phase advance between errors is random-like, so we can change the $cos[\Delta\mu]$ for the average value 1/2.

We can change the integral for a sum over the different elements, this last equation can be written as:

$$\left\langle D(s_0)^2 \right\rangle = \frac{\beta(s_0)}{4\sin^2(\pi Q)} \frac{1}{2} \sum_j \beta_j F_j^2 L_j^2 \quad (9)$$

and from this last expression ,

$$\frac{\left\langle D^2 \right\rangle}{\beta} = \frac{1}{4\sin^2(\pi Q)} \frac{1}{2} \sum_j \beta_j F_j^2 L_j^2 \quad (10)$$

Using the definition of ε_y:

$$\varepsilon_y = \frac{C_q E^2}{2\pi\rho^2} \frac{1}{J_y} \int_{bending} \left(\beta D'^2 + 2\alpha D D' + \gamma D^2 \right) ds \approx \approx \frac{C_q E^2}{2\pi\rho^2} \frac{1}{J_y} \frac{\left\langle D_y^2 \right\rangle}{\beta_y} \Bigg|_{bending} \quad (11)$$

We can find now the contribution of each kind of errors to the vertical emittance. The generating function F for the different types of errors is [2]:

a) Dipole rotation error $\Delta\theta$, F= $\Delta\theta/\rho$.
b) Quad rotation error $\Delta\theta$, F= $K_1 D_x \Delta\theta$.
c) Vertical quad displacement Δz, F= $K_1 \Delta z$.
d) Vertical sextupole displacement Δz, F=$K_2 D_x \Delta z$.

where $\Delta\theta$ and Δz are the rms values of the errors.

The value of $\left\langle D^2 \right\rangle / \beta$ for each one of previous errors is:

a) Dipole rotation error:

$$\frac{\left\langle D^2 \right\rangle}{\beta} = \frac{1}{8\sin^2(\pi Q)} \sum_j \beta_j \left(\frac{1}{\rho} L_j \right)^2 \Delta\theta^2 \quad (12)$$

b) Quad rotation error:

$$\frac{\left\langle D^2 \right\rangle}{\beta} = \frac{1}{8\sin^2(\pi Q)} \sum_j \beta_j D_x^2 \left(K_{1,j} L_j \right)^2 \Delta\theta^2 \quad (13)$$

c) Vertical quad displacement:

$$\frac{\left\langle D^2 \right\rangle}{\beta} = \frac{1}{8\sin^2(\pi Q)} \sum_j \beta_j \left(K_{1,j} L_j \right)^2 \Delta z^2 \quad (14)$$

d) Vertical sextupole displacement:

$$\frac{\left\langle D^2 \right\rangle}{\beta} = \frac{1}{8\sin^2(\pi Q)} \sum_j \beta_j D_x^2 \left(K_{2,j} L_j \right)^2 \Delta z^2 \quad (15)$$

3 LINEAR COUPLING

The other important phenomenon that affects the emittance ratio is the linear coupling of the oscillations in the two transversal planes. Assuming that only the difference resonance Q_x-Q_y=n contributes to the coupling, the emittance ratio is given by:

$$\chi = \frac{\left| \kappa / \Delta \right|^2}{\left| \kappa / \Delta \right|^2 + 1/2} \quad (16)$$

where κ is the coupling coefficient

$$\kappa = \frac{1}{4\pi} \oint K_s(s) \sqrt{\beta_x \beta_y} \exp\left[i \left(\mu_x - \mu_y - s\Delta / R \right) \right] ds \quad (17)$$

and Δ is the fractional part of Q_x-Q_y. K_s is the skew quadrupole component around the ring. The two principal sources of it are rotation of the quads around the *s* axis and misalignments of the sextupoles.

Making similar assumptions than in the previous case, we can write $<\kappa^2>$ as

$$\left\langle \kappa^2 \right\rangle = \frac{1}{(4\pi)^2} \sum_j \beta_{x,j} \beta_{y,j} \left(K_{s,j} L_j \right)^2 \quad (18)$$

The two errors that would excite the resonance have the following skew quadrupole component [2]

a) Rotation of the quadrupoles around the *s* axis: $K_s=2K_1\Delta\theta$
b) Vertical orbit displacement in sextupoles: $K_s=K_2\Delta z$.

where again $\Delta\theta$ and Δz are the rms values of the errors.

4 RESULTS FOR DIAMOND

A description of the lattice for DIAMOND and of the correction system can be found in [3]. Figure 1 shows the optical functions for a quarter of the lattice. We assume the following values (rms) for the different types of errors:

a) Transverse misalignment of magnets Δx,y=0.1 mm
b) Roll angle misalignment of the dipoles $\Delta\theta=5\times10^{-4}$ rad
c) Field imperfections in the dipoles $\Delta B/B_0=5\times10^{-4}$
d) Roll angle misalignment of the quadrupoles (skew quad error) $\Delta\theta=5\times10^{-4}$ rad

4.1. Machine with the correction system

When we compensate the dipolar errors with the correction system, it is difficult to give an algebraic expression for the emittance ratio. We will use simulation programs (MAD [4] and COUPXY [5]), to simulate the effect of the errors and correction system. The correction system is assumed to correct the orbit up to 0.3 mm. The first contribution is due to the vertical

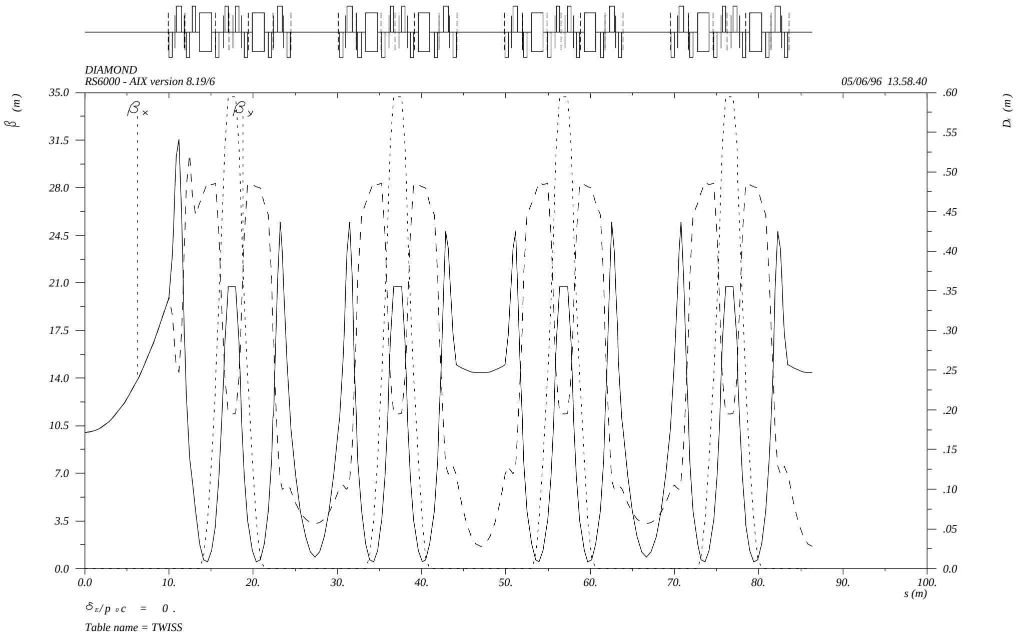

Figure 1: Optical functions for a quarter of DIAMOND.

dispersion. The rms value of the vertical dispersion in the dipoles when the correction system is in place has been found to be of the order of 0.025 cm. This value has been obtained through simulation with MAD for 100 sample machines. The values of the optical functions in the dipoles are:

$$\beta_x = 1.7 \text{ m} \qquad \beta_y = 27 \text{ m}$$

$$\langle D_x \rangle = 0.07 \text{ m} \qquad \langle D_y \rangle = 0.025 \text{ m}$$

And the contribution to the emittance ratio has been found of $\chi \sim 0.8$ %

For the linear coupling, we can give and analytical estimation. In the case of the coupling due to skew quadrupole errors, if we substitute the values of the optical functions and the strength of the quadrupoles, we found:

$$\langle \kappa^2 \rangle = 214.5\, \Delta\theta^2 = 5.4\times10^{-5}$$

$$\chi \sim 0.63 \%$$

In the case of the contribution to the coupling due to vertical orbit displacement on the sextupole, the closed orbit displacement has been found of 0.3 mm, and the coupling coefficient and the emittance ratio:

$$\langle \kappa^2 \rangle = 3575.1\, \Delta z^2 = 3.58\times10^{-4}$$

$$\chi \sim 0.42 \%$$

If we add this three contributions, the total value of the emittance ratio that we find is $\chi \sim 2$ %

4.2. Simulation

We have used MAD and COUPXY to evaluate the effect of the errors and of the correction system on the machine and to calculate the emittance ratio. We have simulate the effect of the errors for 100 sample machines.

The results for the emittance ratio found are:

$$\chi\ (\text{MAD}) \sim 2.8 \%$$

$$\chi\ (\text{COUPXY}) \sim 2.7 \%$$

5 CONCLUSIONS

The present correction system brings the emittance ratio to values of the order of the 3%. This can be improved with a more aggressive correction system and a less pessimistic estimation of the errors of position and angle of the magnets. It should be possible to reach values near the 1% coupling, as achieved in other sources.

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