FRINGING FIELDS IN LOW-BETA MAGNETIC ELEMENTS

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Abstract

The effect of the fringing fields in the low beta regions of DAΦNE are investigated. Due to the crossing angle the beam trajectory passes off axis in low beta quadrupoles and detector solenoids. The modification of the linear optics due to the magnetic field profile and to the linear expansion around the trajectory of the field components is considered. The non linear fringing field multipolar expansion is deduced from the longitudinal behaviour of quadrupole field gradients and solenoidal longitudinal magnetic field. Its effects on the beam dynamics are studied.

1 INTRODUCTION

DAΦNE, the Frascati Φ-factory [1], is a double ring e^+-e^- collider. The two rings share two Interaction Regions (IRs) where the opposite beams travel off-axis and cross at the Interaction Point (IP) at a tunable horizontal angle of θ_{cross} = ±10-15 mrad. Three experiments, DEAR [2], KLOE [3] and FI.NU.DA. [4], will be installed at different times in the IRs. While the DEAR experiment is transparent from the optics machine point of view, the presence of solenoids in the other two experiments is a strong lattice perturbation and the corresponding IR lattice designs [5] are determined by each detector characteristics.

The first approach to the IR linear optics was done with the usual rectangular model for solenoids and quadrupoles. To improve the model we use a very powerful and already known property: the knowledge of the fundamental functions on the axis of a magnet, i.e. the gradient for a quadrupole and the longitudinal field for a solenoid, is enough to deduce the off axis field components at all higher order terms. This global approach embraces what is usually referred to as the 'fringing field' problem, from the point of view either of the linear optics and of the non linear effects.

Using largely the theory developed in a recent paper [6], analytical functions, deduced from cylindrical current models, have been used to fit the longitudinal behaviour of the fundamental functions and analytical descriptions of the fields satisfying Maxwell equations inside the cylinder at any order are therefore available.

The magnetic field in the IRs is a superposition of quadrupolar and solenoidal field. The Rotating Frame Method (R.F.M.) [7] is applied to compensate the coupling and to produce uncoupled beams at the IP and outside the IRs: the quadrupoles immersed in the solenoid are tilted following the rotation of the transverse plane introduced by the longitudinal magnetic field. The integral of the longitudinal component of the magnetic field is cancelled by two superconducting solenoids placed in the IRs outside the detector. The residual coupling due to the fact that the quadrupole tilts do not follow continuously the longitudinal magnetic field component integral is corrected by using four parameters per each side around the IP: three additional quadrupole tilts of the order of few mrad and a correction on the field integral of the compensator.

2 POTENTIAL AND FIELD REPRESENTATION

We describe the IR magnetic fields deducing them from the expressions of the scalar potential of each magnetic element, and adding the contribution of the different element acting on each point. The scalar potential of a solenoid in cylindrical coordinates is:

$$P_0(r,z) = \left\{ G_{00}(z) + G_{02}(z) r^2 + G_{04}(z) r^4 + \ldots \right\}$$

G_{00} is the integral of the longitudinal component on the axis and the functions G_{02k} are related to it by:

$$G_{02k}(z) = (-1)^k \frac{1}{4^k (k!)^2} \frac{d^{2k} G_{00}(z)}{dz^{2k}}$$

The scalar potential of a quadrupole is:

$$P_2(r,\theta,z) = \frac{r^2 \sin 2\theta}{2} \left\{ G_{20}(z) + G_{22}(z) r^2 + \ldots \right\}$$

while the one corresponding to a quadrupole tilted by an angle θ_T is:

$$P_2(r,\theta,z) = \frac{r^2 \sin 2(\theta+\theta_T)}{2} \left\{ G_{20}(z) + G_{22}(z) r^2 + \ldots \right\}$$

G_{20} is the gradient on the axis and G_{22k} are given by:

$$G_{22k}(z) = (-1)^k \frac{2}{4^k k!(2+k)!} \frac{d^{2k} G_{20}(z)}{dz^{2k}}$$

The fundamental functions $B_z(0,z) = \frac{dG_{00}(z)}{dz}$ and $G_{20}(z)$ describe the fringing field behaviour at lower order. Functions $G_{n2k}(z), m > 0$ describe the higher order fringing fields. B_z and G_{20} are deduced from magnetic code calculations in the design stage of the magnet and from measurements on the real magnet.

3 ON-AXIS LINEAR OPTICS

For every IR magnetic component we have fitted the available measures or the magnetic computations with analytical and differentiable expressions, which can be found in [6]. Fig. 1 shows as an example the superconducting compensator measured field and the corresponding fitting function, which in this case was originated by two superimposed cylindrical currents differing in radius, length and intensity.

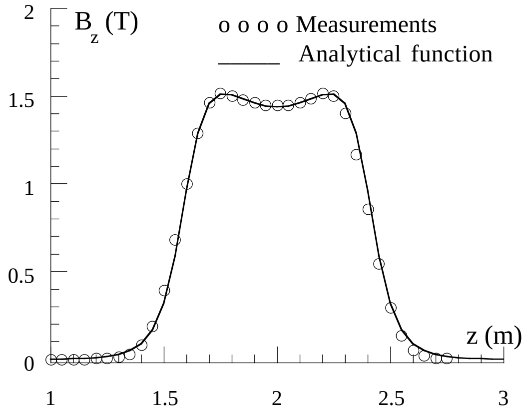

Figure 1 : B_z on the axis of the compensator solenoid.

Figure 2 shows the fundamental functions of quadrupoles and solenoids along half IR for the design corresponding to KLOE experiment.

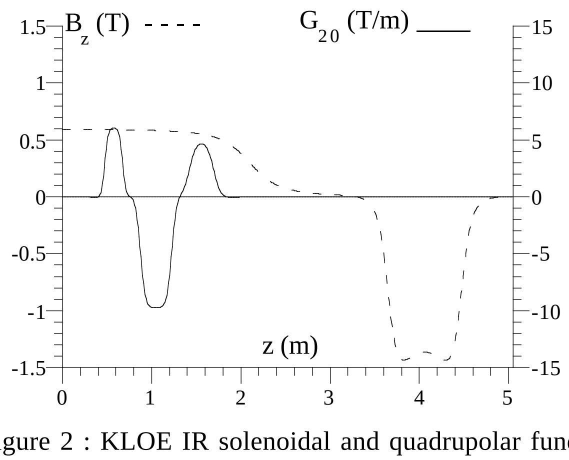

Figure 2 : KLOE IR solenoidal and quadrupolar fundamental functions.

As a first step to improve the rectangular model, the 'sliced' model [8] has been applied, which consists in computing the total IR transport matrix as the product of N linear rectangular model transport matrices, whose characteristic parameters follow the quadrupole and solenoid fundamental functions.

An analysis on the DAΦNE different types of quadrupoles and solenoids has been done separately on each element, before using the method on the whole transport matrix. For quick optical calculations, as for example control system codes, the transport matrix obtained with the sliced model can be still written on each plane as a quadrupole matrix, with the two characteristic parameters different in the two planes, their difference increasing with the fringing extension and with the gradient. In the focused plane the quadrupole strength decreases if compared to the rectangular model strength with the same integrated gradient; in the defocused one it increases.

The solenoid rectangular model transport matrix can be written as the product of a rotation matrix and of a focusing in both planes quadrupole matrix. The rotation matrix corresponding to the sliced model is equal to the rectangular model one, while the focusing matrix corresponds to a weaker focusing strength.

The differences between the rectangular and the sliced model are in general negligible in the overall optics of a ring, and can be summarized as a negative tune shift. They become relevant when quadrupoles with large aperture and strong gradient are placed in high beta position, as is the case of DAΦNE low beta quadrupoles. In KLOE and FI.NU.DA the quadrupoles are permanent magnet ones, and their specifications have been defined by applying the sliced model to the preliminary magnetic calculations. In fact by applying the sliced model to the DAΦNE optics we have found tune shifts of - 0.01 in both planes due to the quadrupoles outside the IRs, while the differences in tunes when the IR quads are included increase up to (-0.03,-0.07).

4 OFF-AXIS LINEAR OPTICS

Beams travel off-axis in the IRs. The nominal trajectory has been computed integrating the equations of motion from the IP to the IR end. The linear jacobian around the trajectory has been successively computed. Let's remind that if a magnetic vector potential **A** is present, the generalized canonical variables associated to x, y are $P_{x,y} = p_{x,y} + eA_{x,y}$, instead of the variables $p_{x,y}$, normally used in optics calculations. We have integrated the motion in the usual variables and transformed them to the generalized ones knowing the vector potential at the initial and final point of our system. At the IP, which corresponds to the center of the detector solenoid, the magnetic vector potential is [9]:

$$A_x = -\frac{1}{2} y B_z$$

$$A_y = \frac{1}{2} x B_z$$

while it is of course negligible at the IR end. We checked that the jacobian computed as the transformation between the generalized canonical variables is simplectic.

As expected, there is a very good agreement between the linear IR transport matrix computed with the 'sliced' model and the jacobian computed around the axis.

Information about the phase advance and the optical functions at the IR end are deduced from the jacobian. The jacobians computed at trajectories crossing at different angles are slightly different: for the nominal optical parameters at the IP (β_x = 4.5 m, β_y = 4.5 cm), as θ_{cross} increases the phase advance along the IR increases, especially in the vertical plane. In fact around the off axis trajectory the quadrupoles add an alternate bending action, like a wiggler, which, as it is well know, gives a vertical focusing. In the presence of solenoids this focusing acts in the direction perpendicular to the trajectory plane point by point. At the IR end, where the normal modes become horizontal and vertical because of the RFM method, the increase in phase advance appears in the vertical plane.

There is also another change in the off-axis linear optics due to the pseudo-octupole (G_{22}) appearing in the quadrupole fringing, which expanded around the off-axis trajectory modifies the linear quadrupole gradient. While the integral of G_{22} on the axis is zero, it is non null when integrated on the off-axis trajectory. Its influence on the IR optics is however much smaller than the previous effect.

The dependence of the machine tune on the crossing angle is shown in Fig. 3, for the DAΦNE configuration corresponding to KLOE + DEAR experiments.

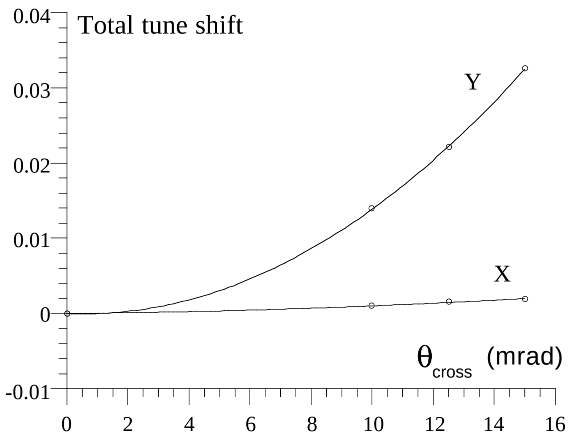

Figure 3 : Tune shift with crossing angle due to linear optics modification.

The R.F.M. had been applied up to now only on axis, and in fact if only linear terms of the field are included, the on-axis or off-axis methods are equivalent.

The four parameters which are used for decoupling the motion between the IP and the IR end, i.e. the three quadrupole tilting angles and the integral of the longitudinal magnetic field on the compensator solenoid depend in principle on the crossing angle.

Using the linear terms of the jacobian computed around the trajectory, it is possible to readjust the values of the decoupling parameters by minimizing the non diagonal elements of the jacobian, i.e.:

$$\frac{\partial x}{\partial y}, \frac{\partial x}{\partial P_y}, \frac{\partial P_x}{\partial y}, \frac{\partial P_x}{\partial P_y}, \frac{\partial y}{\partial x}, \frac{\partial y}{\partial P_x}, \frac{\partial P_y}{\partial x}, \frac{\partial P_y}{\partial P_x}$$

Since the jacobian is simplectic, vanishing four elements implies the vanishing of the remnant ones.

The differences between the decoupling parameters with θ_{cross} in the nominal range have resulted smaller than the alignment tolerances, which are set to 0.1°.

5 CONCLUSIONS

The fringing field effect due to the IR quadrupoles and solenoids on the linear optics has been investigated with the 'sliced' model.

The main trajectory in presence of all significant higher order terms and the jacobian around it has been computed. The dependence of the RFM parameters on the crossing angle has been studied, and shows to be negligible if compared with the coupling tolerances requested.

Presently multiturn tracking is being performed by integrating particle motion along the IR and applying the linear matrix from the IR end to its beginning, matched to the values obtained from the jacobian computation. First results show the appearance of resonances due to the non-linear fringing terms, which give rise to emittance exchange between the two transverse modes, with a strength which depends of course on the tune values. When far from the resonances the coupling parameter is kept under the design values (κ=1%). To understand deeply the real extent of the phenomena more investigation is needed.

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