FRINGING FIELD EFFECTS OF MAGNETS IN CYCLIC ACCELERATORS

S.Efimov, KFTI, Kharkov, Ukraine

Abstract

Expressions for the tune shift of vertical betatron oscillations versus their amplitude are obtained by using Hamilton mechanics methods. Conditions of resonance excitation are considered. Expressions to compute the fringing field effects in the codes simulating beam dynamics, and the appropriate coefficients for use in practice are given. The fringing field effect on a dynamic aperture is illustrated by the example of the synchrotron radiation source ISI-800 designed in Kharkov.

1 INTRODUCTION

In calculations of accelerator magnet lattices it is general practice to take into account only effective lengths of magnet elements and the linear tune shift due to fringing field effects of dipole magnets. However, in low- and medium- energy (hundreds of MeV for electrons) accelerators higher-order edge effects may appear essential and give rise to an increase in effective beam emittance (luminosity), a decrease in the dynamic aperture of the device (lifetime); they restrict the application of inserts such as wigglers and undulators in storage rings as synchrotron radiation sources.

2 CALCULATION OF DIPOLE-MAGNET AND MULTIPOLE-LENS EDGE EFFECTS

2.1 The dipole magnet

The Hamiltonian of perturbed motion can be written in the form [1]:

$$H_1 = -\frac{R^2 A_\vartheta}{c|B\rho|} - \frac{R}{c|B\rho|}\left(p_x A_x + p_z A_z\right), \qquad (1)$$

where R is the average radius of machine; $B\rho$ is the magnetic rigidity; $A_{x,z,\vartheta}$ are the perturbing magnetic vector potential components; $p_{x,z}$ are the transverse momenta.

The perturbation is here represented by the fringing (edge) field. Because of beam trajectory distortion at the dipole magnet edges (fig.1) the fringing field is described not by one potential component but by two components which are written in the beam co-moving coordinate frame as [2]

$$\begin{cases} A_x{}' = B_0 \sum\limits_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!\,R^{2k-1}}\, b_0^{(2k-1)}(\vartheta) z^{2k}; \\ A_\vartheta{}' = B_0 \alpha(\vartheta) \sum\limits_{k} \frac{(-1)^{k}}{(2k)!\,R^{2k-1}}\, b_0^{(2k-1)}(\vartheta) z^{2k}, \end{cases} \qquad (2)$$

where B_0 is the field in the magnet gap;

$$\alpha(\vartheta) = \frac{B_0}{B\rho}\int_{\vartheta}^{2\pi} b_0(\vartheta)\, d\vartheta + \alpha_0;$$

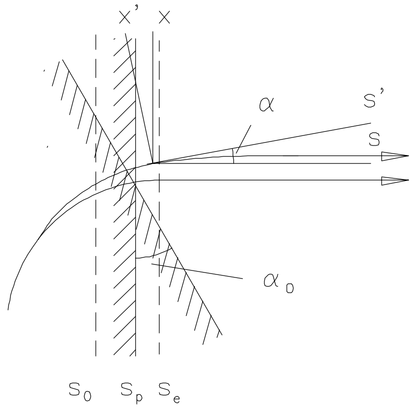

Fig.1. Equilibrium particle trajectories near the outer magnet edge. s_0 is the calculated origin of the fringing field, s_p is the mechanical boundary of the pole, s_e is the effective boundary of the field.

After substituting (2) into (1), expanding in azimuthal harmonics, averaging over quickly oscillating variables, separating the particle amplitude and phase ($z = a_z V e^{i\nu_z\vartheta} + c.c.$, V being the vertical Floquet function), Hamiltonian (1) can be presented to consist of two parts: the resonant part that comprises the azimuthal angular dependence, and the stabilizing part where this dependence is absent. The analysis of the resonant part shows that the fringing fields of the dipole magnet can generate resonances of types $(2j-1)\nu_z = p$ and $(2j-1)\nu_z + \nu_x = p$, (j and p are the integers). From the stabilizing part we derive the expression for the vertical tune shift:

$$\nu_z = \nu_{zo} + \frac{R^2 B_0}{B\rho}\sum_{k,j}\frac{(-1)^{k+1}}{\left(2(k-j)\right)!\, j!\,(j-1)!}\, r_z^{2(j-1)} z_0^{2(k-j)} J_{kj} \qquad (3)$$

where ν_{x0}, ν_{z0} is the working betatron frequency of the machine;

$$J_{kj} = \frac{1}{2\pi R^{2k-1}}\sum_m \left|\frac{\beta_z(\vartheta_m)}{2R}\right|^j \int_0^{2\pi} b_0^{(2k-1)}(\vartheta)\,\alpha(\vartheta)\, d\vartheta$$

$\beta_z(\vartheta_m)$ is the amplitude function of vertical motion at the m-th edge of magnet ($|V|=\sqrt{\beta_z/2R}$); z_0 is the vertical displacement of orbit; $r_z = a_z e^{i\varphi}$.

The summation is made over all 2M edges of M magnets. It is seen from expression (3) that the effect of fringing fields can be reduced with the use of the edge cutoff angle.

In practice, it is generally sufficient to be restricted to the efficient cubic nonlinearity in field:

$$\Delta\nu_{z1} = \frac{B_0^2}{4\pi B^2\rho^2} k_1 \sum_m \beta_m \text{ ; (linear shift)} \quad (4)$$

$$\Delta\nu_{z2} = -\frac{B_0^2 R\beta_{max}}{16\pi B^2\rho^2} k_2 \sum_m \frac{z_m^2}{\beta_m} \text{ , (nonlinear shift)}$$

where $k_1 = \int_0^\infty b_0'(s)\int_s^\infty b_0(s)dsds$,

$$k_2 = \int_0^\infty b_0^{(3)}(s)\int_s^\infty b_0(s)dsds .$$

In the thin-lens approximation, expressions (4) are equivalented the form:

$$\begin{pmatrix} z \\ z' \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 \\ \frac{1}{\rho^2}\left(k_1 - \frac{k_2}{6}z^2\right) & 1 \end{pmatrix} \begin{pmatrix} z \\ z' \end{pmatrix}_0 \quad (5)$$

The coefficients $k_{1,2}$ can be calculated through the use of the function $b_0(s) = 1/1+\exp(f(s))$, ($f(s) = a + bs + cs^2 + \ldots$)[3] written for the outer edge of the magnet. Figure 2 shows these coefficients as functions of the gap g for the two-dimensional model of the field. It is evident from the figure that with an increasing length of the dipole-magnet fringing field the linear vertical tune shift increases, while the nonlinear one decreases.

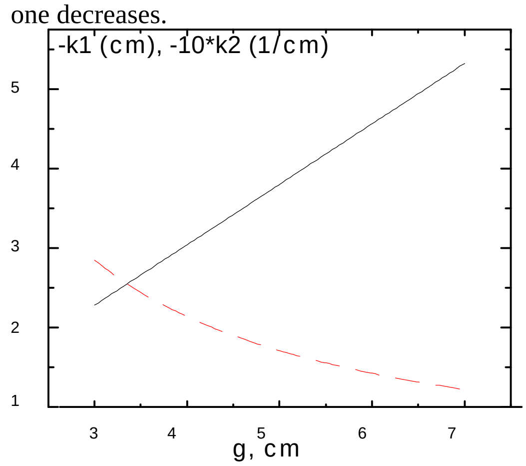

Fig.2.Edge coefficients versus dipole magnet gap. k_1 - solid line, k_2 - dashed line.

2.2 The multipole lens

The magnetic potential components of multipole lenses can be written in the form [4]:

$$\begin{cases} A_x = i\sum_{n=M-1}^{\infty} \frac{nx}{M(n+1)!} b_n'(s)F_n(x,z); \\ A_z = i\sum_n^{\infty} \frac{nz}{M(n+1)!} b_n'(s)F_n(x,z); \\ A_s = -i\sum_n^{\infty} \frac{1}{M(n-1)!} b_n(s)F_n(x,z), \end{cases} \quad (6)$$

with $F_n(x,z) = \left(x^2+z^2\right)^{\frac{n+1-M}{2}}(x+iz)^M$.

Here M is the number of lens pole pairs, n is the multipole order. The imaginary part of expressions (6) corresponds to normal lenses, the real part refers to skew lenses.

After substituting (6) into (1) and appropriate manipulations it becomes evident from the stabilizing part of the Hamiltonian that the edge fields of quadrupole lenses lead to square amplitude dependence of betatron frequency. The betatron amplitude value is given by difference between the products to the 3rd power of the modulus of the Floquet function by its derivatives with respect to inner- and outer- edge azimuths. The analysis of the resonant part of the Hamiltonian shows that the edge fields of quadrupole lenses can excite resonances of type $2n\nu_{x,z} = k$, $2n(\pm\nu_x \pm\nu_z) = k$. The edge fields of sextupole lenses can excite resonances of types $(2n+1)\nu_x = k$, $(2n+1)\nu_x + 2n\nu_z = k$, $n_1\nu_{x,z} + n_2\nu_{x,z} = k$, $n\nu_{x,z} = k$. The stabilizing part of the perturbation Hamiltonian is identically equal to zero for sextupole lenses.

2.3 Plane Insertion Devices (ID)

In the infinitely wide-lens approximation with a cosine variation of the vertical field component along the azimuth, we obtain the expression to describe the tune shift under the action of plane ID fringing fields that manifests itself to an accuracy of the squared amplitude as:

$$\delta\nu_z = \frac{\beta_z L_u B_0^2}{8\pi B^2\rho^2}\left(1 + \frac{k_u^2 a_z^2 \beta_z}{4R}\right), \quad (7)$$

where L_u is the ID length; $k_u = 2\pi/\lambda$, (λ is the period); a_z is the oscillation amplitude.

The corresponding expression to calculate the effects considered in computer programs simulating the beam dynamics is of the form

$$\begin{pmatrix} z \\ z' \end{pmatrix}_1 = \begin{pmatrix} 1 & 0 \\ \frac{K}{k_u \rho^2}\left(1+k_u^2 z^2\right) & 1 \end{pmatrix} \begin{pmatrix} z \\ z' \end{pmatrix}_0 , \qquad (8)$$

where $K = 1 - \pi / 4 - \rho\alpha_0 k_u$, so, in the case of sector magnets ($\alpha_0 = 0$) we have $K = 1 - \pi / 4$, and for rectangular magnets ($\alpha_0 = 1 / k_u \rho$) $K = -\pi / 4$.

For a fixed beam energy value the effect of ID fringing fields can be compensated by means of edge cutoff angles at each of the magnet units.

It has been demonstrated in [5] that the fringing fields of dipole magnets and wigglers appreciably reduce the dynamic aperture of the storage ring ISI-800 being designed [6] in a vertical direction, yet it remains larger geometric in this case.

3 EXPERIMENT

In the Kharkov storage ring N-100 the betatron frequency of the beam was measured as a function of the oscillation amplitude. The measurements were carried out at a beam energy of 100 MeV and a stored current of 1μA.

Fig. 3 shows the measured and calculated squared amplitude dependences of the vertical betatron frequency.

Experiments were also made to investigate the fringing field effect of N-100 dipole magnets on the process of slow beam extraction in a vertical direction at resonance ν_z=2/3. The specific feature of these experiments is that the electrostatic septum was located not at the maximum of the vertical amplitude function, which is in the azimuth of the centre of the magnet. The computer simulation of the extraction process has shown that without considering the fringing field effect nearly all the beam would be lost on vacuum chamber walls at the maximum of the amplitude function. At the same time, if the stabilizing effect of fringing fields is taken into account, the slow extraction efficiency is computed to be close to the experimental one [7].

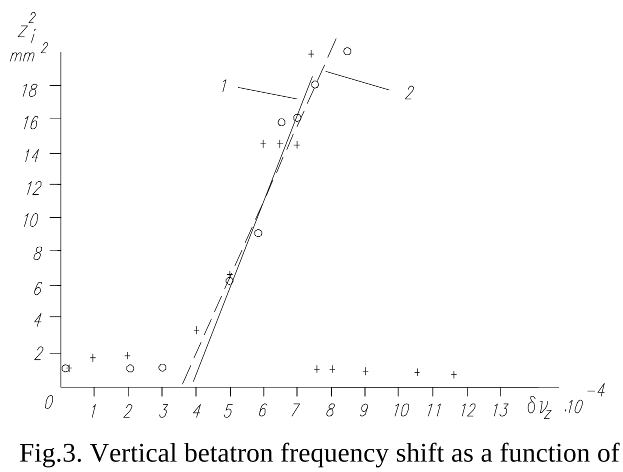

Fig.3. Vertical betatron frequency shift as a function of squared amplitude. The circles (crosses) show the experimental points at intersection of resonance on the side of lower (higher) frequencies. 1.- calculation by expression (5), 2.- experimental results.

REFERENCES

[1]. G. Guignard, *A general treatment of resonances in accelerators*, CERN 78-11, 1978.

[2]. E.V.Bulyak, S.V.Efimov, *Nonlinear effects due to fringe fields of cyclic accelerator dipoles*, Proc. EPAC-90.

[3]. S.Kowalski, H.A.Enge. *RAYTRACE* , Cambridge, Massachusetts 02139, USA, 1987, 73p.

[4]. E.V.Bulyak, S.V.Efimov, *Fringing field effects of multipole magnet lenses on transverse motion of charged particles in cyclic accelerators* (in Russian), Zh.Tekh.Fiz., 57, n.7, 1987, p.p.1324-1327.

[5] S.Efimov, I.Karnaukhov, S.Kononenko, et al., *The dynamical aperture of ISI-800*. Proc. PAC-95, Dallas, 1995.

[6] V.Androsov, V.Bar'yakhtar, E.Bulyak, et al., *Synchrotron Radiation Complex ISI-800*. Journal of Electron Spectroscopy and Related Phenomena, 68, 1994, p.747-755.

[7] E.V.Bulyak, P.I.Gladkikh, S.V.Efimov, et al., *Experiments on continuous electron beam extraction from the storage ring N-100* (in Russian), Trudy IX Vsesoyuznogo soveshchaniya po uskoritelyam (Dubna, 1984), v.2, p.p.250-253.