

NON-LINEAR TUNING AND HALO TRANSPORT IN BEAM EXPANDERS

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1 INTRODUCTION

Beam expanders are optical assemblies which provide a transverse expansion of a presumably high intensity particle beam, in order to obtain a large size footprint at an irradiation target. Two main concerns in these installations are the obtainment of a uniform irradiation over the all surface of the extended target, and the minimization of the irradiation by particle loss along the beam line.

Following pioneering work [1-3], it is now common to consider employing non-linear lenses to achieve the transverse uniformization. In previous reports [4,5] we gave an original analytical treatment of the uniformization of transverse beam densities by non-linear lenses in terms of the transport of random variables and their probability density functions.

We now apply the method to the mastering of the halo transport and particle losses along the beam line, in an analytical way [6]. This in particular allows specifying the aperture of the chamber and optical elements upon the criterion of a tolerable loss.

The paper is divided as follows. In section 2 we recall the main aspects of the analytical material used. In section 3 we give the behaviour of the beam halo and show that the dodecapole and higher odd-order multipoles strongly modify the extent of the transverse distribution tails, and can thus be used to master the beam halo/chamber acceptance. In section 4, we provide the calculation of the non-linear beam envelopes downstream the uniformization lens. In section 5, we show an application in the realistic optical structure of a two-dimensional beam expander.

2 NON-LINEAR TRANSPORT

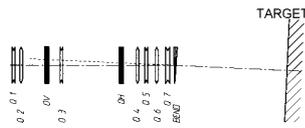


Figure 1.

We refer here to the analytical material developed in refs. [4,5]. The developments that follow are based on two main hypothesis. 1) The transverse motions are supposed to be fully uncoupled. 2) The particle distributions in both transverse phase-spaces are supposed to be flat at the non-linear lens upstream end.

2.1 The non-linear transport

Considering the first hypothesis above, the problem is a 2-dimensional one. In the sequel, the transverse phase space will be denoted (y,t) , with $t = dy/ds$ and s is the longitudinal coordinate. Referring to fig. 1 that schemes the optical design of a beam expander (Q1-Q7 are quadrupoles, OV and OH are respectively the vertical and horizontal non-linear lenses, BEND is a final shielding bend magnet [6]), the first order

transport downstream the non linear lens (either OV or OH, for respectively the vertical or horizontal motion) writes

$$\begin{pmatrix} y(s) \\ t(s) \end{pmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{pmatrix} y_2 \\ t_2 \end{pmatrix} \quad (1)$$

where (y_2, t_2) [respectively $(y(s), t(s))$] are the phase space coordinates at the right end of the lens (respectively at s downstream the lens) and the R_{ij} are the transfer coefficients.

The lens adds the non-linearities of concern in beam uniformization : the octupole component which determines the size of the footprint at target and the bulk of the uniformization, and the higher odd-order non-linearities which mostly amount to improved uniformization [1-3], and transverse tail extent [7]. A thin lens model is considered, with integrated strengths $K_{2p+1}L$ ($p=1$: octupole, $p=2$: dodecapole, etc...), leading to the following kick at the lens

$$\begin{pmatrix} y_2 \\ t_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ t_1 - \sum_{p=1}^n K_{2p+1} L y_1^{2p+1} \end{pmatrix} \quad (2)$$

Now, let the beam ellipse frontier be

$$\gamma_1 y_1^2 + 2\alpha_1 y_1 t_1 + \beta_1 t_1^2 = \varepsilon / \pi \quad (3)$$

where β_1 , α_1 , and $\gamma_1 = (1 + \alpha_1^2) / \beta_1$ are the optical functions at the lens, ε is the surface of the ellipse. Following the second hypothesis above, the beam ellipse is flat, so that in terms of statistical variables, its population is amenable to the linear regression $y_1 = r_1 t_1$ with $r_1 = -\beta_1 / \alpha_1$. Thus, from eqs. (1,2) the position random variable at s is given by the polynomial (schemed in fig. 2)

$$y(s) = \sum_{p=0}^n \lambda_{2p+1} t_1^{2p+1} \quad (4)$$

with $\lambda_1 = r_1 R_{11} + R_{12}$, and $\lambda_{2p+1} = -R_{12} K_{2p+1} L r_1^{2p+1}$.

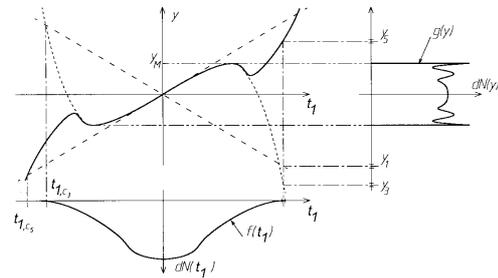


Figure 2.

2.2 Transverse beam density

Now, it can be demonstrated [4] that, given the probability density function $f(t_1)$ of the initial angle t_1 (e.g., bell-shaped as schemed in fig. 2), the probability density function of the random variable $y(s)$, which represents the transverse beam density, writes

$$g[y(s)] = \sum_{i=1}^N f[t_{1,i}(y)] / \left| \sum_{p=0}^n (2p+1)\lambda_{2p+1}t_{1,i}^{2p}(y) \right| \quad (5)$$

where the $t_{1,i}$ ($i=1,N$) are the N real roots of eq. (4).

2.3 Non-linear optical tuning

The multipole strengths for adequate tuning of the beam footprint at target are obtained from the expressions above, as follows. The footprint size with the octupole alone is (fig. 2)

$$y_M \equiv y_{M,3} = \frac{2}{3}\lambda_1[-\lambda_1/3\lambda_3]^{1/2} \quad (6)$$

(considering that $\lambda_1\lambda_3 < 0$), or in the case of a combined octupole + dodecapole non-linearity (with $\lambda_1\lambda_5 > 0$)

$$y_M \equiv y_{M,5} = \lambda_1 t_M + \lambda_3 t_M^3 + \lambda_5 t_M^5 \quad \text{with}$$

$$t_M^2 = \frac{-3\lambda_3}{10\lambda_5} \left(1 - \sqrt{1 - 20\lambda_1\lambda_5/9\lambda_3^2} \right) \quad (7)$$

with presumably $y_{M,3} \approx y_{M,5}$ since the dodecapole is supposed to simply provide a fine tuning of $g(y)$, and negligible effect of eventual higher order multipole components.

From these and given the footprint size y_M at the target, the tuning of the multipole components comes out, namely, by introducing α , β = optical functions at the target, φ = betatron phase advance from the lens to the target [5],

- first order tuning : it determines the β value at target or equivalently λ_1 , and is provided by $\sigma_y \approx 3y_M/4$,

- octupole :

$$K_3L = \frac{4}{27} \frac{1}{y_M^2} \frac{\beta \cos^3 \varphi}{\beta_1^2 \sin \varphi} \quad (8)$$

- dodecapole : an adequate starting value of λ_5 for further fine smoothing of the transverse density is $\lambda_5 = \lambda_3^2/4\lambda_1$, thus giving

$$K_5L = -\frac{(K_3L)^2}{4} \frac{\beta_1}{\alpha_1} t_{g(\varphi)} \quad (9)$$

3 HALO CONFINEMENT

It can be observed (fig. 2) [7] that exists a critical value $t_{1,c}$ of the starting angle defined in the case of the octupole alone by $t_{1,c} \equiv t_{1,c3}$ such that $-\lambda_1 t_{1,c3} = \lambda_1 t_{1,c3} + \lambda_3 t_{1,c3}^3$ that is, considering that $\lambda_1\lambda_3 < 0$

$$t_{1,c3} = \sqrt{-2\lambda_1/\lambda_3} \quad (10)$$

or, in the case octupole + dodecapole, by $t_{1,c} \equiv t_{1,c5}$ such

that $\lambda_1 t_{1,c5} = \lambda_1 t_{1,c5} + \lambda_3 t_{1,c5}^3 + \lambda_5 t_{1,c5}^5$ that is, with $\lambda_1\lambda_5 < 0$

$$t_{1,c5} = \sqrt{-\lambda_3/\lambda_5} \quad (11)$$

This critical angle $t_{1,c}$ defines two regions in the incident density $f(t_1)$: under the effect of the octupole alone, any particle with incidence $t_1 < t_{1,c3}$ (a contrario, $t_1 > t_{1,c3}$) is

confined within the region $y < y_{c3} = |\lambda_1| \sqrt{(-2\lambda_1/\lambda_3)}$ in the neighbouring of the optical axis (a contrario, deconfined) ;

under the effect of the combination octupole + dodecapole, any particle with incidence $t_1 < t_{1,c5}$ (a contrario, $t_1 > t_{1,c5}$)

is confined within the region $y < y_{c5} = |\lambda_1| \sqrt{(-\lambda_3/\lambda_5)}$ (a

contrario, deconfined). This shows in particular that an adequate dodecapole component is likely to increase the vacuum chamber acceptance, or equivalently to permit the reduction of the transverse design aperture of the optical elements along the beam line.

The fig. 3 represents this effect at the quadrupole Q4 (fig. 1) in the mixed phase space (y, t_1) for a population sorted in the range $[6\sigma_{t_1}, 7\sigma_{t_1}]$ of an incident gaussian distribution $f(t_1)$ (relative population $< 10^{-8}$). The deconfinement by the octupole (curves 3) can be observed : the particles show transverse excursions larger than in the linear case (curves 1), while the combined octupole + dodecapole lens induces a confinement (curves 5) : the particles show transverse excursions smaller than in the linear case.

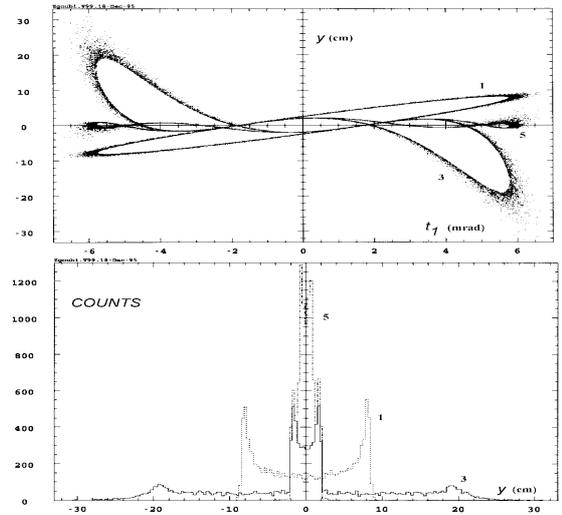


Figure 3.

4 NON-LINEAR ENVELOPES

Beam envelopes are normally obtained by the transport of the optical functions $\alpha(s)$, $\beta(s)$ and $\gamma(s)$. Assuming zero dispersion, the r.m.s. beam size (so-called « σ -envelope ») is given along the line by $\sigma(s) = \sqrt{\beta(s)/\epsilon/\pi}$. The transverse apertures (chamber, pole tip radii, etc...) are thus defined as the $n\sigma$ -envelope, with usually n = a few units depending on the loss tolerance. The parameter of concern is actually the loss ratio, assumed to be the population beyond the $n\sigma$ width of a gaussian distribution, that is,

$$\tau = 1 - erf(n\sigma) \quad (12)$$

In the case of beam expanders, the non-linearities introduced by the uniformization lens are so strong that it is no longer relevant to address τ in terms of the $n\sigma$ envelope, because the transverse density $g[y(s)]$ (eq. 5) is much too far from gaussian, as observed in fig. 2. In fact, given the tolerable loss ratio τ (of the order of 10^{-7} - 10^{-9} in high energy, high intensity LINAC installations), the non-linear envelope $Y(s)$ is given by

$$\int_0^{Y(s)} g(y) dy = \int_0^{Y(s)} \sum_{i=1}^N \frac{f[t_{1,i}(y)]}{\left| \sum_{p=0}^n (2p+1)\lambda_{2p+1}t_{1,i}^{2p}(y) \right|} dy = 1 - \tau \quad (13)$$

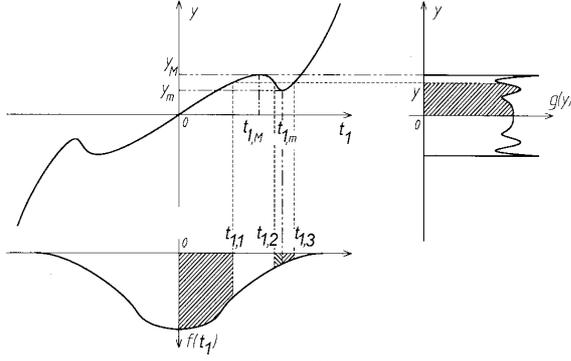


Figure 4.

The fig. 4 gives a geometrical interpretation of this integral : $\int g(y)dy$ builds up from contributions of the N roots of eq. (4): the shaded area under $g(y)$ is the sum of the three separate shaded areas under $f(t_j)$. For instance, if $f(t_j)$ is gaussian and centered, the integral to solve for $Y(s)$ writes

$$\text{erf}[t_{1,1}(Y(s))] - \text{erf}[t_{1,2}(Y(s))] + \text{erf}[t_{1,3}(Y(s))] = 1 - \tau \quad (14)$$

5 EXAMPLE

The $\tau=10^{-7}$ non-linear envelopes in a structure as schemed in fig. 1 that provides a $0.45 \times 0.45 \text{ m}^2$ uniform footprint at target, have been computed by the method above [8], with the octupole alone (curves 3), and octupole + dodecapole (curves 5) [7]. They are displayed in fig. 5 together with the linear envelopes (curves 1) for comparison.

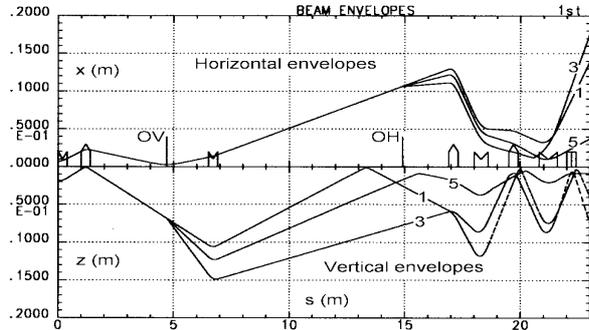


Figure 5.

On the other hand this analytical material has been validated by comparison with numerical stepwise ray-tracing of a population of 1000 particles through that expander [9]. In order to obtain the beam envelopes from stepwise ray-tracing, the 1000 particles are launched with their initial invariant lying lying in the range $\varepsilon / \pi \in [5.433^2 \varepsilon_o / \pi, 5.434^2 \varepsilon_o / \pi]$ ($\varepsilon_o / \pi = 610^{-7} \text{ m.rad}$) which materialises the $\tau=10^{-7}$ loss limit. Namely, the initial phase space coordinates at the left end of the structure of fig. 1 are correlated by

$$\gamma_o y_o^2 + 2\alpha_o y_o t_o + \beta_o t_o^2 = \varepsilon / \pi \quad (15)$$

It is clear from this that the non-linear beam envelope is given by the maximum excursion $Y(s)$ of the 1000-particle set along the beam line. The particle trajectories in the vertical plane are shown in fig. 6, with the solid lines of the analytical envelopes of fig. 5 superimposed, for comparison.

Note the strong differences in the transverse beam size, and the confinement generally induced by the octupole + dodecapole configuration, w.r.t. both the linear and octupole alone cases.

In terms of the transverse apertures of the optical elements along the beam line, it means that one can afford much smaller pole tip radii when using a combined octupole + dodecapole lens. In this respect, things may still be improved by an additive 7-th order multipole component, that would introduce a further distortion and a next transverse τ limit at y_7 , leading to $y_7 < y_3$ (see fig. 2). This might be of interest if even lower loss ratios were to be achieved.

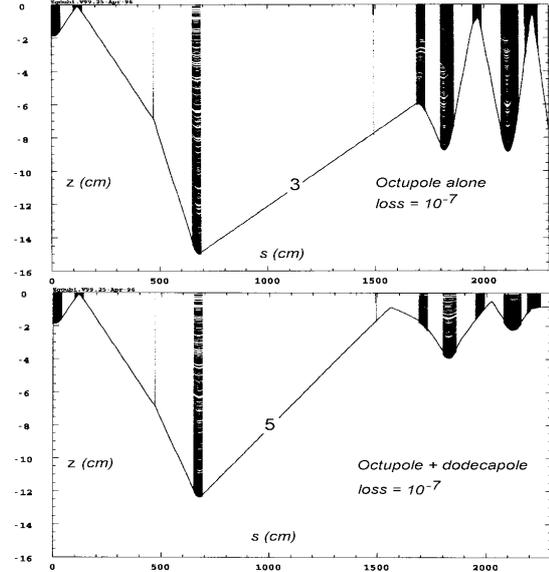


Figure 6.

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