

# COUPLING AND DISPERSION CORRECTION IN ELETTRA

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*Abstract*

The paper describes the importance of correction of spurious dispersion for the beam quality control in ELETTRA together with the ways how the correction is performed.

## 1. INTRODUCTION

Correction of spurious vertical dispersion is one of the most important operations for the beam quality control in ELETTRA which is running as a new generation low emittance light source [1]. While the presence of combined function dipoles allows a moderately focused optics with a furthermore reduced horizontal emittance, it is expected to enhance simultaneously the sensitivity of the spurious dispersion against orbit displacements particularly in the vertical plane in which the dipoles are focusing. As the phase relation among different sources is crucial in the evaluation of spurious dispersion, the rms's of the spurious vertical dispersion are evaluated in the simulation at BPM positions in Fig. 1 versus corresponding orbit rms for 60 different cases of ELETTRA by including the random misalignment of quadrupoles. It is seen that the rms of the dispersion may

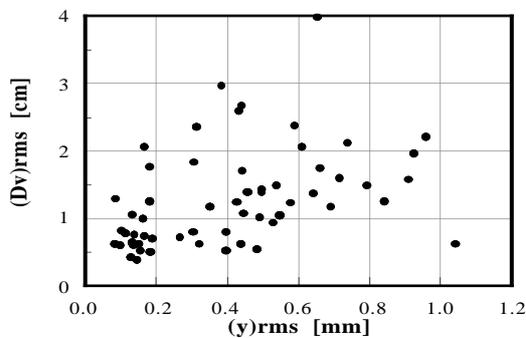


Fig. 1. The rms spurious vertical dispersion versus rms vertical orbit for ELETTRA, obtained from the simulation.

exceed 2 cm for a few tenths of a millimetre of the orbit rms, which is what is actually encountered in reality. The radiation integral is calculated for a generated spurious vertical dispersion to evaluate its contribution to the emittance coupling in Fig. 2. As the emittance coupling due to the betatron coupling is estimated from the measurement to be less than 1% [2], the spurious vertical dispersion is considered to be the major source of the

emittance coupling in ELETTRA, which in turn becomes a key parameter for the lifetime as it is dominated by the Touschek scattering.

The correction is performed with steerer magnets used for the orbit correction, by elaborating on the sensitivity matrix for the dispersion which takes into account the feed down effects of the orbit through sextupoles and thick quadrupoles. The obtained matrix is applied in ways that also seek to keep the orbit from being distorted.

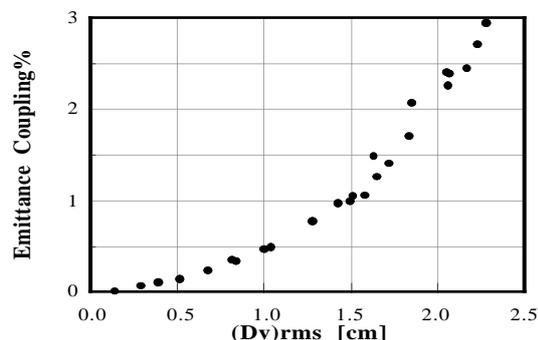


Fig. 2. Emittance coupling due to spurious vertical dispersion for various simulated cases of ELETTRA. The rms's are calculated at BPM positions. Radiation integral takes 20 points per dipole.

## 2. CORRECTION METHOD

As mentioned above, we attempt to correct the spurious dispersion with the steerer magnets used for the orbit correction, by providing ourselves with the sensitivity matrix  $D_{ij}$  for the dispersion. Here the suffix  $i$  stands for a monitor and  $j$  for a corrector. The procedure for constructing the matrices will firstly be described below [3]. Starting from the basic equations of motion for the dispersion, we consider changes on the dispersion  $\Delta D_u$  caused by an additional kick  $\Delta \kappa_u$  given by a corrector ( $u = x, y$ ),

$$\Delta D_x'' + (b_1 + \kappa^2) \cdot \Delta D_x = -\Delta \kappa_x + (b_1 - 2\kappa^2) \cdot \Delta x_{co} - b_2 \cdot \Delta x_{co} \cdot D_x, \quad (1)$$

$$\Delta D_y'' - b_1 \cdot \Delta D_y = -\Delta \kappa_y - b_1 \cdot \Delta y_{co} + b_2 \cdot \Delta y_{co} \cdot D_x, \quad (2)$$

in which  $b_n \equiv e/p_0 \cdot \partial^n B_y / \partial x^n$  and  $\kappa = b_0$ , and  $\Delta x_{co}$  and  $\Delta y_{co}$  denote associated changes in the horizontal and vertical closed orbit. There are also other terms such as coupling of the closed orbit or terms related to  $b_2$ , but have been omitted as they can be assumed to be small. Consequently there are three major sources for the spurious dispersion: 1) Corrector kicks themselves. 2)

\*Present address: ESRF, BP220, F-38043 Grenoble Cedex, France, which supports the presentation of this work.

Feed down effects of  $\Delta u_{c0}$  through chromaticity correcting sextupoles. 3) Feed down effects of  $\Delta u_{c0}$  through quadrupoles. Since the effects add up linearly, we may write

$$D_{ij} = \sum_{n=1}^3 D_{ij}^{(n)}. \quad (3)$$

Let us consider different contributions separately. 1) The matrix  $D_{ij}^{(1)}$  is simply the negative of the usual sensitivity matrix for the orbit  $A_{ij}$  given by

$$A_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi Q} \cdot \cos(\pi Q - |\psi_i - \psi_j|) \quad (4)$$

where  $\psi$  is the phase advance and  $Q$  is the tune. 2) The kick angles from the sextupoles are seen to be  $g \cdot D_x \cdot \Delta u_{c0}$ , where  $g$  is defined by  $g = \mp b_2 \cdot l_s$  with the upper sign for horizontal and  $l_s$  the sextupole length. Extending the use of Eq. 4, we have,

$$D_{ij}^{(2)} = \sum_{k \in \text{sext}} A_{ik} \cdot (g \cdot D_x)_k \cdot A_{kj}. \quad (5)$$

3) The evaluation of this contribution needs a little more care since quadrupoles or combined function dipoles in most cases cannot be treated as thin lens as above where any change in the displacement is ignored. Thus, if  $(d, d')$  represents the amount of increment on the dispersion at the exit of an element, the corresponding closed solution is given by

$$\begin{pmatrix} \Delta D_u \\ \Delta D'_u \end{pmatrix} = M(s+C | s) \cdot \begin{pmatrix} \Delta D_u \\ \Delta D'_u \end{pmatrix} + \begin{pmatrix} d \\ d' \end{pmatrix} \quad (6)$$

where  $M(s+C | s)$  is a single turn transfer matrix defined at the exit. The solution to the equation

$$\Delta D_u'' + K \cdot \Delta D_u = K \cdot \Delta u_{c0} \quad (7)$$

can be solved analytically, from which we obtain,

$$\begin{pmatrix} d \\ d' \end{pmatrix} = \frac{K}{2} \begin{bmatrix} \Delta u_0 L a_{12} - \Delta u'_0 (L a_{11} - a_{12}) / K \\ \Delta u_0 (L a_{11} + a_{12}) + \Delta u'_0 L a_{12} \end{bmatrix} \quad (8)$$

where  $L$  denotes the element length,  $\Delta u_0$  and  $\Delta u'_0$  are the closed orbit and its slope at the entrance of an element, which can be derived from Eq. 4. Coefficients  $a_{ij}$  ( $i, j = 1, 2$ ) are the elements of the usual  $2 \times 2$  transfer matrices of a focusing element having the strength  $K$ . The rest is just to transform  $(\Delta D_u, \Delta D'_u)$  obtained from Eqs. 6 and 8 to the position of monitor  $i$ :

$$D_{ij}^{(3)} = \sum_{\substack{q \in \text{quad,} \\ \text{bend}}} [\sqrt{\beta_i / \beta_q} (\cos \Delta \psi_{iq} + \alpha_q \sin \Delta \psi_{iq}) \cdot \Delta D_u + \sqrt{\beta_i \beta_q} \sin \Delta \psi_{iq} \cdot \Delta D'_u], \quad (10)$$

where  $\Delta \psi_{iq}$  is given by

$$\begin{aligned} \Delta \psi_{iq} &= \psi_i - \psi_q & (\psi_i > \psi_q) \\ &= 2\pi Q + \psi_i - \psi_q & (\psi_i < \psi_q) \end{aligned} \quad (11)$$

and those with the suffix  $q$  are to be evaluated at the exit of an element.

The above analytical expressions for the sensitivity matrix are numerically evaluated and compared with those calculated rigorously by solving the equation of motion for a closed solution [4]. The results calculated for ELETTRA show first of all that the contribution from the correctors is small with respect to those from sextupoles and quadrupoles, and secondly that there is a large cancellation between the latter two, as shown for a corrector versus BPMs in the vertical plane in Fig. 3.

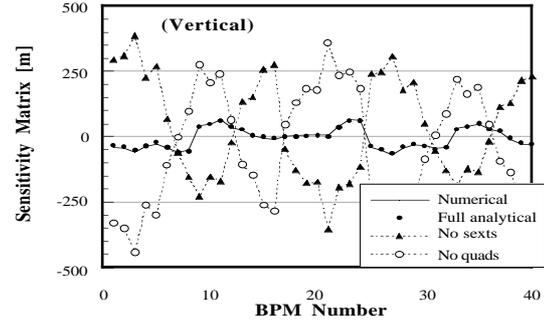


Fig. 3. Comparison of calculated sensitivity matrices for different sources. Triangles include only the contributions of 1 and 3, while open circles include only 1 and 2.

The full analytical calculation (dark circles) is seen to agree well with the rigorous result. As the matrix consists of cancellation of two large quantities, the results depend sensitively on the approximations made. An example is shown in Fig. 4 for the horizontal case where the full calculation is compared with the thin lens treatment for the quadrupoles (triangles), and with the thick treatment however neglecting the difference in the optical values at the entrance and the exit (open circles).

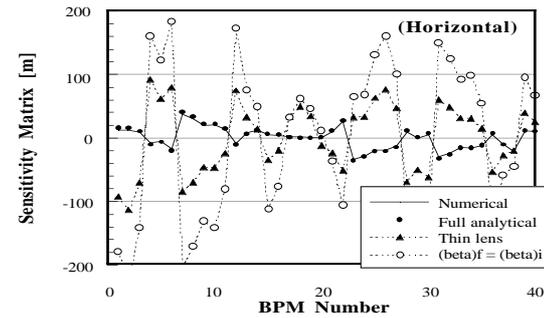


Fig. 4. Comparison of matrices under different treatments.

The approximations used are seen to cause even sign errors in the above example.

### 3. APPLICATION TO THE MACHINE

The sensitivity matrices as described in the previous section are calculated in the orbit correction program Orbit [5] in its dispersion correction routine, which is implemented in parallel with other routines for the orbit correction. So far, the matrices are applied to two

algorithms, the best corrector and the SVD (Single Value Decomposition) methods. In both, a parameter is introduced which the operator can vary with a scroll bar on the control panel to define a weighting between the orbit and the dispersion so that a simultaneous optimisation of both quantities can be made. An important variable in the SVD method is the number of eigenvalues, i.e. how many eigenvalues to take into account, which characterises the correction. The most recent development extends the matrix dimension to perform a simultaneous inversion of both the orbit and the dispersion, which, in the horizontal plane, furthermore includes a constraint to fix the path length [6].

An example of achieved corrections is shown in Figs. 5 for the vertical case, which is obtained using the best corrector

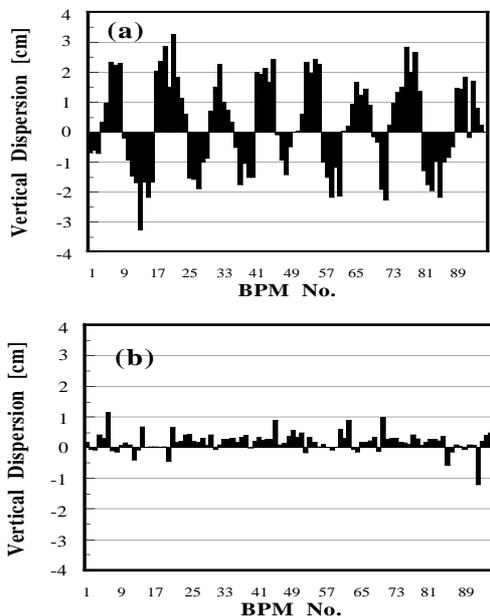


Fig. 5. Spurious vertical dispersion (a) before and (b) after correction.

Table 1. Statistics before and after correction. ptop: peak to peak

$y_{co}$ [mm]			$D_v$ [cm]			
ave	rms	ptop	ave	rms	ptop	
Before:	-0.01	0.13	0.89	0.20	2.21	7.93
After:	-0.05	0.34	1.77	0.19	0.36	2.43

method. The rms of the dispersion is seen to exceed 2 cm for a very well corrected orbit. A similar degree of correction can be achieved with the SVD method with a higher efficiency and with a more reduced corrector strength, as the same trend is noticed in the orbit correction. In the 96 BPM  $\times$  82 Corrector system of ELETTRA, the correction tends to saturate with the number of eigenvalues around 20. As in the example, the orbit rms under the best corrected dispersion is usually slightly larger than its lowest achievable value. This aspect is more pronounced horizontally where the best rms achieved so far of 0.6 cm for the spurious dispersion

associates an orbit distortion of rms larger than 1 mm. In view of a possible mispositioning of the machine components, however, it may well be that the best corrected dispersion with a degraded orbit rms better represents the designed machine, the analysis of which is yet to be carried out.

In the course of dispersion correction, a large reduction, mainly vertically, can often be observed in the SRPM (Synchrotron Radiation Profile Monitor) image. This is understood to be due to a particularly high sensitivity of the vertical dispersion at the SRPM position which is at the entrance of a dipole where the horizontal dispersion is much smaller. The image becomes a small round spot after correction. As the beam lifetime in ELETTRA is dominated by the Touschek scattering, the correction level of the spurious vertical dispersion is sensitively reflected on the lifetime through the emittance coupling (Fig. 6) [7]. The correction of spurious dispersion in ELETTRA thus becomes a key beam quality control in the daily operation for the users.

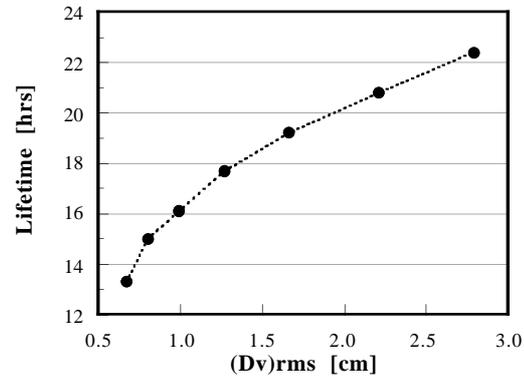


Fig. 6. Measured lifetime versus rms spurious vertical dispersion.

#### 4. ACKNOWLEDGEMENT

The author thanks C.J. Bocchetta, A. Fabris, F. Iazzourene, E. Karantzoulis, M. Svandrlík, L. Tosi, R.P. Walker, A. Wrulich and all other ELETTRA shift crew for their contribution in the machine operation and measurements.

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