SUPPRESSION OF SPACE CHARGE INDUCED BEAM EMITTANCE GROWTH IN TRANSPORT LINE

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ABSTRACT

Nonlinear space charge forces of high intensity beam produce strong emittance growth in linear focusing channel due to mismatching of the beam profile with focusing field. To obtain matching conditions for a beam with an arbitrary distribution function, it is necessary to accept that the potential of the external focusing field is a highly nonlinear function of radius. The solution for external potential is obtained from the stationary Vlasov's equation for beam distribution function and Poisson's equation for electrostatic beam potential. Ideal way to create required potential distribution is a plasma lens with specific distribution of the opposite charged particles. Another variant is a quadrupole four vanes structure with higher order multipole component. In that case the matched beam profile has to be close to square, instead of the conventional circle beam cross section. An analytical approach is illustrated by results of a particle-in-cell simulation.

1 INTRODUCTION

Intense particle beams exhibit strong emittance growth and beam halo formation in the linear focusing channel due to a mismatch between the beam profile and the focusing field (see fig. 1). This phenomenon limits the value of beam brightness and results in particle losses. In the next generation of intense particle accelerators the beam emittance growth will have to be suppressed. Recently [1,2] it was shown, that the emittance of a high brightness beam can be conserved in a highly nonlinear focusing field. Such a focusing field requires a linear function of radius near the axis that drops to a nonlinear function further from the axis. The ideal way to create the required potential distribution is by using a plasma lens with a specific distribution of particles with the opposite charge. Another way utilizes a quadrupole structure with a higher order (duodecapole) field component, where approximate matched conditions for the beam can be obtained. This paper studies the behavior of a space charge dominated beam in an uniform four vanes quadrupole line with small value of phase advance per period of particle oscillation. In such systems the envelopes of the matched beams are close to constant. This makes it possible to consider the z-independent process and treat the problem analytically. As shown below, to match the non-uniform beam with quadrupole line , two problems have to be solved: (i) to introduce the multipole field component of 6th order; (ii) to truncate beam profile as a 45^{o} skewed square.

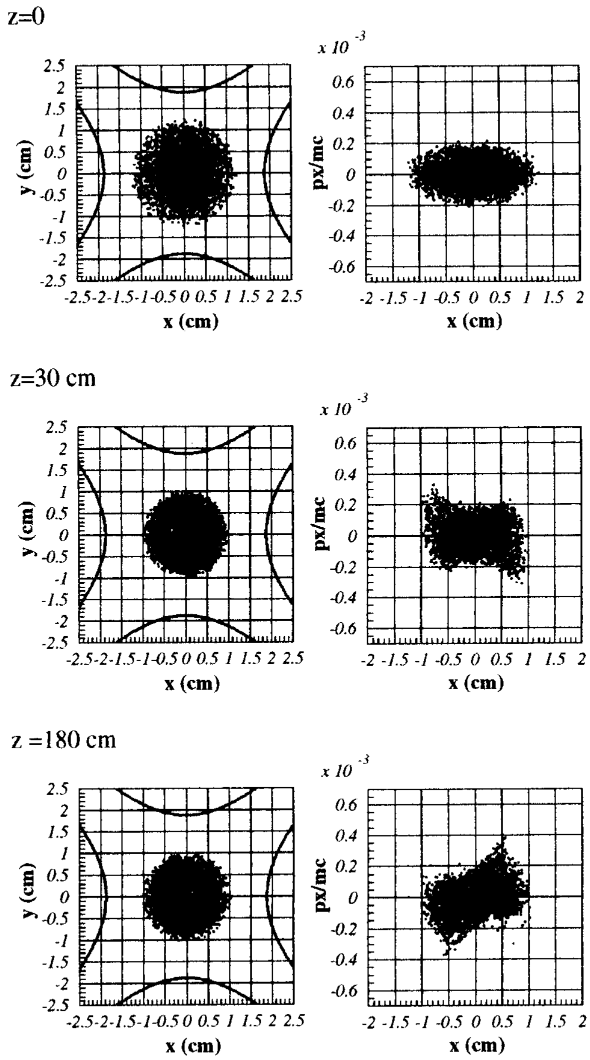

Fig. 1. Emittance growth of the 150keV, 100mA, 015πcm mrad proton beam in a uniform four vanes quadrupole line with field gradient 50kV/cm^2 and frequency 270MHz.

2 MATCHING OF A BEAM INTO THE UNIFORM FOCUSING CHANNEL

Self-consistent matched conditions for a beam with an arbitrary distribution function in a uniform focusing channel have been obtained from two principles: Vlasov's equation for time-independent distribution function and Poisson's equation for space charge potential of the beam [1,2]. The realistic beam distribution is characterized by a high concentration of particles near the axis and declining particle density towards the periphery of the beam. Let us consider a beam of particles with a parabolic distribution function, charge q and mass m, which is close to an experimentally observed beam distribution. In this distribution, the phase space density of particles monotonically decreases from the center of the beam until reaching the boundary of 4-dimensional hypervolume:

$$f = f_o \left(1 - \frac{x^2 + y^2}{2R^2} - \frac{p_x^2 + p_y^2}{2 p_o^2}\right), \qquad (1)$$

where I is a beam current, β is a longitudinal particle velocity, $R = 2\sqrt{<x^2>}$ is a beam envelope, $p_o = 2\sqrt{<p_x^2>}$ is a double rms beam size in phase space, and $\varepsilon = R\, p_o/(mc)$ is a normalized RMS beam emittance. Space charge density ρ_b and space charge potential of the beam U_b are defined by the expressions:

$$\rho_b = \frac{3I}{2\pi c \beta R^2}\left(1 - \frac{r^2}{2R^2}\right)^2; \qquad (2)$$

$$U_b = -\frac{3}{2}\frac{mc^2}{q}\frac{I}{I_c \beta}\left(\frac{r^2}{R^2} - \frac{r^4}{4R^4} + \frac{r^6}{36 R^6}\right), \qquad (3)$$

where $I_c = 4\pi\varepsilon_o mc^3/q = (A/Z)\ 3.13 \times 10^7$ Ampere is a characteristic value of beam current. In ref. [2] the required potential of the focusing structure to maintain given distribution function was found:

$$U_{ext} = \frac{mc^2}{q}\left[\frac{r^2}{2R^2}\left(\frac{\varepsilon^2}{R^2} + \frac{3I}{I_c\beta}\right) + \frac{3I}{8 I_c \beta}\left(-\frac{r^4}{R^4} + \frac{r^6}{9R^6}\right)\right]. \quad (4)$$

The required nonlinear focusing field can be created by a distribution of the opposite charged particles along the focuser (plasma lens). A more simple way employs a quadrupole channel with multipole components.

3 MATCHING OF THE BEAM INTO THE FOUR-CONDUCTOR QUADRUPOLE LINE WITH DUODECAPOLE COMPONENT

In Ref.[2] it was shown that introducing duodecapole component in pure quadrupole alternating-gradient structure results in better matching of the beam with the transport channel. Higher order terms in the potential distribution produce nonlinear components, which can be used to compensate for nonlinear space charge forces. Let us consider an uniform four vanes structure with potential

$$U(r,\varphi,t) = \left(\frac{G_2}{2} r^2 \cos 2\varphi + \frac{G_6}{6} r^6 \cos 6\varphi\right) \sin \omega_o t, \qquad (5)$$

where G_2 is a quadrupole gradient, G_6 is a duodecapole component and $\omega_o = 2\pi c/\lambda$ is a RF frequency. Electrical field of the structure is given by:

$$\vec{E}(r,\varphi,t) = [-\vec{i}_r (G_2 r \cos 2\varphi + G_6 r^5 \cos 6\varphi) + \vec{i}_\varphi (G_2 r \sin 2\varphi + G_6 r^5 \sin 6\varphi)] \sin\omega_o t \qquad (6)$$

Particle trajectories in the field (6) can be represented as a combination of the slow variation of particle position and fast oscillations with small amplitude. The problem of averaging of particle motion in fast oscillating field $\vec{E}(\vec{r},t) = \vec{E}_o(\vec{r}) \sin \omega_o t$ was analyzed in ref. [3, 4, 5].The oscillating field creates an effective scalar potential:

$$U_{eff}(\vec{r}) = \frac{q}{4m}\frac{E_o^2(\vec{r})}{\omega_o^2}, \qquad (7)$$

which describes the averaged (slow) motion of particle. For considered structure the effective potential is:

$$U_{eff} = \frac{mc^2}{q}\mu_o^2 \left(\frac{a}{\lambda}\right)^2 F(r,\varphi); \qquad (8)$$

$$F(r,\varphi) = \frac{1}{2}\left(\frac{r}{a}\right)^2 + \xi\left(\frac{r}{a}\right)^6 (\cos 2\varphi \cos 6\varphi + \sin 2\varphi \sin 6\varphi) + \frac{\xi^2}{2}\left(\frac{r}{a}\right)^{10}$$

where a is a radius of structure, μ_o is a smoothed transverse oscillation frequency and ξ is a normalized ratio of field components:

$$\mu_o = \frac{q G_2 \lambda^2}{\sqrt{8}\,\pi mc^2}; \qquad \xi = \frac{G_6\, a^4}{G_2}. \qquad (9)$$

The effective potential is an axial - nonsymmetrtic and a highly nonlinear radius function. Let us compare potential (8) for $\varphi=0$ with required axial -symmetric potential (4). Linear focusing parts of the field have to be the same, which gives the value of quadrupole gradient:

$$G_2 = \frac{\pi^2}{2\sqrt{2}}\frac{mc^2}{q\lambda R}\sqrt{\frac{\varepsilon^2}{R^2} + \frac{3I}{I_c\beta}}. \qquad (10)$$

To define the value of G_6, let us assume, that the values of electric fields $E_{ext} = -\partial U_{ext}/\partial r$ and $E_{eff} = -\partial U_{eff}/\partial r$ are equal at the boundary of the beam distribution $r = \sqrt{2}R$. The terms proportional to r^2, vanish due to the adopted condition (10). The remaining terms give the expression for duodecapole component:

$$G_6 = -\frac{G_2}{R^4}\frac{I}{I_c}\frac{1}{12\beta\left(\frac{\varepsilon^2}{R^2} + \frac{3I}{I_c\beta}\right)}. \qquad (11)$$

Note that the duodecapole component has to be the opposite of the quadrupole component, i.e. the absolute value of the field is reduced in the x and y directions as compared with linear function of radius.

Analysis of the potential function $F(r,\varphi)$ shows, that for a small radius, the equipotential lines $F(r,\varphi) = C$ are close to circles, because only the quadratic term r^2 in (8) is essential. With larger radius the equipotential is close to a 45^o skewed square. This suggests, that the matched beam should also have the square shape (see fig. 2).

Self-consistent computer simulations using particle-in-cell code BEAMPATH were done to verify the matched conditions of the beam, obtained from the above consideration. Parameters of the beam and of the structure were chosen the same as in fig.1. Introducing required duodecapole component with the value of G_6=-1.4kV/cm^6 results in change of the shape of electrodes (see fig.2). The beam profile in real space x-y was truncated to be a skewed square with the maximum beam size $R_{max} = 2.5\sqrt{<x^2>} = 1.25$ R. The beam sizes as well as the value of beam current and initial value of beam emittance were the same in simulations, presented in figs. 1,2.

As shown at figs.1, 3 the beam in pure quadrupole channel experience noticeable emittance growth. Initial value of emittance growth rate is $7\cdot10^{-4}\ \frac{\pi \text{ cm mrad}}{\text{cm}}$. After transport distance z=30 cm, which correspond to one quarter of transverse oscillation, the beam emittance achieved the value 0.177 π cm mrad. Finally the beam emittance oscillates around stable value of 0.17 π cm mrad. In a nonlinear structure (see figs. 2, 3), the initial value of emittance growth rate is $2.5\cdot10^{-4}\ \frac{\pi \text{ cm mrad}}{\text{cm}}$, which is substantially smaller than for beam transport in a pure quadrupole channel. The final value of beam emittance is 0.162 π cm mrad, which indicates the better matching conditions than in a pure quadrupole focusing channel. The final beam emittance and beam profile are matched without serious phase space portrait distortion.

4 CONCLUSIONS

Matching of a bright beam into a transport focusing channel, while avoiding beam halo formation in phase space, is considered. A plasma lens with specific distribution of particles along the channel provides an ideal way to create an appropriate non-linear focusing field. Another way employs a four-conductor quadrupole structure with a multipole component of the 6th order (duodecapole component). In that case, the initial matched beam profile has to be close to square, instead of the conventional circle beam cross section.

5 REFERENCES

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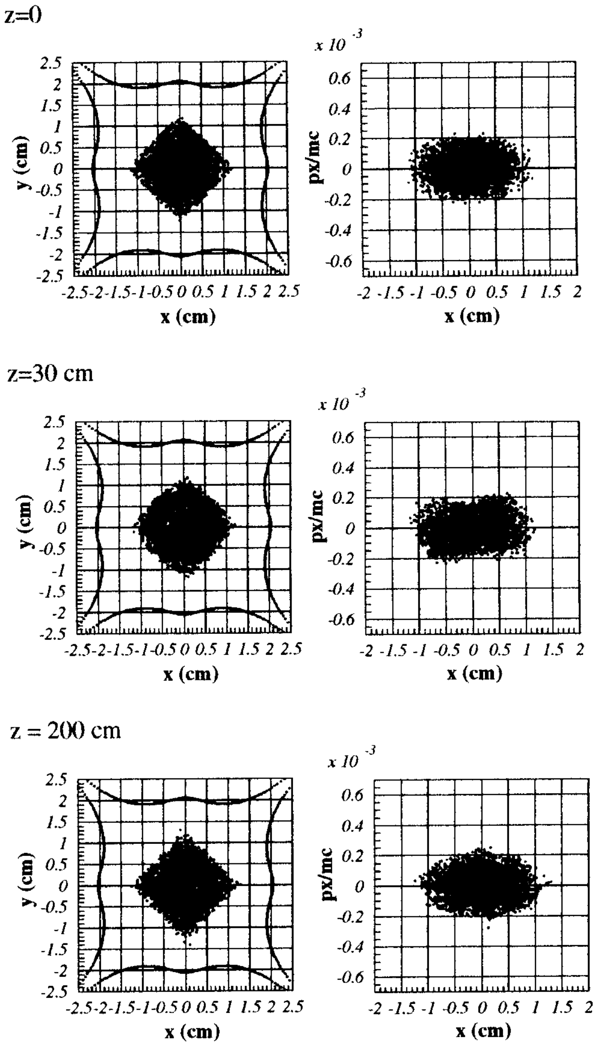

Fig. 2. Matching of the 150keV, 100mA, 015 π cm mrad proton beam in a uniform four-conductor quadrupole line with field gradient 50kV/cm^2, duodecapole component -1.4 kV/cm^6 and frequency 270MHz.

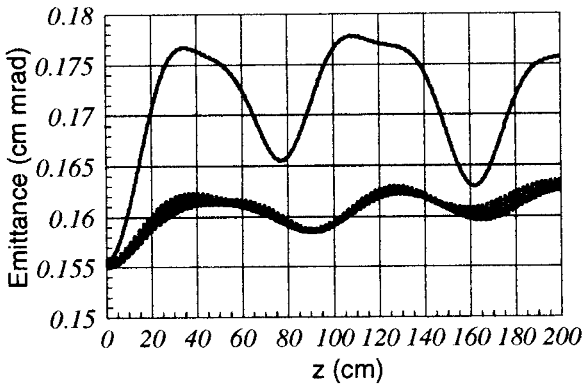

Fig. 3. Beam emittance growth in a four vane structure with pure quadrupole field (up) and in quadrupole field with duodecapole component (bottom).