CALCULATION OF TRANSVERSE RESISTIVE IMPEDANCE FOR VACUUM CHAMBERS WITH ARBITRARY CROSS SECTIONS

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Abstract

Considering the resistive instability of betatron oscillations of an multibunch beam in the storage ring, one can see that the growth rate of the most unstable oscillation mode is mainly determined by the transverse resistive impedance at the most dangerous frequency. This frequency is $\omega_d = \omega_0(1-\nu')$, where ω_0 is a beam revolution frequency and ν' is the fractional part of the betatron tune ν. At this very low frequency, the skin depth can be sufficiently big to be of order or even more than the vacuum chamber walls thickness, therefore, the model of infinitely thick walls for the surface impedance can not be applied for the transverse impedance calculation. A computer code for calculating the electromagnetic field of an source - a dipole current - in the vacuum chamber with lossy walls of arbitrary geometry was used for dipole ohmic losses calculations. These losses determine the actual transverse resistive impedance in approach of a stationary current.

1 GROWTH RATES OF A MULTIBUNCH BEAM

For symmetric multibunch beam transverse oscillation, the growth rates due to resistive instability is determined by the resistive impedance [1]:

$$\sigma^l = -\frac{1}{4\pi}\omega_0\frac{I}{V_s}Re\left\{\sum_{p=-\infty}^{\infty} Z_t(\omega_0(pn_b+l+\nu))\right\},$$

where σ^l is the growth rate of the l-th symmetric oscillation mode with a phase difference between neighbour bunches oscillation phases $\Delta\phi_l = 2\pi l/n_b, \;\; l = 0,...,n_b-1$; n_b is the number of bunches in the beam; ω_0 is the revolution frequency; $\omega_0 n_b$ is the bunch repetition frequency; $\Omega = \nu\omega_0$ is the betatron oscillation frequency; I is the average beam current; V_s is the synchronous particle energy devided by the electron charge; Z_t is the resistive transverse impedance of the vacuum chamber multiplied by the averaged beta function (R/ν).

Usually, the transverse impedance is determined for a beam deflected from the chamber axis as a whole, i.e. with infinite longitudinal phase velocity v_{ph} of the current harmonic. This definition works for the systems with the length much shorter than the wave length, for example, for RF cavities. But considering the resistive instability of the beam, we deal with the whole length of the storage ring with ν oscillations along circumference and one should still check the validity of applying the traditionally defined transverse impedance.

In this traditional static approach, for a round vacuum chamber with walls much thicker than the skin depth at lowest spectrum frequency,

$$Z_t(\omega) = Z_{t0}(\omega) = \frac{Z_0 w(\omega)}{\pi(\omega/c)a^3}\frac{2\pi R^2}{\nu},$$

where R is the average radius of the storage ring; c is the light velocity; a is the inner radius of the vacuum chamber; Z_0 is the free space wave inpedance; $w(\omega) = \sqrt{\epsilon_m/\mu_m}$, ϵ_m and μ_m are the magnetic permeability and dielectric permittivity of metal chamber walls (with the account of walls conductivity).

Due to the very rarefied spectrum of the multibunch beam (with the dictance between neighbour spectrum lines $n_b\omega_0$), the growth rate of the most unstable mode of betatron oscillations can be estimated by the main term at the most dangerous frequency $\omega_d = \omega_0(1-\nu')$ (ν' is the fractional part of ν).

But, looking at the spectrum of the current of the most unstable mode, one can see that the most dangerous for the instability current harmonic $I\exp(i(\omega_d t - k_z z))$ has a phase velocity $v_{\mathrm{ph}} = \beta c$ with

$$\beta = k/k_z = \frac{1-\nu'}{1+[\nu]}\beta_p \ll 1,$$

where $[\nu]$ is the integer part of ν, $k = \omega/c$ and $\beta_p = v_p/c$, $v_p \sim c$ is the velocity of particles.

For example, for LHC $\nu = 70.3$, $\omega_d = 8.5$kHz and $\beta = 0.01 \ll 1$, the case very different from the static model with $\beta \gg 1$, for which the transverse impedance is usually considered.

Thus, there arises a problem to define the transverse impedance for arbitrary β and, in particular, for $\beta \ll 1$. For that, one should turn back to the growth rates and to see, in which form it appears if phase velocities of all current harmonics are taken into account.

In fact, we should find the fields induced by the dipole current harmonic with arbitrary phase velocity and their influence on beam dynamics.

2 FIELD EQUATIONS FOR ARBITRARY PHASE VELOCITIES

We consider Maxwell equations with a dipole current as an source:

$$\mathrm{rot}\vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_e, \ \mathrm{rot}\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0,$$

$$\vec{J}_e = J_z(\vec{r}_\perp) e^{i(\omega t - k_z z)} \vec{e}_z,$$

$$\vec{D} = \epsilon\epsilon_0 \vec{E}, \vec{B} = \mu\mu_0 \vec{H}$$

In metals with a conductivity σ $\mu_m = 1$, $\epsilon_m = 1 - i\frac{\sigma}{\omega\epsilon_0}$.

Writing separately longitudinal and transverse components (relative z-direction) of the Maxwell equations, one can express transverse fields components via longitudinal one and get the equations for only longitudinal components (denoting here $\tilde{\gamma}^2 = 1/(1-\beta^2\epsilon\mu)$):

$$\vec{H}_\perp = -\frac{\tilde{\gamma}^2}{ik_z}\left(\vec{\nabla}_\perp H_z + \beta\epsilon(\vec{e}_z \times \vec{\nabla}_\perp)(E_z/Z_0)\right),$$

$$\vec{E}_\perp = -\frac{\tilde{\gamma}^2}{ik_z}\left(\vec{\nabla}_\perp E_z - \beta\mu(\vec{e}_z \times \vec{\nabla}_\perp)(H_z Z_0)\right),$$

$$\tilde{\gamma}^2\beta^2 \nabla_\perp^2 E_z - k^2 E_z = J_{e\,z} i Z_0 k/\epsilon,$$

$$\tilde{\gamma}^2\beta^2 \nabla_\perp^2 H_z - k^2 H_z = 0.$$

The static case corresponds to $\beta \gg 1$, $H_z = 0$, $E_\perp = 0$, the boundary conditions are connect $H_\perp$ and E_z.

But at finite phase velocity, the solution should take into account both E_z and H_z components.

3 THE GROWTH RATES WITH THE ACCOUNT OF PHASE VELOCITIES.

Analogously to the method used in [1], we can analyse the multibunch beam dynamics, but now, instead of the method of eigen functions, we will assume that the fields induced by all current harmonics are found in a way given in the previous section, with the account of their phase velocities. In this way, we get the growth rates of symmetric multibunch beam oscillations modes:

$$\sigma^l = -\frac{1}{4\pi}\omega_0 \frac{I}{V_s}\cdot$$

$$\cdot Re\left\{\sum_{p=-\infty}^{\infty} Z_t(k_z = \frac{pn_b + l}{R}, \omega = \omega_0(pn_b + l + \nu))\right\},$$

$$Z_t(k_z, \omega) = \frac{\gamma^2(1-\beta)2\pi R^2}{k_z \nu}\left(Z_\parallel^E(k_z,\omega) - Z_\parallel^H(k_z,\omega)\right),$$

$$Z_\parallel^E(k_z,\omega) = \frac{\partial(E_z - E_z^0)}{\partial x}|_{r\to 0}, \ Z_\parallel^H(k_z,\omega) = \frac{\partial H_z}{\partial y}|_{r\to 0}.$$

Here E_z and H_z are the fields induced by the dipole current harmonic oscillating in the plane xz with the unit dipole moment; E_z^0 is the electric field induced in the chamber with ideal walls. $Z_t(k_z, \omega)$ defined here is the transverse impedance with the account of its dependence on $\beta = k/k_z$.

4 A MODEL OF A MULTILAYER ROUND METAL TUBE

As an example, we will consider a a round metal tube with thick walls with a coaxial tube with thin walls inside it (see fig.1) and a dipole current (with a unit dipole moment) in the middle with a dencity

$$J_z(\vec{r}_\perp) = \frac{1}{\pi r_0^2}\delta(r - r_0)\cos\varphi, \ r_0 \ll a$$

We assume that for low frequencies, at $\frac{ka}{\beta\gamma} \ll 1$, the solution in vacuum regions has a static form (the terms in E_z and H_z proportional to $r^{\pm 1}$). In metal walls we assume a wall curvature radius being much bigger than the skin depth, therefore the plane solution can be used ($\exp(\pm ikr/w)$).

Matching the tangential fields components at $r = r_0$, with the account of the source current, and at the metal boundaries and imposing the condition of fields decreasing at $r \to \infty$, we get the solution, which in common case has a not very transparent form. But, one can simplify the result in some important cases:

1. The case of a thick wall, $|ik(b-a)/w| \gg 1$

In this case, it appears for any $\beta \gg |w|$

$$Z_t = Z_{t0} = \frac{2Z_0 w R^2}{\nu(\omega/c)a^3}.$$

The impedance has no dependence on β in the case of thick walls, which corresponds to the previous results ([1]).

2. The static case, $\beta = \infty$.

In this case, $H_z = 0$. If $|\frac{w}{ika}| \ll 1$; $(b-a), (c-b) \ll a$, then, denoting $T = \mathrm{tg}(-k(b-a)/w)$, we get

$$Z_t = Z_{t0}\cdot\frac{1+F}{1-FiT}, \ F = -\frac{ik(c-b)/w}{1-iT}.$$

The impedance has a week dependence on the width of the vacuum gap between walls.

3. The case of $\beta \ll 1$.

We assume $\beta \sim 0.01$, i.e. $\beta \ll 1$, but still $\beta \gg |w|, |ikb|$; In this case, the vacuum gap affects strongly on the fields. It appears that for $(c-b) = (b-a) = 1$mm, $a = 30$mm, $f = 8.5$kHz, $|\frac{w}{ika}| \sim 0.1$ (steel walls)

$$Z_t = Z_{t0}\cdot\frac{1-F}{\beta + FiT} \approx Z_{t0}\cdot\frac{1-F}{FiT}.$$

The impedance differs very much from the case of thick walls. In the case of thin inner wall ($|iT| \ll 1$) it is inversly proportional to its thickness.

5 THE FEM METHOD FOR THE EXCITATION PROBLEM WITH ALL FIELD COMPONENTS

The numerical method for the solution of the problem formulated in the section 2, is modified on the basis of the method given in [2] for determining eigen modes of RF cavities. The main distinctions are:

1. The problem with an source current is solved instead of the eigen value problem.

2. The two component problem is solved, implying at the same time both E_z and H_z components, which are coupled because the transverse fields matched at the boundaries are expressed via both components, E_z and H_z.

3. The second order equations for E_z and H_z in the section 2 are written in the form suitable for the FEM method, which conserves the continuity of the tangential fielfs components and does not contain the normal derivatives of E_z and H_z.

6 AN EXAMPLE OF CALCULATIONS

As an example, we offer the picture of the magnetic field lines in the vacuum chamber of LHC in static approach (fig.2). The cross section has a complicated structure: the steel vacuum chamber itself, the steel beam screen inside it with copper coated inner wall, the outer surrounding modelled as thick steel wall. The thickness of the steel walls of the beam screen and of the vacuum chamber and the thickness of the copper coating are sufficiently less than the corresponding skin depths in steel and copper at considered frequency (8.5kHz). The field penetrates through the beam screen at the regions without copper coating. The structure of the fields depends on the vacuum gaps between the beam screen, the vacuum chamber and outer surrounding and also on the current harmonic phase velocity.

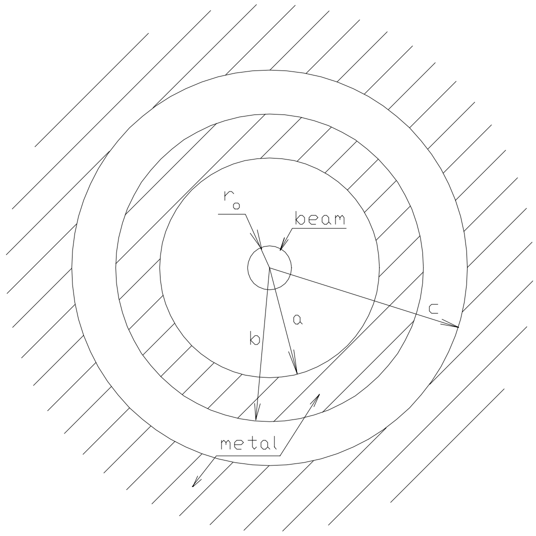

Figure 1: Cross section of the round multilayer tube.

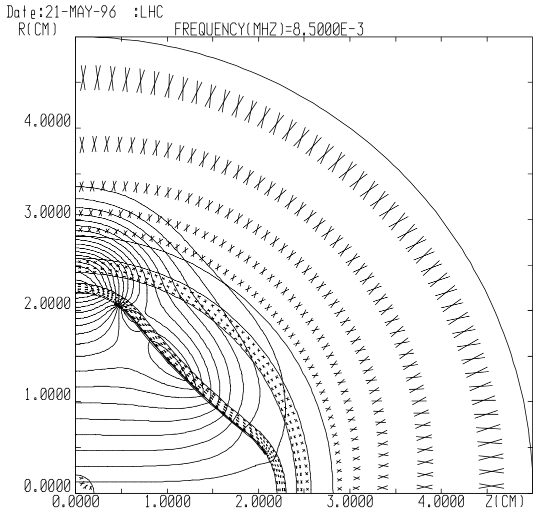

Figure 2: The field map of the field penetration between copper coating stripes.

7 CONCLUSION

1. The necessity of taking into account the phase velocity of the current harmonic is shown.

2. The small phase velocity of the dangerous current harmonic is mostly important in the case of the multilayer wall with the vacuum gap between layers, which are thinner than the skin depth. For the vacuum chamber of LHC, it is important in the case of not everywhere copper coating.

3. The most unstable mode of the transverse oscillations of the multibunch beam and the most dangerous frequency should be found with the account of the phase velocities of the current harmonics.

4. The FEM method is developed for the numerical solution of the excitation problem for arbitrary phase velocity of the exciting current harmonic, taking into account both E_z anf H_z field components.

5. One should note that at small phase velocities, the addition due to the transverse components of the current can be sufficient and should not be neglected. In future, this fact should be proved and taken into account.

8 REFERENCES

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