ANALYTICAL CALCULATION OF RADIATION IN A SIDE COUPLED MM-WAVE ACCELERATING STRUCTURE

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Abstract

After the proposal of a planar muffin-tin cavity structure for mm-wave accelerators three years ago, many calculations were done using both, analytical methods and numerical codes. Applying mode matching technique, a double sided muffin-tin as well as open structures of a single chain of cavities were analyzed. The present paper gives an extension of this analysis to side coupled cavity arrangements consisting of a chain of acceleration cells and a chain of coupling cells. Dispersion diagrams and shunt impedances are given and compared to existing numerical codes.

1 INTRODUCTION

The first proposal for accelerating structures in the mm–wave region ([1]) was a double sided muffin–tin designed for a 120 GHz traveling wave mode with a $\frac{2\pi}{3}$ phase advance per cell. The calculations were done using both numerical codes like MAFIA and GdfidL as well as analytical methods based on mode–matching technique ([3], [4]). Later, geometries with a confluent π–mode were considered in order to achieve an accelerating structure with not vanishing group velocity and good mode separation which are expected to be insensitive against errors. Such properties can be realized with double periodic structures ([2]). As an extension of the mode–matching analysis of single periodic muffin–tins the present paper treats side coupled cavity arrangements consisting of a chain of accelerating cells and an alternating chain of coupling cells as shown in Fig. 1.

2 SYMMETRY CONSIDERATIONS

In [3] it was shown, that a proper symmetry consideration, whenever possible, can essentially reduce the CPU time consumption especially for 3D boundary value problems. Looking at the special arrangement in Fig. 1 one can expect that the knowledge of the field distribution in the cavity regions 1 and 2 is sufficient to determine the fields in region 1', 2' and 2" via a simple transformation. In order to find this transformation rule we consider an arbitrary periodic waveguide, which for example may look like in Fig. 2. Further we consider two field states where either the electric or the magnetic field strength is y–directed. Then the Floquet theorem can be written as

$$\begin{aligned} \mathbf{e}_y \times \mathbf{E}(-x, z+L_e) &= -\underline{\mathcal{D}}\,[\mathbf{e}_y \times \mathbf{E}(x,z)]\ \mathrm{e}^{-\mathrm{j}\beta_0 L_e} \\ \mathbf{e}_y \times \mathbf{H}(-x, z+L_e) &= +\underline{\mathcal{D}}\,[\mathbf{e}_y \times \mathbf{H}(x,z)]\ \mathrm{e}^{-\mathrm{j}\beta_0 L_e} \end{aligned}$$

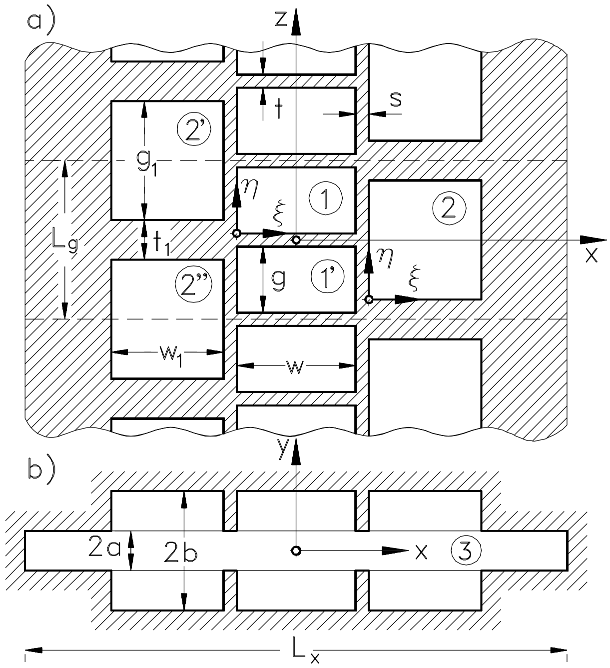


Figure 1: The biperiodic muffin–tin structure. a) Top–view b) transverse cut.

$$\underline{\mathcal{D}} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} , \qquad (1)$$

where $L_e = \frac{L_g}{2}$ is the effective period length.

3 FIELD EXPANSION

In [3] the mode–matching technique applied to planar mm–wave structures was described in detail. We use the same procedure and split the field in transverse electric (TE_y) and transverse magnetic (TM_y) components w.r.t. the y–direction. Further local coordinates ξ, η are introduced in each cavity. Due to lack of space only the field expansions in the areas 1,2,3 are presented, the other ones follow from (1).

$$\mathbf{E}_t^{\substack{(1)\\(2)}}/Z_0 = \sum_{m,n,TX} B_{mn}^{\substack{(1)\\(2)}} \mathbf{G}_{mn}^{\substack{(1)\\(2)}}(\xi,\eta) W_{mn}^{\substack{(1)\\(2)}}(y) \qquad (2)$$

$$\mathbf{e}_y \times \mathbf{H}^{\substack{(1)\\(2)}} = \sum_{m,n,TX} B_{mn}^{\substack{(1)\\(2)}} \mathbf{G}_{mn}^{\substack{(1)\\(2)}}(\xi,\eta) W_{mn}^{\prime\,\substack{(1)\\(2)}}(y) Y_{mn}^{\substack{(1)\\(2)}}$$

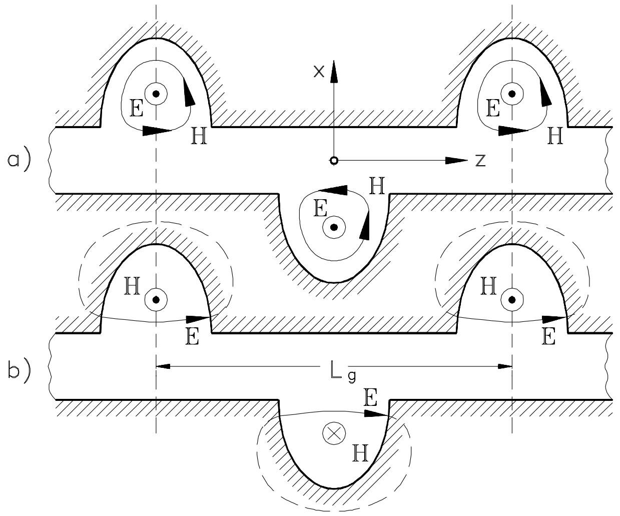


Figure 2: An alternating periodic structure with a) y–directed electric field, b) y–directed magnetic field.

$$\mathbf{E}_t^{(3)}/Z_0 = \sum_{m,n,TX} A_{mn}\mathbf{F}_{mn}(x,z)V_{mn}(y) \tag{3}$$

$$\mathbf{e}_y \times \mathbf{H}^{(3)} = \sum_{m,n,TX} A_{mn}\mathbf{F}_{mn}(x,z)V'_{mn}(y)Y_{mn}^{(3)}$$

where $\mathbf{E}_t = -\mathbf{e}_y \times (\mathbf{e}_y \times \mathbf{E})$ and the following abbreviation has been used

$$\sum_{m,n,TX} = \sum_{\substack{m=0\\m+n\neq 0}}^{\infty}\sum_{n=0}^{\infty} \underset{\text{terms}}{TE_y-} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \underset{\text{terms}}{TM_y-}$$

The vector functions $\mathbf{G}_{mn}^{(i)}(\xi,\eta)W_{mn}^{(i)}(y)$ are well known standing wave solutions of the wave equation in cartesian coordinates and $\mathbf{F}_{mn}(x,z)V_{mn}(y)$ are traveling wave solutions in z–direction with a phase constant of the n'th space harmonic

$$\beta_n = \beta_0 + \frac{2\pi n}{g_1 + t_1} \quad , \quad n = 0, \pm 1, \pm 2, \dots$$

For more details about these functions see ref. [3]. The $Y_{mn}^{(i)}$ describe waveguide admittances normalized with $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ for waves traveling in the y–direction.

4 THE DRIVING FIELD

Since we will later ask for the beam coupling impedance we superimpose the magnetic field produced by a charged particle q traveling with constant velocity $\mathbf{v} = \beta c_0 \mathbf{e}_z$ in a rectangular waveguide (see Fig. 3). The so–called magnetic source field of this charge reads in the plane $y = a$

$$\mathbf{e}_y \times \mathbf{H}^{(s)}(y=a) = \frac{2q}{L_x}\sum_{m=1}^{\infty}\frac{\sin\eta_m(d_y+a)}{\sin 2\eta_m a}\,\cdot$$
$$\cdot \sin\frac{m\pi\left(x+\frac{L_x}{2}\right)}{L_x}\sin\frac{m\pi\left(d_x+\frac{L_x}{2}\right)}{L_x}\,\mathrm{e}^{-\mathrm{j}\frac{\omega z}{v}} \tag{4}$$

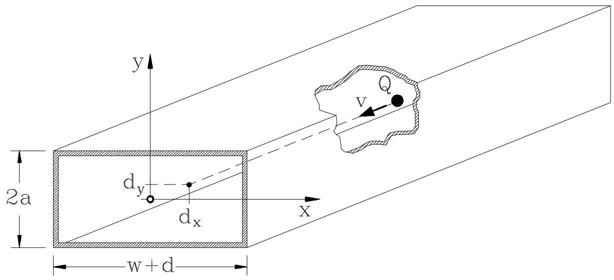


Figure 3: Point charge in a rectangular waveguide.

with $\eta_m = \mathrm{j}\sqrt{k^2\left(\frac{1}{(\gamma\beta)^2} + 2\mathrm{j}\delta\right) + \left(\frac{m\pi}{L_x}\right)^2}$ and $\beta = \frac{v}{c_0}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $k = \frac{\omega}{c_0}$.
In addition we allow for small dielectric losses yielding a complex wave number $\underline{k} = k\,(1-\mathrm{j}\delta)$, $\delta \ll 1$.

5 FIELD MATCHING

The boundary and continuity condition in the plane $y = a$ yield a system of linear equations for the unknown coefficients $B_{mn}^{(1)}$, $B_{mn}^{(2)}$ and A_{mn} via an usual orthogonal expansion

$$\begin{aligned}
A_{pq}I_{pq} &= \sum_{m,n,TX}\left\{(-)^{m+q} - 1\right\} M_{mn/pq}^{(1)} W_{mn}^{(1)}(a) B_{mn}^{(1)} + \\
&+ \sum_{m,n,TX}\left\{(-)^{p+q} - 1\right\} M_{mn/pq}^{(2)} W_{mn}^{(2)}(a) B_{mn}^{(2)} \\
S_{pq}^{\binom{(1)}{(2)}} &= Y_{pq}^{\binom{(1)}{(2)}} J_{pq}^{\binom{(1)}{(2)}} B_{pq}^{\binom{(1)}{(2)}} - \\
&- \sum_{m,n,TX} \overset{*}{M}{}_{pq/mn}^{\binom{(1)}{(2)}} V'_{mn}(a) Y_{mn}^{(3)} A_{mn}
\end{aligned} \tag{5}$$

with

$$\begin{aligned}
J_{pq}^{\binom{(1)}{(2)}} &= \int_0^{g,g_1}\int_0^{w,w_1}\left|\mathbf{G}_{pq}^{\binom{(1)}{(2)}}(\xi\eta)\right|^2 \mathrm{d}\xi\,\mathrm{d}\eta \\
I_{pq} &= \int_0^{g_1+t_1}\int_0^{L_x}\mathbf{F}_{pq}(x,z)\cdot\overset{*}{\mathbf{F}}_{pq}(x,z)\,\mathrm{d}x\,\mathrm{d}z \\
M_{mn/pq}^{\binom{(1)}{(2)}} &= \int_0^{g,g_1}\int_0^{w,w_1}\mathbf{G}_{mn}^{\binom{(1)}{(2)}}(\xi,\eta)\cdot\overset{*}{\mathbf{F}}_{pq}(x,z)\,\mathrm{d}\xi\,\mathrm{d}\eta \\
S_{pq}^{\binom{(1)}{(2)}} &= \int_0^{g,g_1}\int_0^{w,w_1}\left[\mathbf{e}_y\times\mathbf{H}^{(s)}\right]\cdot\mathbf{G}_{mn}^{\binom{(1)}{(2)}}(\xi,\eta)\,\mathrm{d}\xi\,\mathrm{d}\eta
\end{aligned}$$

6 THE COUPLING IMPEDANCE

The beam coupling impedance of a periodic structure with period length L_g is given by the definition

$$Z'_{\parallel}(\omega) = -\frac{1}{qL_g}\int_0^{L_g} E_{\parallel}(\omega)\,\mathrm{e}^{\mathrm{j}\frac{k}{\beta}z}\,\mathrm{d}z\,,\;\mathbf{Z}'_{\perp} = \frac{\beta}{k}\nabla Z'_{\parallel} \tag{6}$$

where $Z'_{\parallel}(\omega)$ denotes the longitudinal and $\mathbf{Z}'_{\perp}(\omega)$ the transverse part w.r.t. the particles trajectory. For a particle moving in the plane $y = 0$ we get

$$\begin{aligned} \frac{Z'_{\parallel}}{Z_0} &= \sum_{m=1}^{\infty} \left(\frac{m\pi}{L_x} A^{TE}_{m0} + \frac{\mathrm{j}k}{\beta} A^{TM}_{m0} \right) \cdot \\ &\cdot \quad \sin \frac{m\pi\,(x + L_x/2)}{L_x} V_{m0}(0). \end{aligned} \tag{7}$$

For an infinite periodic structure the real part of this impedance becomes δ–function like at resonance frequencies ω_r for which the phase velocities in the structures passband are equal to the speed of the driving particle. Since we have allowed for small dielectric losses the resonances get a finite bandwidth. The behaviour of the real part of the impedance can then well be approximated in the neighborhood of a resonance frequency f_r by

$$\Re\{Z\} = \frac{R_s}{2} \frac{1}{1 + V^2 Q^2}\,,\; V = \left(\frac{f}{f_r} - \frac{f_r}{f} \right). \tag{8}$$

In order to obtain the shuntimpedance R_s the Q–value and the resonance frequency f_r it is sufficient to evaluate the impedance at three different frequency points.

7 NUMERICAL RESULTS

For an accelerating structure without side coupling cells ($g_1 = 0$) operating with a 94 GHz π–mode the following dimensions are chosen (all in mm):

$$\begin{array}{lll} 2a = 0.75 & 2b = 2.29 & L_x = 10 \\ g = 1.346 & t = 0.25 & w = 2.261 \end{array}$$

On the other hand for a structure without main cells ($g = 0$) one finds a 94 GHz π–mode with

$$g_1 = 2.4 \quad t_1 = 0.791 \quad w_1 = 2.105$$

and the other dimensions unchanged. Fig.4 shows the corresponding field pattern. Let us now consider the complete

a)
b)

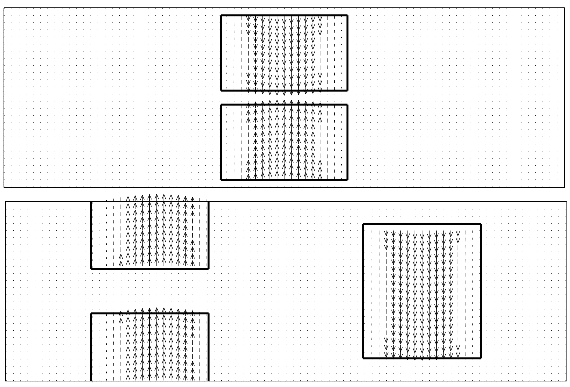
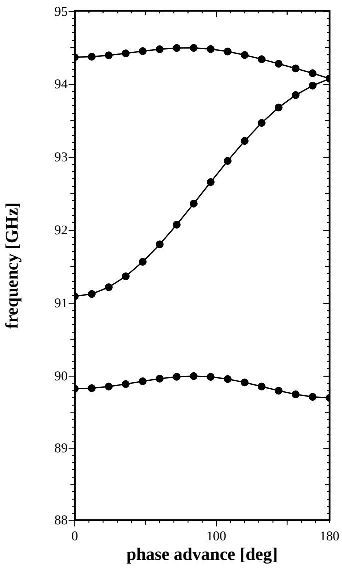


Figure 4: The 94 GHz π–mode field pattern in a structure a) without side coupling cells b) without main cells.

side coupled accelerating structure. Changing the cavity width w and w_1 slightly the pass bands in Fig. 5 arise with confluence of the upper two pass bands in the π–mode. The resulting parameters are

$$\frac{f_\pi}{\mathrm{GHz}} = 94.07\,,\; \frac{R_s}{Q} = 79.6\,\frac{\mathrm{k}\Omega}{\mathrm{m}}\,,\; \frac{v_{\mathrm{gr}}}{c_0} = 1\%\,.$$

These values agree quite good with results from the FD computer code GdfidL [5].



Number of TE/TM-modes ($m \times n$):

$$\begin{array}{l} \text{region } 1:\; 5 \times 3 \\ \text{region } 2:\; 5 \times 5 \\ \text{region } 3:\; 22 \times 7 \end{array}$$

$$\begin{array}{lcll} 2a & = & 0.75 & \mathrm{mm} \\ 2b & = & 2.29 & \mathrm{mm} \\ L_x & = & 10.0 & \mathrm{mm} \\ w & = & 2.253 & \mathrm{mm} \\ w_1 & = & 2.1 & \mathrm{mm} \\ g & = & 1.346 & \mathrm{mm} \\ g_1 & = & 2.4 & \mathrm{mm} \\ t & = & 0.25 & \mathrm{mm} \\ t_1 & = & 0.791 & \mathrm{mm} \\ s & = & 0.25 & \mathrm{mm} \end{array}$$

Figure 5: Dispersion relation of a side coupled accelerating structure with confluence in the π–mode.

8 REFERENCES

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