

# BEAM TRANSMISSION EFFICIENCY STUDY AT RIKEN RFQ LINAC

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## Abstract

Beam dynamics study in an RFQ linac using 3D particle-in-cell code BEAMPATH was performed to analyze beam parameters due to variation of RFQ field. The study shows that linear approximation of RFQ parameters adopted in PARMTEQ code along every cell requires correct treatment of vane cutting in order to reproduce the original PARMTEQ design. Our RFQ layout [1] has non-adiabatically changing region from buncher to accelerating section. For that we noticed that inappropriate treatment of PARMTEQ design can provide mismatching of the beam with the channel and particle loss. Among the linearly varied functions of aperture and electrode modulation along every cell we choose the pair of that parameters which represent the initial PARMTEQ design in the most appropriate way. Final selection of RFQ structure is characterized by stable values of beam transmission efficiency (around 90%) both for zero-current mode and for space charge dominated regime. Effect of vane errors on beam parameters was studied as well to define the engineering tolerance for RFQ vane machining and alignment.

## 1 INTRODUCTION

In many experimental RFQ tests the observed value of beam transmission efficiency in RFQ linac is smaller than it is expected from theoretical predictions. Possible sources for it were found to be higher order terms in RFQ potential and image charges [2]. However, our study of beam dynamics indicates that transmission efficiency and other beam parameters can be affected even in the case of two lowest order RFQ potential terms

$$U(r, \theta, z) = \frac{U_0}{2} [X \left(\frac{r}{a}\right)^2 \cos 2\theta + A I_0(kr) \sin(kz)], \quad (1)$$

due to different treatment of PARMTEQ vane tip design. It comes from the fact that in PARMTEQ design the quadrupole gradient  $XU_0/a^2$  and the axial voltage  $AU_0$  vary linearly over each cell and the final listing of RFQ parameters contains the geometry design values (aperture  $a$  and electrode modulation  $m$ ) at the end of each cell. Different interpretation of output list parameters results in systematic deviation from original design. We examined this effect as well as effect of vane tip manufacturing errors on beam transmission efficiency for RFQ structure designed for upgrading the RIKEN Linear Accelerator (RILAC) [1].

## 2 EFFECT OF FIELD VARIATION

Initial design of the RFQ linac was made with standard approach using PARMTEQ code [3]. In PARMTEQ code every cell is generated with interpolation of input parameters: normalized quadrupole gradient  $B$ , synchronous phase  $\phi_s$ , vane modulation  $m$  and intervane voltage  $U_0$ . Those initial dependencies are transformed in geometry parameters (cell length  $L_j$ , aperture  $a_j$  and electrode modulation  $m_j$ ) by iterative procedure to adjust cell length with local electric field and required phase shift of synchronous particle per cell.

Output listing of PARMTEQ contains cell lengths  $L$  as well as aperture  $a$  and electrode modulation  $m$  at the end of each cell. During the vane tip machinery the monotonous functions  $m(z)$ ,  $a(z)$  could be approximated in different ways which result in deviation of parameters  $A(z)$ ,  $X(z)$  from original design. Additional reason for deviation from original design is that coefficients  $X$  and  $A$  should be considered as  $z$ -independent values at every cell due to their definition as Fourier-Bessel coefficients:

$$X = \frac{1}{U_0 \pi^2} \int_0^{2\pi} \int_0^{2\pi} U(a, \theta, z) \cos 2\theta \, d\theta \, d(kz) \quad (2)$$

$$A = \frac{2}{\pi^2 U_0 I_0(ka)} \int_0^{2\pi} \int_0^{2\pi} U(a, \theta, z) \sin kz \, d\theta \, d(kz) .$$

In order to verify the effect of different representation of  $X(z)$ ,  $A(z)$ ,  $a(z)$  and  $m(z)$  we checked the following approximations of original PARMTEQ design (see fig. 1):

- 1) linear variation of acceleration efficiency  $A$  and focusing efficiency  $X$  along each cell (PARMTEQ - type field);
- 2) a step-wise function of above parameters assuming that the values of  $\langle X_i \rangle = 0.5 (X_{i-1} + X_i)$  and  $\langle A_i \rangle = 0.5 (A_{i-1} + A_i)$  equal to mean values of that parameters over each cell;
- 3) a step-wise function of  $A, X$  assuming that at every cell aperture and electrode modulation are equal to their average values  $\langle a_i \rangle = 0.5 (a_i + a_{i+1})$ ,  $\langle m_i \rangle = 0.5(m_i + m_{i+1})$ ;
- 4) a step-wise function of  $A, X$  assuming that parameters equal to the PARMTEQ designed values at the end of each cell  $A_j, X_j$ .

Calculations were done using general-purpose particle-in-cell code BEAMPATH [4]. Beam was represented as a collection of  $10^4$  model particles. Trajectories were calculated using 20 steps per cell in the

field which is a combination of RFQ potential (1) and self field of the train of bunches. Space charge field was found at each time step from the 3-dimensional Poisson's equation on the  $N_X \times N_Y \times N_Z = 64 \times 64 \times 256$  mesh with the Dirichlet boundary conditions for potential on the surface of square pipe and periodic conditions in longitudinal direction.

Results of calculations for the beam with mass/charge ratio  $A/Z = 5$  are presented in Table 1. For linear - varied field at every cell (PARMTEQ - type field) the beam transmission efficiency calculated from BEAMPATH code is close to PARMTEQ predictions (see results 1,2 in Table 1). Almost the same values of beam transmission efficiency were observed with step-wise field representation of RFQ field where parameters were supposed to be equal to the average values  $\langle X_i \rangle, \langle A_i \rangle$  at each cell (result 3 in Table 1). Smaller value of beam transmission efficiency for  $I = 1\text{mA}$  can be explained by different technique in space charge calculations: in BEAMPATH the 3D space charge field is renewed every time step while in PARMTEQ 2D space charge field appear as a kick once per cell.

The other approximations of original design (results 4,5 in Table 1) show decreasing of originally expected beam transmission efficiencies, especially for the case where  $a, m$  at each cell were supposed to be equal to the designed values at the end of the cell (result 5 in Table 1). Systematic changes of parameters  $A, X$  at every cell due to different representation of RFQ field have the effect of permanent error in vane geometry design which results in deviation in synchronous phase of the bunch and mismatch of the beam with the channel. This deviation is the most serious in the region between buncher and accelerating section. This region is characterized by a sharp increase of electrode modulation which results in a quick change of acceleration gradient along the structure. Most of the particle losses are observed in this area. Beam losses in the longitudinal direction are much smaller (typically 2% for zero beam current and 4% for beam current  $I = 1\text{mA}$ ).

Table 1. Beam transmission efficiency in RFQ structure ( $A/Z = 5, f = 40\text{MHz}$ ).

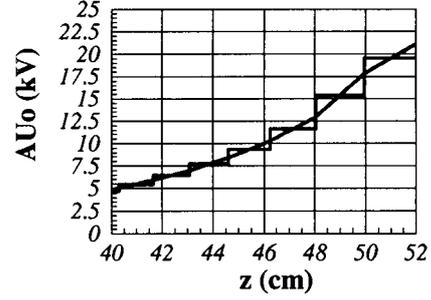
Code	Field Variation over each cell	$I=0$	$I=1\text{mA}$
1. PARMTEQ	Linear	0.94	0.90
2. BEAMPATH	Linear (PARMTEQ-type)	0.94	0.87
3. BEAMPATH	Step-wise ( $\langle X_i \rangle, \langle A_i \rangle$ )	0.94	0.86
4. BEAMPATH	Step-wise ( $\langle a_i \rangle, \langle m_i \rangle$ )	0.92	0.78
5. BEAMPATH	Step-wise ( $X_i, A_i$ )	0.86	0.70

### 3 VANE MANUFACTURING ERRORS

Random errors in manufacturing RFQ vane tips result in the amplitude growth of transverse and longitudinal oscillations. Analytical treatment [5] shows monotonous enlargement of transverse oscillation amplitude  $r_{\text{max}}$  and vertical size of separatrix  $\langle \Delta g \rangle =$

$\langle \Delta P \rangle / v_s$  after passing through the RFQ section with  $N$  cells:

a



b

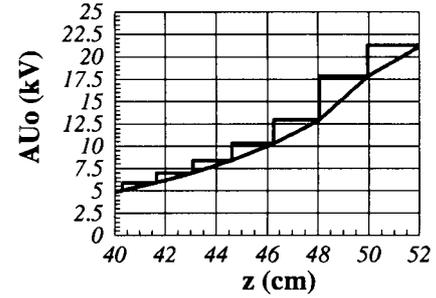


Fig. 1. Different approximation of PARMTEQ axial voltage design: a - average step-wise approximation at each cell; b - step-wise approximation at the end of each cell.

$$\langle \delta r_{\text{max}} \rangle^2 = 2N [\langle \delta r_o \rangle^2 + r_{\text{max}}^2 (\langle \frac{\delta U}{U} \rangle^2 + 4 \langle \frac{\delta R_o}{R_o} \rangle^2)]; \quad (3)$$

$$\begin{aligned} \langle \Delta g \rangle^2 = & \pi^2 \left( \frac{\Omega_f}{\omega} \right)^2 \left( \frac{W_o}{\Delta W} + N \right) \left\{ \left( \frac{\Omega_f}{\omega} \right)^2 \text{ctg}^2 \varphi_n \right. \\ & \left. \left[ 1 - \sqrt{\frac{W_o}{W_f}} \right] \left[ \frac{\langle \delta A \rangle^2}{A^2} + \frac{\langle \delta U \rangle^2}{U^2} \right] + \right. \\ & \left. + \frac{1}{3} \left[ 1 - \left( \frac{W_o}{W_k} \right)^{3/2} \right] \frac{\langle \delta L \rangle^2}{L^2} \right\}, \quad (4) \end{aligned}$$

where  $\delta r_o$  is an axis displacement,  $\delta R_o$  is an error in average radius of the structure  $R_o = a / \sqrt{X}$ ,  $\delta L$  is an error in cell length,  $\delta U$  is an intervane voltage instability,  $\Omega_f$  is a longitudinal oscillation frequency,  $\omega$  is an RF frequency,  $W$  is a particle energy,  $A$  is an acceleration efficiency and  $\varphi$  is a synchronous phase. To study this effect via computer simulation, the following parameters were randomly distributed at every cell within the max error  $\pm \delta$ : cell lengths  $L_i$ , aperture radius  $a_i$ , max distance from axis to electrodes  $m_i$  and axis displacement  $\delta r_{oi}$ . Results of simulation (see Table 2 and Fig. 2) demonstrate that the error of 50 microns does not create any serious degradation of the beam parameters while error of 100 microns could cause notable decreasing of beam transmission efficiency. The engineering tolerance of 50 microns was thus adopted for vane tips fabrications.

Table 2 . Beam transmission efficiency due to errors in vane fabrication.

	$\delta$ , microns	I = 0	I = 1 mA
1.	0	0.94	0.87
2.	50	0.92	0.85
3.	100	0.80	0.70

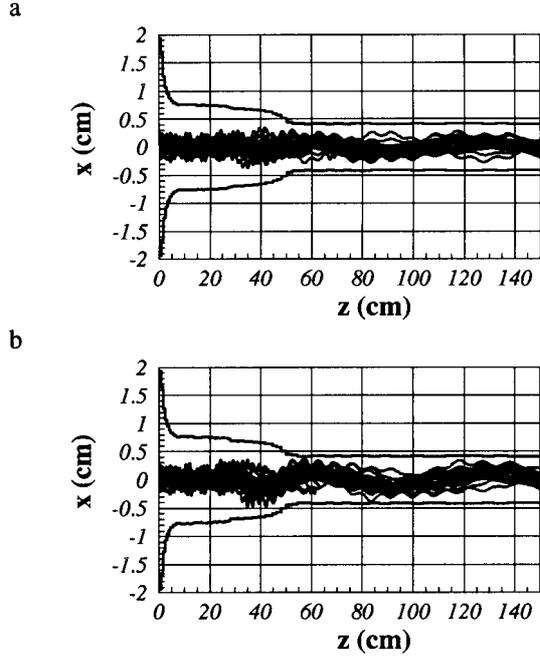


Fig. 2. Particle trajectories in RFQ structure with various value of error in vane fabrication: a)  $\pm 50$  microns; b)  $\pm 100$  microns.

#### 4 VANES ASYMMETRY

Disturbance of quadrupole RFQ symmetry result in appearance of dipole field component. This effect was studied for the case when one of the RFQ electrodes is shifted from its ideal position at the distance  $\Delta$ . It means that extra terms containing  $\cos\theta$ ,  $\cos 2\theta$ ,  $\cos 3\theta$ ,... appear in the potential expansion. We restrict our consideration by adding two lowest order terms in RFQ potential:

$$U(r,\theta,z) = -\frac{U_0}{2} \left[ X \left(\frac{r}{a}\right)^2 \cos 2\theta + A I_0(kr) \sin(kz) + A_{01} \frac{r}{a} \cos \theta + A_{11} I_1(kr) \cos \theta \sin(kz) \right] . \quad (5)$$

Coefficients  $A_{01}$ ,  $A_{11}$  are defined by new boundary conditions where potential is kept constant at the surface of electrodes  $U(r,\theta,z) = -U_0/2$ :

$$\begin{aligned} U(r,\theta,z) &= -U_0/2 & x &= -a(z) , \\ U(r,\theta,z) &= -U_0/2 & x &= a(z) + \Delta , \end{aligned} \quad (6)$$

where  $a(z)$  is the vane-tip profile of the modulated electrodes defined by equation:

$$X \left[ \frac{a(z)}{a} \right]^2 + A I_0[k a(z)] \sin(kz) = 1 . \quad (7)$$

From boundary conditions (6) the expression for RFQ potential with dipole component is given by:

$$\begin{aligned} V(r,\theta,z) &= -\frac{U_0}{2} \left[ X \left(\frac{r}{a}\right)^2 \cos 2\theta + A I_0(kr) \sin(kz) \right] + \\ &+ \frac{U_0}{2} \frac{[x + a(z)] \Delta}{a^2} \left[ X + \frac{A}{4} (ka)^2 \sin(kz) \right] . \end{aligned} \quad (8)$$

Dipole component decreases the focusing strength of pure RFQ field if electrode is shifted outside the channel  $\Delta > 0$  (and vice versa if  $\Delta < 0$ ):

$$E_{dipole}^x = -\frac{U_0}{2} \frac{\Delta}{a^2} \left[ X + \frac{A}{4} (ka)^2 \sin(kz) \right] . \quad (9)$$

Dipole component results in shifting of stable oscillation point from the axis . From equation (8) the new stability point  $E_x(x_0) = 0$  is defined as  $x_0 = \Delta/2$ . Beam dynamics calculation performed for electrode displacement of  $\Delta = 50$  microns shows that center mass of the beam oscillates around the new stability point  $x_0 = 25$  microns. In this case no significant changes of beam parameters were observed.

#### 5 CONCLUSIONS

In summary performed study showed the significant sensitivity of beam dynamics on different representation of RFQ field even in the case of two lower terms in RFQ potential. The vanes of our RFQ were designed in the way to adopt values  $X$  and  $A$  as average values from original linear function in PARMTEQ design. Final version of RFQ structure is characterized by the initially expected value of beam transmission efficiency. Dipole component originating from shifting of the RFQ electrode results in displacement of stability oscillation point and does not influence beam dynamics when the value of electrode shift is much smaller than aperture radius of the channel.

#### 6 REFERENCES

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