

# THE NEW BNL AGS PHASE, RADIAL AND SYNCHRONIZATION LOOPS\*

E. ONILLON, J. M. BRENNAN, Brookhaven National Laboratory,  
AGS Department, Upton, NY 11973, USA

## 1. INTRODUCTION

The AGS and the RHIC must be synchronized before bunch-to-bucket transfer of the beam. A feedback loop has been designed and an improvement has been made to the AGS phase and radial loops. In both cases, the design uses a state variable representation to achieve greater stability and smaller errors. The state variables are beam phase, frequency and radius, the integral of the difference between the radius and its reference and the phase deviation of the bunch from the synchronous phase. Furthermore, the feedback gains are programmed as a function of the beam parameters to keep the same loop performances through the acceleration cycle.

## 2. DESCRIPTION OF THE LOOPS

### 2.1 Variables and transfer functions

The main variables used to describe the system are:

- $\phi_b$  the phase of the beam with respect to the RF
- $\phi$  the instantaneous phase deviation of the bunch from the synchronous phase
- $\delta\omega_b$  the variations of the beam frequency
- $\delta R$  the variations of the beam radius
- $\phi_s$  the synchronous phase
- $E$  the total energy
- $V_{rf}$  the accelerating voltage
- $\phi_{rf}$  the RF phase
- $\omega_{rf}$  the RF frequency
- $f_\infty$  the asymptotic revolution frequency
- $\xi$  the beam damping coefficient (all the calculations have been performed with  $\xi = 0.01$ ).

The cavity, around which an RF feedback is closed, can be described by its pole  $s_c$ .

These variables are related by the three following transfer functions (Fig. 1) [1]:

$$\phi = B_\phi \delta\omega_{rf}$$

$$\delta\omega_b = B_\omega \delta\omega_{rf}$$

$$\delta R = B_R \delta\omega_{rf}$$

$$\text{with: } B_\phi = \frac{s}{s^2 + 2\xi\omega_s s + \omega_s^2}, \quad B_\omega = \frac{\omega_s}{s^2 + 2\xi\omega_s s + \omega_s^2},$$

$$B_R = \frac{b}{s^2 + 2\xi\omega_s s + \omega_s^2} \quad \text{where } b = \frac{ceV_{rf} \cos \phi_s}{2\pi\beta\gamma_{tr} E} \quad \text{and}$$

$$\omega_s = f_\infty \sqrt{\frac{2\pi e V_{rf} \cos \phi_s h |\eta|}{E}}.$$

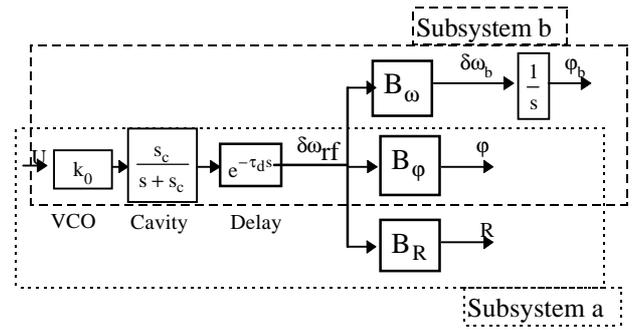


Fig. 1 Model of the system

### 2.2 The phase and radial loop

In a first approximation, we presume that the cavity transfer function is one, the beam damping term  $\xi$  is zero, and the delays of the system are neglected. The corresponding subsystem is a on Fig.1.

It can be described using two state variables 
$$\begin{cases} x_1 = \frac{R}{b} = \frac{k_0}{s^2 + \omega_s^2} U \\ x_2 = \dot{x}_1 = \phi \end{cases}$$
. A third one, corresponding to

the integral of the difference of the radius and its reference, is introduced to force the radius to follow its reference  $R_0$ :  $x_3 = z = \int (R - R_0) dt$ . These state variables lead to the following state space representation:

\* Work performed under the auspices of the U.S. Department of Energy.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ -\omega_s^2 & 0 & 0 \\ b & 0 & 0 \end{pmatrix}}_{A_{\varphi R}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{X_{\varphi R}} + \underbrace{\begin{pmatrix} 0 \\ k_0 \\ 0 \end{pmatrix}}_{B_{\varphi R}} U + \begin{pmatrix} 0 \\ 0 \\ -R_0 \end{pmatrix}$$

All three state variables are observed.

As the rank of the matrix  $\begin{bmatrix} B_{\varphi R} & A_{\varphi R} B_{\varphi R} & A_{\varphi R}^2 B_{\varphi R} \end{bmatrix}$  is 3, it is possible to determine a feedback using pole placement [2].

An analytical expression has been found for the three feedback gains. If  $l_1$ ,  $l_2$  and  $l_3$  are the desired poles, the three state gains are:

$$\begin{cases} k_R = (l_1 l_2 + l_1 l_3 + l_2 l_3 - \omega_s^2) / (b k_0) \\ k_\varphi = -(l_1 + l_2 + l_3) / k_0 \\ k_f = -(l_1 + l_2 + l_3) / (b k_0) \end{cases}$$

These expressions will allow the programming of the gain as a function of E.

The command is a linear combination of the state variables (Fig. 2):  $U = -k_R x_1 - k_\varphi x_2 - k_f x_3$ .

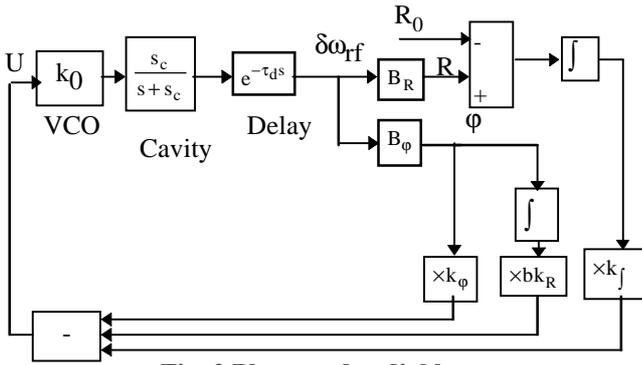


Fig. 2 Phase and radial loop

Simulations have been performed using Matlab™. The following results have been obtained with the three following desired poles:  $-7.7 \cdot 10^4$ ,  $10^3(-5.4-5.5j)$ ,  $10^3(-5.4+5.5j)$ . The reference is a 1 mm radius step.

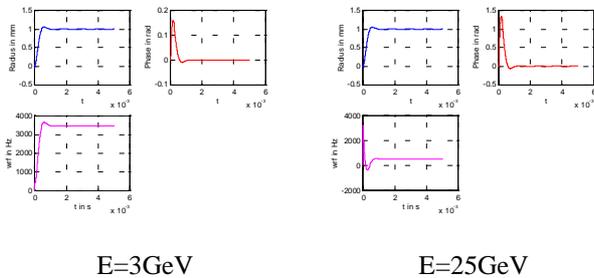


Fig. 3 Radius step response

The radius reaches its final value and the phase is brought back to zero in roughly 2 ms. The corresponding Bode plots are given in Fig. 4.

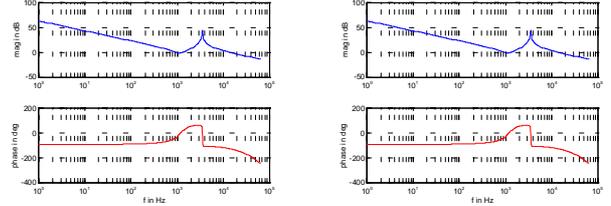


Fig. 4 Phase and Radial loop step response, Bode Plots

In both cases, the phase margin is approximately  $55^\circ$  and the amplitude margin 15dB.

The transition jump is simulated by adding a  $180^\circ$  phase perturbation on the phase. The transfer function between  $\omega_{rf}$  and R has to be made equal to zero, the transition jump having no effect on the radius (Fig. 5). After jumping to  $\pi$ , the phase comes back to zero in less than 0.1ms.

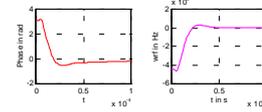


Fig. 5 Simulation of transition

### 2.3 The synchronization loop

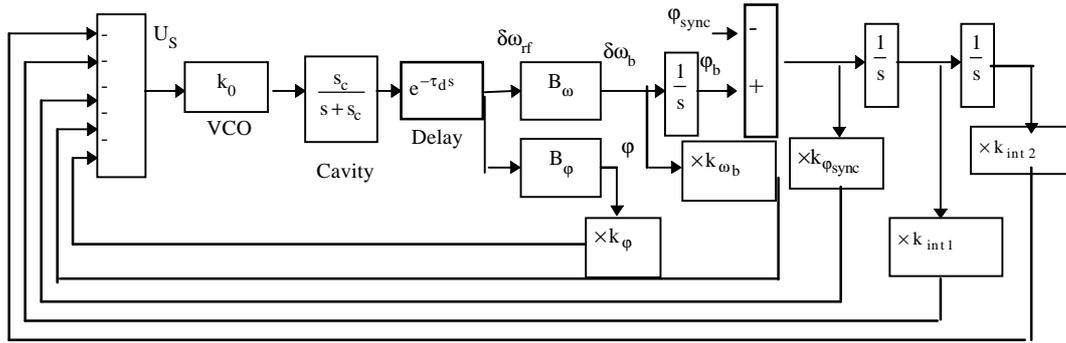
Using the same approximations as in paragraph 2.2, the subsystem b in Fig. 1 can be described with three state variables,  $\varphi_b$ ,  $\omega_b$ ,  $\varphi$ . As we want  $\varphi_b$  to follow a ramp, a double integration is added on  $\varphi_b$ . The state vector is thus :

$$X_s = \begin{bmatrix} \varphi_b \\ \omega_b \\ \varphi \\ \int \varphi_b - \varphi_{sync} \\ \int x_7 \end{bmatrix}^T$$

with  $\varphi_{sync}$  the loop reference. The evolution of  $X_s$  is given by:

$$\begin{pmatrix} \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\omega_s^2 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}}_{A_S} \underbrace{\begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{pmatrix}}_{X_S} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{B_S} U + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\varphi_{sync} \end{pmatrix}$$

All the state variables are observed. As the rank of the matrix  $\begin{bmatrix} B_S & A_S B_S & A_S^2 B_S & A_S^3 B_S & A_S^4 B_S \end{bmatrix}$  is 5, it is again possible to determine a feedback using pole placement, leading to the schematic given in Fig. 6.

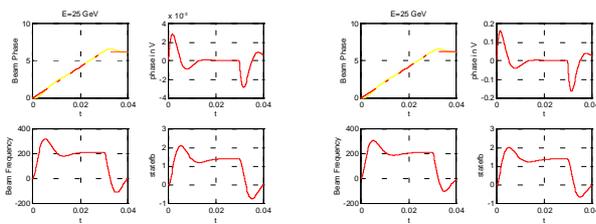


**Fig. 6 Synchronization loop**

The following results have been obtained with the following desired poles:  $-7.6 \cdot 10^4$ ,  $10^2(-2.7 + 3.2j)$ ,  $10^2(-2.7 - 3.2j)$ ,  $-1.9 \cdot 10^2$ ,  $-7.7 \cdot 10^2$ .

## REFERENCES

- [1] RF Beam Control System for the AGS Booster, J. M. Brennan, BNL - 52438 Formal Report.
- [2] Feedback Control of Dynamic Systems, G. F. Franklin, J. D. Powell, Addison-Wesley Publishing Company.



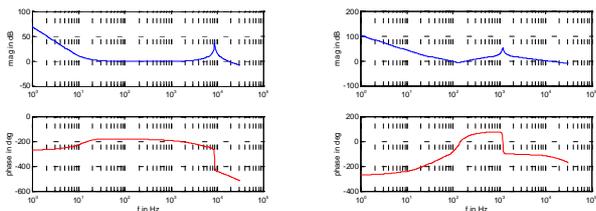
E=3GeV

E=25GeV

**Fig. 7 Synchronization loop, response to a ramp**

The corresponding Bode plots are given in Fig. 8.

In both cases, the phase margin is approximately  $60^\circ$  and the amplitude margin 13dB.



**Fig. 8 Synchronization loop, Bode Plots**

## 3. CONCLUSION

The use of a state space representation has lead to the design of two new feedback loops. These loops should provide us with good stability, a good robustness and performance, despite the system delays. The practical realization only requires gains and summations. Moreover, the analytical expression of the feedback gains will allow the programming of these as a function of energy, so that the loops will keep the same behavior throughout the accelerating cycle.