SENSITIVITY STUDIES OF CRYSTALLINE BEAMS *

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Abstract

The equations of motion are presented, appropriate to interacting charged particles of diverse charge and mass, subject to the external forces produced by various kinds of magnetic fields and radio-frequency (rf) electric fields in storage rings. These equations have been employed in the molecular dynamics simulations for sensitivity studies of crystalline beams. The two necessary conditions for the formation and maintenance of crystalline beams are summarized. The effects of lattice shear and AG focusing, magnetic field imperfection, and ion neutralization on crystalline beam heating is presented.

1 INTRODUCTION

In previous work, the nature of crystalline beams has been studied by many investigators.[1]In particular, we have made a systematic study in a series of papers[2] which has consisted of deriving the equations of motion for charged particles undergoing Coulomb interaction in a real storage ring, and then using them to explore many properties of crystalline beams.

In this paper, we carry on our investigation with sensitivity studies of crystalline beams. We summarize first in Section 2 the generalized equations of motion, and then in Section 3 the conditions for the formation and maintenance of crystalline beams. In Sections 4, 5, and 6, we present the results of sensitivity studies of crystalline beams on lattice shear and AG focusing, lattice imperfection, and ion neutralization, respectively. The conclusions are given in Section 7.

2 EQUATIONS OF MOTION

In order to adopt the molecular dynamics (MD) methods, we employ[2] a rotating frame (x, y, z, t) of a reference particle in which the orientation of the axes is rotating so that the axes are constantly aligned to the radial (x), vertical (y), and tangential (z) direction of motion. It is convenient to define a reference particle with charge state Z_0 and atomic mass M_0. We scale dimensions in terms of the characteristic distance ξ, with $\xi^3 = r_0\rho^2/\beta^2\gamma^2$, where $r_0 = Z_0^2e^2/M_0c^2$, the velocity of the reference particle is βc, its energy is γM_0c^2, and it moves on an orbit with bending radius ρ in magnetic field B_0. We measure time in units of $\rho/\beta\gamma c$ and energy in units of $\beta^2\gamma^2 Z_0^2e^2/\xi$. For the ith species of ions with charge state Z_i and mass M_i, we define the relative charge and mass with respect to the reference values

$$\bar{Z}_i \equiv Z_i/Z_0, \text{ and } m_i \equiv M_i/M_0. \tag{1}$$

In a bending region with bending radius ρ, the Hamiltonian for particles of the ith species is derived as

$$\begin{aligned} H_i &= \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) + \frac{1}{2}\left[x^2 + \left(\frac{\bar{Z}_i}{m_i} - 1\right)\gamma^2 z^2\right] \\ &\quad - \left(2 - \frac{\bar{Z}_i}{m_i}\right)\gamma x P_z + \left(\frac{\bar{Z}_i}{m_i} - 1\right)\frac{\rho}{\xi}\, x + V_{Ci}, \end{aligned} \tag{2}$$

where the Coulomb potential is

$$V_{Ci} = \sum_j \frac{\bar{Z}_i\bar{Z}_j}{m_i\sqrt{(x_j - x)^2 + (y_j - y)^2 + (z_j - z)^2}}, \tag{3}$$

and the summation, j, is over all the other particles. In a straight section, where there is no bending of particles, there often are quadrupole magnets for focusing, sextupole magnets for chromatic correction, and electric field for acceleration or bunching, etc.. If the normal quadrupole, skew quadrupole, and sextupole strengths are represented by

$$n_1 = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial x}, \quad n_{1s} = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial y}, \quad n_2 = -\frac{\rho}{B_0}\frac{\partial^2 B_y}{\partial x^2}, \tag{4}$$

respectively, the Hamiltonian can be derived as

$$\begin{aligned} H_i &= \frac{1}{2}\left(P_x^2 + P_y^2 + P_z^2\right) - \frac{n_1}{2}\frac{\bar{Z}_i}{m_i}(x^2 - y^2) \\ &\quad - n_{1s}\frac{\bar{Z}_i}{m_i}xy - \frac{n_2\xi}{6}\frac{\bar{Z}_i}{m_i}(x^3 - 3xy^2) + V_{Ci} + U_s, \end{aligned} \tag{5}$$

where the electrical force F_s in the reduced units can be expressed in terms of electric field E_s measured in the laboratory frame,

$$F_s \equiv -\frac{\partial U_s}{\partial z} = \frac{\bar{Z}_i}{m_i}\frac{Z_0 eE_s\xi}{M_0c^2}\left(\frac{\rho}{\xi\beta\gamma}\right)^2. \tag{6}$$

The equations of motion can be obtained from the Hamilton's equations using Eqs. 2 and 5.

3 CONDITIONS FOR CRYSTALLIZATION

The first condition arises from the criteria of stable transverse motion when particles are crystallized. It requires

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that the ring is AG focusing, and that the energy of the particles is less than the transition energy of the ring.

The second condition arises from the criteria that there is no linear resonance between the phonon modes of the crystalline structure and the machine lattice periodicity. It requires that the ring is designed such that the lattice periodicity is at least $2\sqrt{2}$ as high as the maximum betatron tune to adequately maintain crystalline beams at any beam density.

4 EFFECTS OF LATTICE SHEAR AND AG FOCUSING

According to the first condition, a storage ring for crystalline beams is necessarily AG focusing. In addition to bending sections, the ring also consists of straight sections for cooling, acceleration, and experiments. As the crystalline beam circulates around the ring, its ground state is periodic in time with the period of the ring lattice. Fig. 1 shows an example of the ground state of a crystalline beam.

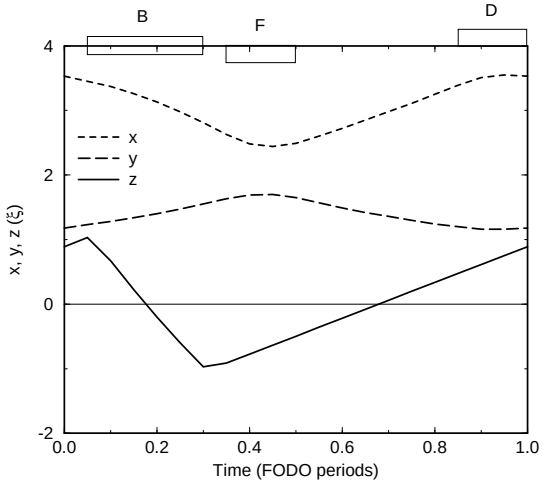

Figure 1: Typical trajectory of a coasting crystalline beam. Lattice components are displayed on the figure: B is a bending section, F and D are focusing and de-focusing sections.

The ground state structure is a single-layer shell. The machine, having a periodicity of 10, consists of 10 FODO cells with 25% bending with $\nu_x = 1.96$, $\nu_y = 2.06$, and $\gamma_T = 1.81$. The beam energy corresponds to $\gamma = 1.4$. The variation in time of the trajectory in the transverse direction (x and y) corresponds to AG focusing, and the variation in the longitudinal direction (z) corresponds to the variation in bending and straight sections (shear).

Because of the time dependence, the crystalline structure absorbs energy from the lattice at non-zero temperature. Define the normalized temperature[2] in terms of the deviation of momenta P_x, P_y and P_z from their ground-state values, squared and averaged over particles for a relatively long time. The + symbol in Fig. 2 shows the heating rate as a function of temperature for a beam at density $N/L = 5$ (5 particles per unit MD cell). In order to investigate the effects of AG focusing and shear on the heating rate, we created two idealized machine lattices with the same transverse tunes and lattice periodicity for comparison: one with the similar AG focusing but constant bending around the ring (as shown by the filled squares in Fig. 2); the other with the same 25% bending section but constant focusing around the ring (as shown by the empty squares in Fig. 2). In all the cases, the phonon spectrum of the beam is similar relative to the lattice modulation frequency (periodicity). Consequently, the heating behavior is similar, i.e., AG focusing and shear cause similar beam heat up. Not surprisingly, when the bending and focusing are both constant with time, the temperature of the crystalline beam stays constant.

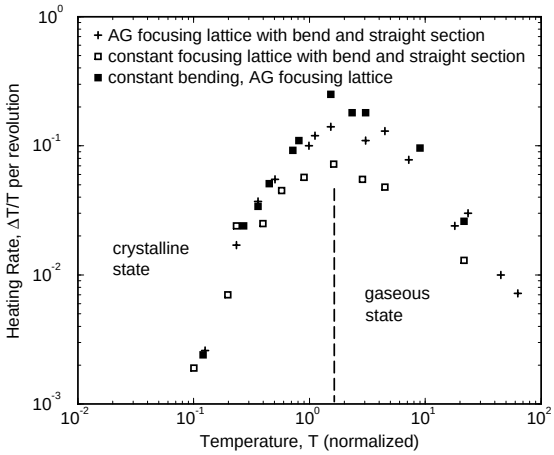

Figure 2: Heating rates as functions of the normalized temperature for crystalline beams of equivalent density but different machine lattices.

5 EFFECTS OF MAGNETIC FIELD IMPERFECTION

In an actual magnet in a storage ring, the fields will differ somewhat from the ideal design, resulting in beam misalignment and focusing gradient errors. Strictly speaking, the machine lattice periodicity is equal to 1, and the second condition for crystallization is not satisfied. In Section 4, we have demonstrated that the effect of variation in bending and focusing on beam heating is similar. Therefore, we here only present the results on the effects of field gradient error in the quadrupole magnets.

With the same machine lattice as in Fig. 1, the + symbol in Fig. 3 shows the heating rate as a function of normalized temperature T for the beam at density $N/L = 5$. With $Z = A = 1$, and $\xi = 1.2$ μm, the conventionally used beam temperature T_B can be obtained from T by $T_B \approx 7.5\,T$ [°K]. To study the effect of gradient errors, we assume that the strength n_{Fi} and n_{Di} of the ith focusing and de-focusing quadrupoles randomly deviate from their design values ($n_F = -1.4$ and $n_D = 2.35$ at $\rho = 1$ m) according to the expression

$$n_{Fi,Di} = n_{F,D}\,(1 + g\,G_i)\,, \qquad (7)$$

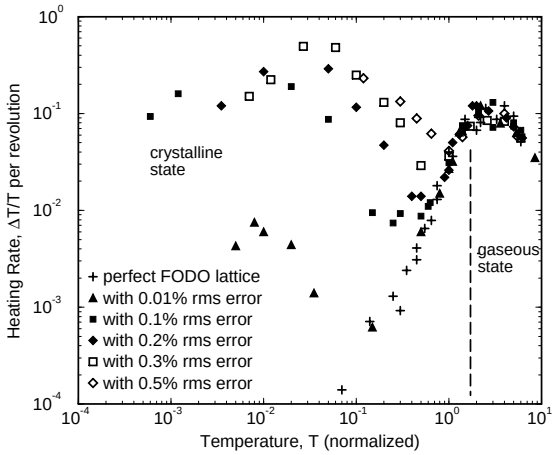

Figure 3: Effects of gradient errors in storage ring quadrupole magnets for a beam of density $N/L = 5$ ($N = 100$, and $L = 20\ \xi$).

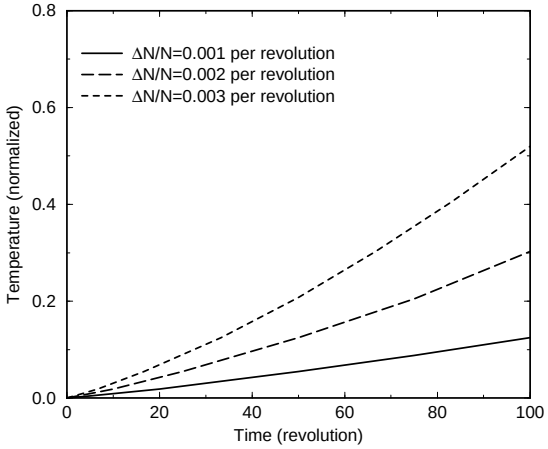

Figure 4: Temperature evolution for ion neutralization rate of $\Delta N/N = 0.001, 0.002,$ and 0.003 per revolution in the absence of cooling, starting from the crystalline beam ground state at density $N/L = 5$.

where g is the amplitude of the random error, and G_i returns a normally distributed deviate with zero mean and unit variance. Fig. 3 shows the change in heating rate when g is increased from 0 to 10^{-4}, 10^{-3}, 2×10^{-3}, 3×10^{-3}, and 5×10^{-3}. As the error increases, the lattice deviates significantly from its design periodicity of 10. Even at low temperature, the beam absorbs energy from the imperfect lattice. Fig. 3 indicates that when the rms error is larger than 10^{-3}, the heating rate at low temperature exceeds the peak rate at $T \approx 2$. On the other hand, the accuracy in the construction of modern storage ring magnets is typically at 10^{-4} level. With this standard, magnetic imperfection is of little concern in crystalline beam maintenance.

6 EFFECTS OF ION NEUTRALIZATION

Ion neutralization caused by gas scattering and other mechanisms can disrupt the crystalline structure and cause beam heating. We simulate this neutralization process by removing the particles from the crystalline structure at a given rate. Starting from the ground state, when a particle is removed, the structure deviates from its ground state. The potential energy is then transferred into kinetic energy, resulting in a finite temperature. With the same lattice as in Fig. 1 and the same beam density of $N/L = 5$, Fig. 4 shows the increase of heating rate for various neutralization rates of 0.1%, 0.2%, 0.3% per revolution. It is found that the peak rate ($T \approx 2$ in Fig. 3) from the lattice heating corresponds to a neutralization rates of about 0.5% per revolution. Therefore, ion neutralization at a rate much less than $\Delta N/N$ =0.5% per revolution is of little concern in crystalline beam formation.

7 CONCLUSIONS

In this paper, we have summarized the conditions for the formation and maintenance of crystalline beams. We have demonstrated that lattice shear and AG focusing have similar effects in beam heating. In an example, we have shown that magnetic field error at less than 0.1% level can be tolerated. Ion neutralization at a rate much less than 0.5% per revolution is also of little concern in crystalline beam formation.

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8 REFERENCES

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