SIMULATION OF FEEDBACK FOR ORBIT CORRECTION

I. Reyzl, Technische Universität Berlin, Institut für Nachrichtentechnik
und Theoretische Elektrotechnik, Sekr. EN 2, Einsteinufer 17, D-10587 Berlin

Abstract

This paper describes the design of a feedback system for the TESLA linear collider study. Based on linear state-space models, the algorithm uses a predictor-corrector formalism of optimal control theory. In order to control the orbit of the beam, the corrector settings are determined (via Linear Quadratic Gaussian Control) by an estimation of the state vector. On the basis of measurements, the state is estimated by the Kalman filter which minimizes the variance of the estimation error. The feedback loop algorithm is given by matrix equations. It is highly advantageous that the applied matrices can be determined before the measurements are available. First results of these numerical simulations are presented.

1 INTRODUCTION

The design studies for a next generation e^+e^- linear collider differ mainly in the choice of bunch charges, rf-frequencies and spot sizes [1]. The TESLA 500 approach uses superconducting Nb accelerating structures operating at 1.3 GHz and is, hence, on the lower end of the rf-frequency scale. The main advantages of this concept are very low wakefields and a high accelerating efficiency, whereas a stable operation with a gradient of 25 MV/m has to be demonstrated. TESLA 500 is aiming for a nominal luminosity of $3.6 \cdot 10^{33}\text{cm}^{-2}\text{s}^{-1}$ at a center of mass energy of 500 GeV and with a repetition rate of 5 Hz. Critical parameters are the requested transverse beamsizes of $\sigma_x^\star \backslash \sigma_y^\star = 845 \backslash 19$ nm and the small vertical emittance of the beam at the interaction point. Several feedback loops for orbit control are required to reduce the influence of disturbances which increase the effective emittance.

The goal of the simulation program to be presented in this paper is to simulate an orbit control in the TESLA beam delivery section by taking into account certain stochastic characteristics of the disturbances. Thus, it is important to develop reasonable models of the noise (typical beam motion) which are adjusted to given rms-values and spectra. It also has to include a control algorithm able to handle random disturbances.

This paper gives a brief introduction to optimal filtering and optimal control. It also describes a method to model the noise as the output of a linear system driven by white noise. Finally, first results of the simulations are presented.

2 CONTROL OF LINEAR DYNAMIC SYSTEMS

The concept of a state space has its roots from cause-and-effect relationships in classical mechanics. The motion of a system is uniquely described by its current state - like position and angle of particles - and the future forces acting on the system. These forces might be well-defined control inputs such as requested corrector settings, as well as stochastically arising disturbances. Furthermore position and angle can not be determined exactly from measurements because of sensor noise. One can only derive an estimation.

The relation between states, forces and measurements is described by a first-order differential equation and an equation of the measurement. By choosing a periodic sampling $t_k = kT,\ k \in \mathbb{N}, T$ sample time, the matrices in these equations are time invariant. The time invariant, discrete state-space formulation [2] reads as follows (the subscript k indicates the k^{th} sample)

$$\begin{aligned} x(k+1) &= A\,x(k) + B\,u(k) + G\,w(k)\,, \\ y(k) &= C\,x(k) + v(k) \quad \text{with} \end{aligned}$$

- $x \in \mathbb{R}^n$: state vector of the system which is controlled. In our case the states are position and angle of the beam and states describing the dynamics of the correctors.
- $u \in \mathbb{R}^m$: control input vector (required corrector settings to steer the beam) which is a known input sequence calculated by linear quadratic Gaussian control (LQG).
- $y \in \mathbb{R}^p$: vector of measurements taken by beam position monitors (BPM).
- $w \in \mathbb{R}^l, v \in \mathbb{R}^p$: system and measurement noise (assumed to be white, Gaussian random sequences with a zero mean).
- $A \in \mathbb{R}^{n\times n}$: is in general the transition matrix derived from the differential equation describing the motion in the observed system. In our case it consists of transfer matrix elements (defined by the linac model), elements describing the disturbances and the time delay for correctors to move to a new required setting.
- $B \in \mathbb{R}^{n\times m}, G \in \mathbb{R}^{n\times l}$: describe the influence of control input and of white noise on the states.

$C \in \mathrm{I\!R}^{p\times n}$: consists of transfer matrix elements defined by the linac model.

2.1 Optimal Filtering

The classical approach to filtering is to suppress unwanted frequency components, whereas the statistical approach uses certain statistical characteristics of the useful signal and of the noise to eliminate as much of the noise as possible. This can be done by processing measured values through the Kalman filter using the least squares ideas of Gauss [3]. The Kalman filter is the best linear filter in the sense that it provides the smallest error covariance by a priori knowledge about the system's uncertainties and measurement noise.
By solving a Riccati difference equation we derive the Kalman gain factor L(k) as a function of time. The current estimate of the state $\hat{x}$ using all measured values till the k^{th} pulse is given by

$$\hat{x}(k) = \bar{x}(k) + L(k)\left(y(k) - C\,\bar{x}(k)\right),$$

where the state x is predicted to be

$$\bar{x}(k+1) = A\,\hat{x}(k) + B\,u(k).$$

2.2 Optimal Control

The control gain $K(k)$ is calculated in a similar way as the applied filter by LQG. The requested control input $u(k)$ depends linearly on states and reference points:

$$u(k) = -K(k)\,\bar{x}(k) + N\,r(k) \quad \text{where}$$

$r \in \mathrm{I\!R}^{q}$: are reference points of position and angle,

$N \in \mathrm{I\!R}^{m\times q}$: maps the reference points to the control input; N is derived from the linac model.

In the calculation of u the time update $\bar{x}$ and not the measurement update $\hat{x}$ is used to simulate a delay of one pulse. Note that the gain matrices $L(k)$ and $K(k)$ depend on time, but they can be derived before the first measurement is obtained [3]. Other advantages are the linear dependence of all important values of the control loop and that the algorithm is recursive.

3 THE DISTURBANCE MODEL

One important part is to investigate the reasons for the beam variation and the characteristics of disturbances. The use of stochastic or random concepts is suitable to describe disturbances appearing in an accelerator (such as measurement errors, beam variation). They are generated as outputs of linear dynamic systems driven by white noise. This shaping filter is designed in such a way that its output matches the spectrum of the assumed disturbances.
Let $W(z)$ and $S(z)$ be the Z-transform of the input signal w_k and of the output signal s_k of the shaping filter, respectively. The relation between input and output is given by $S(z) = H(z)\,W(z)$. $H(z)$ is the transfer function of the shaping filter which is a rational function in z^{-1}:

$$H(z) = \frac{\sum_{n=0}^{N} b_n z^{-n}}{1 + \sum_{m=0}^{M} a_m z^{-m}}.$$

The power spectral density (PSD) of the output, Φ_{ss}, is given by

$$\Phi_{ss}(z) = H(z)\,\Phi_{ww}(z)\,H(z^{-1}),$$

where Φ_{ww} is the PSD of the input white noise. Methods proposed in [4, 5] estimate the coefficients a_m and b_n so that the shaping filter possesses the desired frequency characteristics.

4 THE PROGRAM

The simulation program is written in Fortran 90 which is very suitable for matrix multiplications. It generates the filter parameters used for the noise model, determines the matrices needed for the feedback and simulates the closed loop. In this loop the user can select between different kind of disturbances (dirac, step, white noise) and determine their amplitude and arrival time. Several plots such as the PSD of beam jitter before and after control, the response and the Bode diagram of the loop can be obtained. Before each run the user can vary all values of the assumed disturbance model like rms-values of jitter, frequency range of disturbances and the linac model (i. e. number/position of correctors and BPM's used for feedback; position of control, reference points). It is also possible to obtain plots of the Kalman filter loop comparing the estimated and the actual beam parameters.

5 PRELIMINARY RESULTS

The problem in designing a feedback loop is the wish to satisfy several criteria which often compete with each other. The feedback should provide a good rejection of DC bias and of disturbances, respond quickly to a step perturbation, work even when assumptions about the linac model differ from reality, minimize the variance of the states and should avoid oscillations in the closed loop.
For the simulations presented in this paper we assumed the following: a sample frequency of 5 Hz (repetition rate of TESLA) and a BPM resolution between 100 nm and 200 nm. In the BDS tuning and diagnostic section the spot sizes will be about $\sigma_x \backslash \sigma_y = 17\backslash 3.3\,\mu$m. The rms-values of the beam motion were choosen between 25% and 50% of σ in both transverse planes. The spectrum of the beam motion used in this model resembles a spectrum of a lowpass filter plus some peaks at special frequencies. The results depend strongly on the passband and cut off frequency f_c of the shaping filter.
The feedback loop works quite well for low frequency disturbances. For example, a DC bias rejection of -40 dB

was achieved for noise models with $f_c = 0.005$ Hz. Raising f_c to 0.01 Hz and 0.1 Hz lowers the DC bias rejection to -32 dB and -15 dB. Fig. 1 shows the effect of the shaping filter's f_c on the beam control: for the same incoming disturbances the feedback works essential better by assuming a smaller f_c. For higher f_c the user also has to reckon with

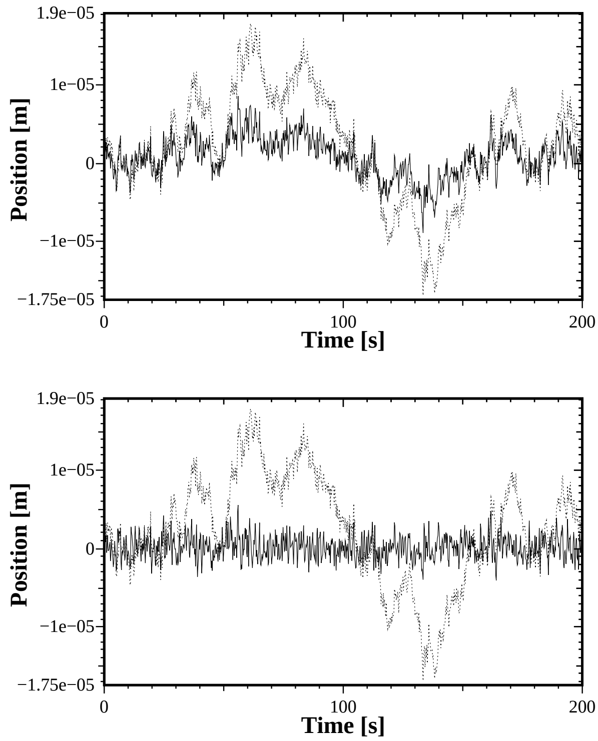

Figure 1: Plots of simulations: Controlled (solid) and uncontrolled (dashed) beam motion for $f_c = 0.05$ Hz (upper plot) and $f_c = 0.005$ Hz (lower plot) in the sampling time scale.

oscillations in the feedback loop and a much longer time to recover states from a step perturbation on the incoming beam.

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7 REFERENCES

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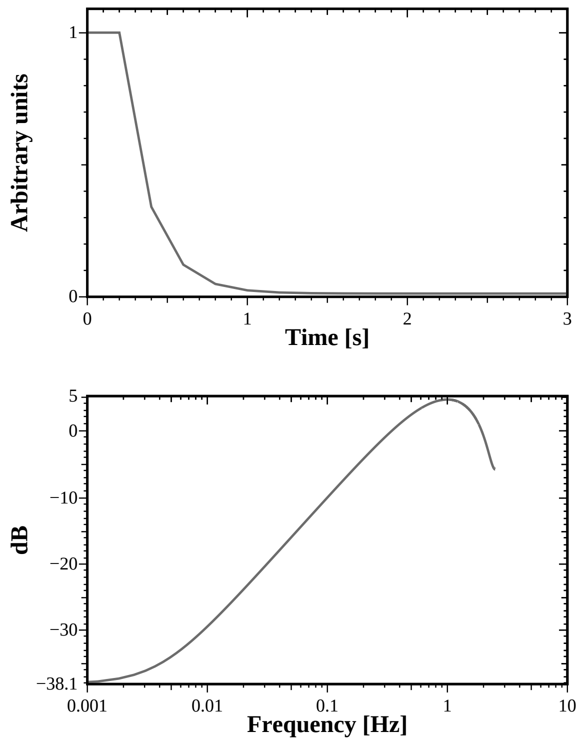

Figure 2: Plots of simulations: 1.) Response on a step disturbance on incoming beam; 2.) Disturbance transfer function of a feedback loop, Nyquist frequency at 2.5 Hz, $f_c = 0.005$ Hz.

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