MEASUREMENT OF THE MAGNETIC FIELD RIPPLE EXPERIENCED BY A STORED BEAM

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Abstract

By analysing beam transfer functions, i.e. the beam response to an RF excitation signal at a harmonic of the revolution frequency or at a betatron sideband, the low frequency ripple of the bending and focusing fields can be deduced. The underlying theory as well as experimental results of ripple measurements for two of CERN's machines (Booster, Antiproton Collector) are presented. In the Antiproton Collector ring for example, fluctuations smaller than 10^{-6} have been observed using this technique.

1 PRINCIPLE

1.1 Longitudinal BTF and CW Excitation

Near a harmonic n of the particle revolution frequency, the current modulation $i(t) = \mathrm{Re}\{I(\omega)e^{j\omega t}\}$ induced on a coasting beam by a sinusoidal excitation $u(t) = \mathrm{Re}\{U(\omega)e^{j\omega t}\}$ in a kicker gap (i.e. the longitudinal BTF, Beam Transfer Function [1]) is given by:

$$g_{\|}(\omega) = \frac{I(\omega)}{U(\omega)} = \frac{g_o(\omega)}{1 - Z_{\|}(\omega) g_o(\omega)} \qquad (1)$$

$$g_o(\omega) = j \frac{I_{DC}\,\eta\,\omega_o^2}{2\pi\beta^2\gamma\, m_o c^2/e} \int \frac{d\Psi/d\omega_\ell}{\omega - n\omega_\ell}\, d\omega_\ell = \frac{I_{DC}\,\eta\,\omega_o^2}{2\pi\beta^2\gamma\, m_o c^2/e} \left\{ \frac{\pi}{n} d\Psi/d\omega_\ell + j \int_{PV} \frac{d\Psi/d\omega_\ell}{\omega - n\omega_\ell} \right\} \qquad (2)$$

Here $\omega_\ell = \omega_o(1+|\eta|\Delta p_\ell / p_o)$ is the revolution frequency of particle ℓ, $p_\ell = p_o + \Delta p_\ell$ its momentum, $\eta = \gamma_t^{-2} - \gamma^{-2}$ the off momentum parameter [2] of the machine, $I_{DC} = N e\, \omega_o / 2\pi$ the circulating beam current, $m_O c^2$ (= 938 MeV for protons) the particle rest energy, $\Psi(\omega_\ell) = (1/N)\, dN/d\omega_\ell$ the density distribution of particle revolution frequencies; $Z_{\|}$ is the longitudinal coupling impedance of the beam environment. In the following we shall assume for simplicity, that the beam is far from the instability threshold so that $|Z_{\|}(\omega)\, g_0(\omega)| << 1$, i.e. $g_{\|} = g_0$.

The measured BTF: $G_{\|}(\omega) = A(\omega)\, g_{\|}(\omega)$ contains a transfer function $A(\omega)$ given by the characteristics of the excitation (kicker, amplifier...) and the acquisition (PU) system. We assume $|A(\omega)|$ to be constant over the band of interest.

For a well behaved ("bell shaped", example in Fig. 1) particle distribution $\Psi(\omega_\ell)$ the BTF $g_0(\omega)$, eq. (2), has the behaviour of a "fourth order resonance" with the resonance at $n\omega_0$ and a width given by the beam frequency spread [1,3]. The response is purely imaginary outside the particle distribution (phase +90 degrees below and above the distribution). The revolution frequencies entering into the resonant denominator of the dispersion integral in (2) are modulated by ripple in the bending field according to:

$$\frac{\Delta\omega_\ell}{\omega_\ell} = \frac{\Delta\omega_o}{\omega_o} = \alpha \frac{\Delta B}{B} = \gamma_t^{-2} \frac{\Delta B}{B} \qquad (3)$$

where $\alpha = \gamma_t^{-2}$ is the momentum compaction factor of the machine [2]. Thus the B-ripple leads to a fluctuation in the position of the resonance curve Fig 1. The method we chose to detect this ripple is to analyse the time behaviour of $|G_{\|}(\omega_{CW})|$ at a fixed frequency ω_{CW}. Then, with small amplitude, low frequency B ripple:

$$\Delta\left|G_{\|}(\omega_{CW})\right|(t) = \omega_{CW}\, \gamma_t^{-2} \frac{\Delta B(t)}{B} \left.\frac{\partial \left|G_{\|}(\omega)\right|}{\partial\omega}\right|_{\omega=\omega_{CW}} \qquad (4)$$

We have to make sure that over the bandwidth and the time of the measurement, parameters (including the distribution function $\Psi(\omega_\ell)$) do not change. Then the fluctuation of $|G(\omega_{CW})|$ is directly proportional to the fluctuation of B. We can calibrate the measurement by either injecting a well determined amount of low frequency modulation into the main bending supply as was done in the AC, or measure the BTF by a network analyser as was done in the PSB.

Compared with other beam based methods like Schottky scans [4], the CW BTF method can achieve an extremely high sensitivity if needed. The choice of the harmonic n and the distance of ω_{CW} to $n\omega_0$ is subject to a number of considerations: the kicker and pick up sensitivity should be high and constant over the band, adjacent harmonics should be well separated and ω_{CW} should be in the tail (imaginary response) not to shake up the distribution, but also in a region where $\partial G/\partial\omega$ is still high so that the signal to noise ratio is acceptable. Similar considerations apply for the excitation level, which must be small enough not to change the distribution but large enough to get a useful signal in spite of the acquisition noise.

1.2 Transverse BTF and CW excitation

The transverse BTF, defined as the beam "dipole moment" $d(\omega)\, e^{j\omega t} = I_{DC} <x(\omega)> e^{j\omega t}$ in response to $U_\perp(\omega) e^{j\omega t} = \int [E(\omega) + v \times B(\omega)]_\perp e^{j\omega t} ds$, which is the integrated deflection per turn, can be written as:

$$g_\perp(\omega) = \frac{g_{\perp o}(\omega)}{1 - Z_\perp(\omega)\, g_{\perp o}(\omega)} \qquad (5)$$

$$g_{\perp o}(\omega) = \mp j \frac{I_{DC}\, c}{4\pi Q \beta^2 \gamma\, m_o c^2 / e} \int \frac{\Psi(\omega_\beta)}{\omega - \omega_\beta} d\omega_\beta \qquad (6)$$

Now the betatron sideband frequencies $\omega_\beta = (n \pm Q)\,\omega_\ell$ and the particle distribution $\psi(\omega_\beta)$ in these frequencies enter into the dispersion integral. Similar to the longitudinal case we choose one of these bands and analyse the time behaviour of the response at a fixed frequency $\omega_{\beta m} \approx (n \pm Q)\,\omega_o$. The ripple in the sideband frequencies is given by both revolution frequency and Q ripple:

$$\Delta\omega_{\beta m} = (n \pm Q)\,\omega_o \gamma_t^{-2} \frac{\Delta B}{B} \pm \omega_o\, \Delta Q \qquad (7)$$

where the first term represents the fluctuation of the revolution frequency due to ripple in the bending field just as in the longitudinal case. The second term, representing the tune ripple $\Delta Q = \Delta Q_B + \Delta Q_{QUAD}$ has contributions from both the bending (via the dependence of Q on the radial position) and the quadrupole fields. Thus the transverse beam response contains both types of ripple. To have sufficient sensitivity for the Q ripple one prefers a low harmonic $n \le Q$.

2 MEASUREMENTS IN THE CERN PS BOOSTER

The CERN PS Booster accelerates normally from 50 MeV to 1 GeV kinetic energy. It is a separated function synchrotron with a common power supply powering 33 bending magnets (of which one is a reference magnet outside the ring), 32 F and 16 D quadrupoles in a series connection. Separate F and D trim current supplies allow small (up to 7%) tune correction from its natural value. The measurements described below were all done with a debunched proton beam during a 50 MeV injection field plateau lasting 400 ms.

2.1 Measurement of Revolution Frequency Modulation

The longitudinal BTF (Fig. 1) was measured using the RF cavity as kicker driven by a sweeping network analyser. A fairly low excitation level (80 V_p) combined with a fast sweep of 25 ms to avoid formation of a "scanning bucket", which modifies the distribution and produces non-linear response dramatically different from the low-level linear response given by (1) and (2).

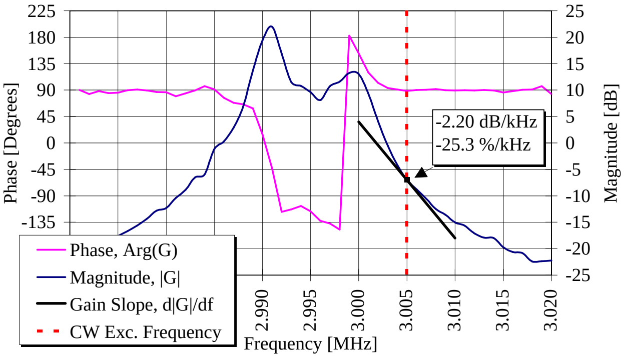

Figure 1: Longitudinal BTF measured in the PSB

The real part of the response corrected for the space-charge dominated $Z_\parallel$ (equation (1)) can be integrated to yield the momentum distribution $\Psi(f)$, as the real part of $g_0(f)$ is proportional to the derivative of $\Psi(f)$, figure 2.

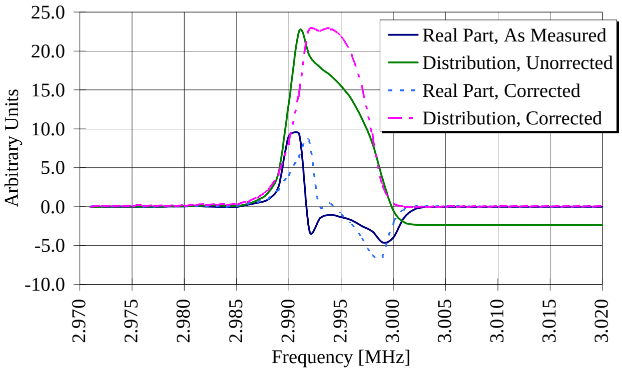

Figure 2: Measured real part of longitudinal BTF and its integral the density distribution $\Psi(f)$

A CW excitation frequency is chosen just outside the momentum distribution where *the response is purely imaginary* (i.e. no net energy transfer from cavity to distribution), such that a much higher excitation level (1 kV_p) and excitation time (400 ms) can be chosen without any observable deviation from a stationary beam response. A long observation time is required to obtain adequate frequency resolution of the modulations to be measured, while a high excitation level improves signal to noise ratio such that small modulations can be measured. The gain slope needed for the FM to AM conversion can be determined from the swept BTF measurement at the chosen frequency.

The time variation of the BTF gain part is then recorded over the observation time and a spectral analysis obtained by FFT of the gain versus time function.

A summary of a few spectral components are shown in table 1, where the beam measured bending ripple is compared with the current ripple obtained by using measured power supply voltage ripple and the known magnet inductance. The differences observed are more likely due to capacitive ground currents, than measurement uncertainty: signal to noise ratio of the FFT of the CW BTF is about 20 dB.

Table 1: Bending field ripple in the CERN PSB

Modulation Frequency	$(\Delta B/B)_p$ from CW BTF	$(\Delta I/I)$ from ΔV and inductance
50 Hz	3.95×10^{-5}	6.44×10^{-5}
100 Hz	1.01×10^{-4}	3.41×10^{-5}
150 Hz	5.78×10^{-5}	4.33×10^{-5}

2.2 Measurements of Tune Modulations

The horizontal BTF was measured using pick-ups and kickers of the transverse damper system. The swept response (Fig. 3) looks noisy due to strong betatron frequency modulation. The CW BTF response was analysed in spectral components by FFT as in the longitudinal case.

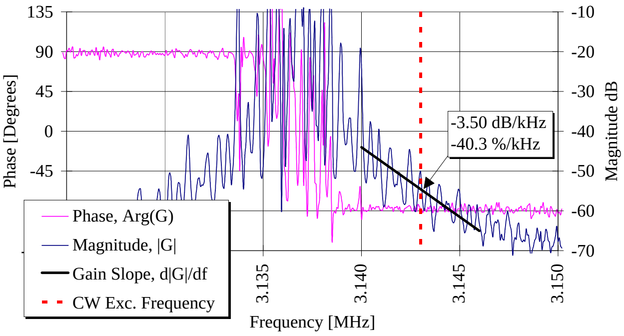

Figure 3: Horizontal BTF measured in the PSB

For the chosen harmonic (n = 1, Q_h = 4.23), the tune modulation dominates over revolution frequency modulation (from table 1), but is different from what can be calculated from the known chromaticity (ξ_h = -1, assuming same field ripple in quadrupoles and bendings), table 2.

Table 2: Horizontal betatron line FM amplitudes

Mod. Freq.	Δf [Hz] by CW BTF	Δf [Hz] from Revol. Freq.	Δf [Hz] from chromaticity
50 Hz	132.1	7.4	94.8
100 Hz	120.7	18.9	242.7
150 Hz	361.0	10.8	138.6
200 Hz	84.2	2.9	37.3
250 Hz	616.2	1.9	24.2

3 MEASUREMENTS IN THE CERN ANTIPROTON COLLECTOR (AC).

The CERN AC [5] is a storage ring for antiproton with stochastic cooling at 3.57 GeV/c (γ_t = 4.67, η = 0.0187). It has a circumference of 182.4 m. For ripple measurements an antiproton beam of 5×10^7 particles cooled to a momentum spread of 0.1% has been used. By changing the bending current (nominal value 2280 A) by +/- 1 A the γ_t of the machine was verified using the relation (3) and neglecting the influence of momentum change due to betatron acceleration. The results agree within 5% with the theoretical values (the current for the quadrupoles has not been changed).

Then a small sinusoidal 1 Hz modulation (modulation frequencies limited by power supply regulation) of known amplitude was applied to the bending power supply to calibrate for the FM to AM conversion factor at the chosen excitation frequency. The CW BTF was measured and analysed by a network analyser connected to a longitudinal stochastic cooling system (bandwidth 0.9 to 1.6 GHz) with highly sensitive pick-ups and kickers. Operating at a high harmonic number in a low η machine has the advantage that the useful frequency range of the imaginary part of the BTF amounts to several kHz (f_{CW} = 1300.181 MHz corresponds to n = 818). The results obtained are summarised in Table 3 and compared with current ripple derived from power supply voltage as in the PSB. Rather large drifts were observed (up to a factor of 3) in the amplitude of the ripple current spectral components of both types of measurements within few days.

Table 3: Measured bending field ripple in the CERN AC

Modulation Frequency	$(\Delta B/B)_{pp}$ from CW BTF	$(\Delta I/I)_{pp}$ from ΔV and inductance
50 Hz	0.76×10^{-6}	1.31×10^{-6}
600 Hz	2.19×10^{-7}	4.38×10^{-7}

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