Betatron Phase Measurements in CESR*

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Abstract

The transverse betatron phase has been measured at the Cornell Electron/positron Storage Ring CESR by shaking the beam and observing the phase of oscillation at detectors located around the ring. From the phase measurements the Twiss parameters can be calculated and this allows beta errors to be corrected.

1 INTRODUCTION

The standard method for measuring the Twiss parameter β at the Cornell Electron/positron Storage Ring CESR is to measure the tune Q as a function of quadrupole strength k_j of the jth quadrupole. The beta at the quadrupole β_j is then obtained from the standard formula[1]

$$\delta Q = \frac{\beta_j}{4\pi} \, \delta k_j \, l_j \ , \tag{1}$$

where l_j is the length of the quadrupole. There are a number of problems with measuring beta in this way. For one, hysteresis in the quadrupoles limits the accuracy and reproducibility of the measurements. Moreover, saturation of the quadrupole iron can introduce errors in the results.

An alternative approach is to shake the beam at some betatron sideband and then measure the phase of the oscillations at the detectors around the ring. This will give the betatron phase ϕ at the detectors which can then be related to the beta function via[1]

$$\phi(s) = \int^s \frac{d\tilde{s}}{\beta(\tilde{s})} \ . \tag{2}$$

This alternative approach has recently been implemented in CESR and is proving to be a valuable tool.

2 THEORY

The experimental setup is shown schematically in figure 1. A shaker situated at position $s = 0$ shakes the beam at a betatron sideband

$$\omega_s = n\,\omega_0 + \omega_\beta \ , \tag{3}$$

where ω_β is the betatron frequency, ω_0 the revolution frequency, and n is an integer. For a horizontal phase measurement the beam is shaken horizontally and for the vertical phase the beam is shaken vertically. The resulting oscillations can be observed at various detectors around the ring. The signal from a given detector is connected to a signal analyzer where it can be compared to a reference signal from the shaker. Heterodyne bandpass filters – which give good filtering in the stop band – are used to filter out unwanted frequency components. The heterodyne filter also has the property that its center frequency can be computer controlled which is desirable when switching between horizontal and vertical measurements.

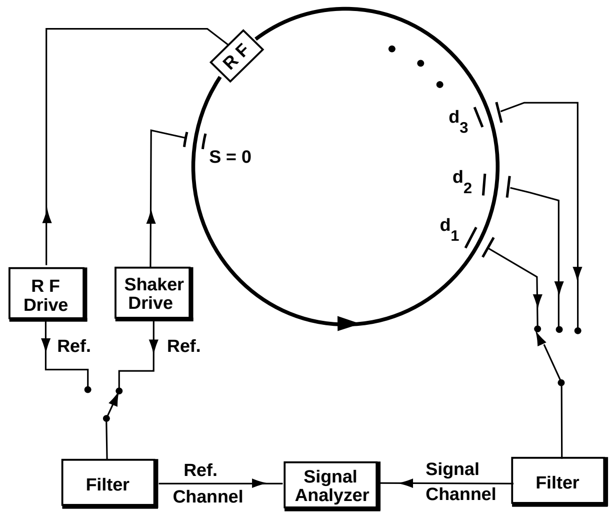

Figure 1: Schematic illustration of the experimental setup.

The phase $\theta_s(i)$ of the beam signal from the ith detector relative to the shaker reference signal gives a measure of the betatron phase at the detector:

$$\theta_s(i) = \phi(i) - \left(\frac{s_i}{c} + t_c(i)\right) \omega_s + 2\,\pi\,m_i + \theta_a \ , \tag{4}$$

where $\phi(i)$ is the betatron phase at detector i, s_i is the distance from the shaker to the detector, c is the speed of light, $t_c(i)$ is the time delay for the beam signal going from the detector to the analyzer, m_i is an integer, and θ_a is a constant. The s_i and $t_c(i)$ terms are due to the time delays in the beam going from the shaker to the detector and for the signal going from the detector to the signal analyzer. the negative sign is because delays represent negative phase. The m_i term is due to the fact that phases are always measured modulo 2π. Finally, the θ_a term, which is a constant independent of the detector being used, takes care of such things as time delays in the reference channel, etc.

In order to be able to subtract off the terms due to the transit delays the shaking is turned off and the filters are ad-

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justed to pass signals at the revolution frequency. The reference channel is also switched to a reference signal which, because it is derived from the RF system (cf. figure 1), is synchronous with the longitudinal motion of the beam. For the ith detector the phase $\theta_{\rm rev}(i)$ of the beam signal at frequency ω_0 relative to the reference is

$$\theta_{\rm rev}(i) = -\left(\frac{s_i}{c} + t_c(i)\right)\,\omega_0 + 2\,\pi\,p_i + \theta_b\;, \qquad (5)$$

where p_i is an integer, and θ_b, which is a constant independent of i, is due to various factors such as delays in the reference channel, etc. Only the changes in p_i between detectors are needed since any constant part can be absorbed into θ_b. Differences in $2\pi p_i - t_c(i)\omega_0$ between detectors are due to differences in cable lengths. A change of 1 unit in p_i represents a cable length change of (very roughly) $2\pi c/\omega_0 = L_0$ where L_0 is the ring circumference. For CESR L_0 is 768 m. This represents an enormous change in cable length and, since the approximate cable lengths are known, it is a simple matter to ascertain the p_i.

Using Eq. (5) in Eq. (4) gives

$$\phi(i) = \theta_{\rm s}(i) + 2\,\pi\,m_i + (\theta_{\rm rev}(i) - 2\,\pi\,p_i)\,\frac{\omega_s}{\omega_0} + \theta_c\;, \quad (6)$$

where $\theta_c = \theta_a - \theta_b\omega_s/\omega_0$ is a constant. The value of θ_c is immaterial since phases are always calculated relative to some arbitrary zero point. The tricky part here is that if ω_s is less than 0 (*i.e.* $n < -\omega_\beta/\omega_0$ in Eq. (3)) then the *measured* $\theta_{\rm s}$ will have a reversed sign from what it should be since a spectrum analyzer will always calculate the phase under the implicit assumption that the input frequency is positive. In this case $-\theta_{\rm s}$ should be substituted for $\theta_{\rm s}$ in Eq. (6). In Eq. (6) m_i can be obtained using knowledge of the phases from the theoretical design lattice. For CESR, the measured phase typically differs from the theoretical phase by of order $2\pi/10$. Thus m_i is simply chosen such that the measured phase most closely matches the theoretical phase.

Each detector in CESR consists of 4 button electrodes labeled 1 through 4 as shown in figure 2. In theory, for a horizontal phase measurement at a given detector, the measured phase at buttons 2 and 4 will be 180° opposite the phase of buttons 1 and 3. A similar situation occurs with the vertical measurement. The appropriate average of the button phases to obtain $\theta_{\rm s}$ is thus

$$\theta_{\rm s} = \begin{cases} \frac{1}{4}(\theta_4 - \theta_3 + \theta_2 - \theta_1) & \text{Horizontal} \\ \frac{1}{4}(\theta_4 + \theta_3 - \theta_2 - \theta_1) & \text{Vertical.} \end{cases} \qquad (7)$$

For the measurement of $\theta_{\rm rev}$ all the button phases should be the same so the appropriate average is

$$\theta_{\rm rev} = \frac{1}{4}(\theta_4 + \theta_3 + \theta_2 + \theta_1)\;. \qquad (8)$$

Because of the symmetry of the button configuration coupling between the horizontal and vertical motions caused by skewquads, etc. does not cause any changes in $\theta_{\rm s}$ as long as the beam centroid is centered between the buttons. Even when the beam is off–center there is good cancellation of the phase shifts of the individual button signals due to coupling. This was a necessary requirement since for CESR the phase shift at the individual buttons due to coupling can easily be more than the phase deviations from the theoretical that need to be measured. For $\theta_{\rm rev}$ there is no affect due to coupling even with an off-center beam.

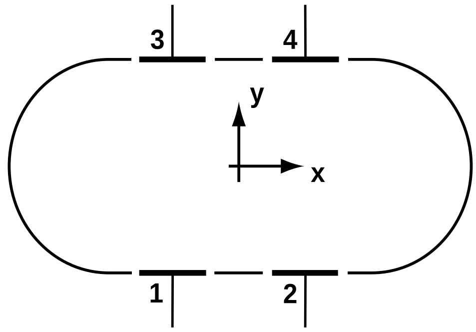

Figure 2: Beam button layout at a detector.

3 EXPERIMENTAL RESULTS

Figure 3 shows an initial (before correction) measurement of the horizontal and vertical beta and betatron phase. Plotted in the figure is the deviation of the measured beta and phase from the values of the theoretical design lattice. For the phase data θ_c in Eq. (6) has been adjusted so that the average of $\phi_{\rm meas} - \phi_{\rm theory}$ is 0.

Since the theoretical design lattice has been optimized to give optimum machine performance (in terms of maximal dynamic aperture, maximal luminosity, etc.) the large deviations of the measured beta from the theoretical shown in the figure – over 40% in places – leads to a significant degradation in performance. To correct this the quadrupole strengths k_j in the theoretical model are adjusted so that the calculated beta and/or phase matches as closely as possible the measured values. These fitted k_j's are (presumably) equal to the actual k_j's present in the ring and the correction is made by adjusting the ring quadrupoles by an amount $k_j(\rm theory) - k_j(\rm fit)$. Fitting only to the phase data the measured beta and phase after correction are shown in figure 4. The beta and phase errors have been substantially reduced. In fact, an analysis shows that most of the beta error shown in figure 4 is due to second order effects (change in beta with change in k) not accounted for by Eq. (1) and that the actual RMS of $(\beta_{\rm meas} - \beta_{\rm theory})/\beta_{\rm theory}$ after correction is only about 2% percent. Typically only one round of measurements/corrections are needed to achieve a correction to this level.

4 CONCLUSION

A possible disadvantage with the phase measurement is that it is relatively insensitive in regions of large beta. How-

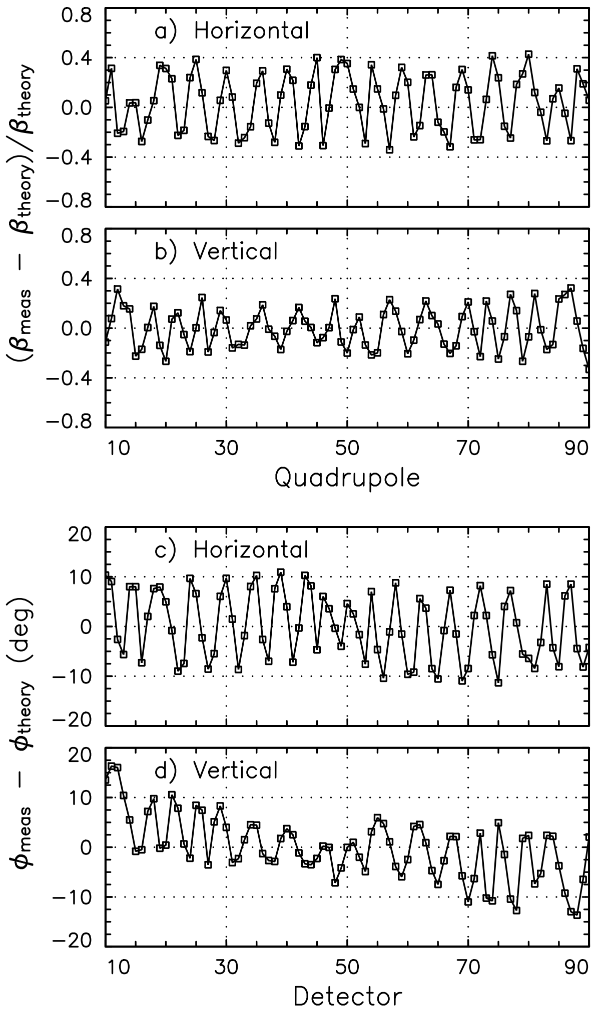

Figure 3: Initial beta and betatron phase relative to the theoretical values. The numbering system in CESR is such that the ith detector is close to the ith quadrupole.

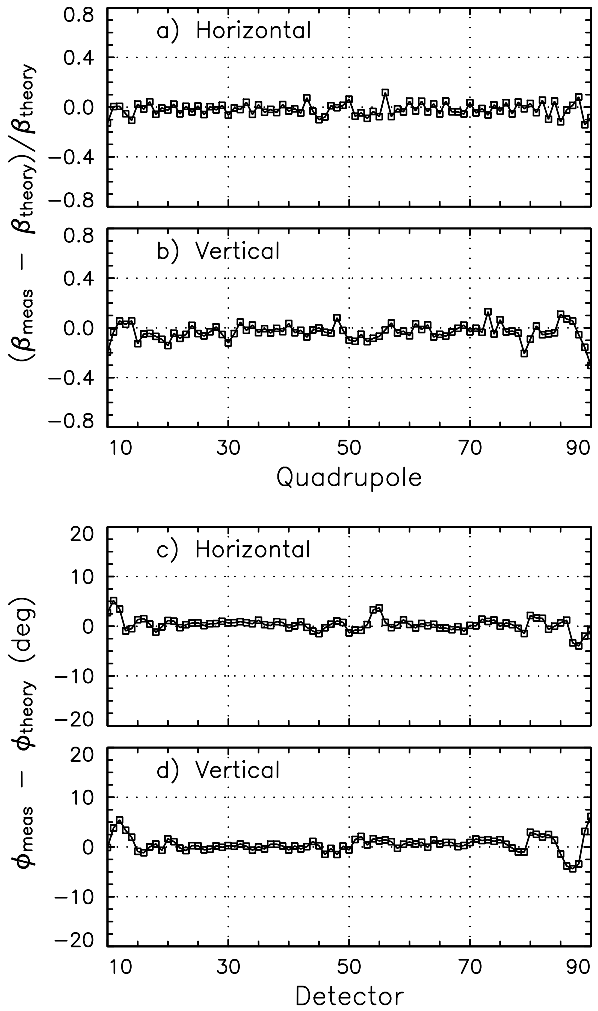

Figure 4: Beta and betatron phase relative to the theoretical values after correcting using only the phase data of the previous figure.

ever, since measurements are made in both the horizontal and vertical planes, and since large β_x usually goes with small β_y (and vice versa), this has not proved to be a problem in CESR. In fact, better results are obtained when correcting the Twiss parameters using the phase measurements compared with using the beta measurements.

One significant advantage of the phase measurement is that, unlike the beta measurement, it is sensitive to variations in the phase over long distances. This can be important for closure of bumps. Additionally, CESR is east/west symmetric and the presence of this symmetry causes the strengths of some resonances to be zero. Since phase errors can break this symmetry it is important to be able to accurately measure and correct the phase.

Another advantage of the phase measurement is that it can be used when there are significant orbit displacements. With a beta measurement there is the problem that a orbit displacement, coupled with a variation in quadrupole strength, will result in a variation in the orbit which, in turn, causes tune variations due to the sextupoles. For CESR, in normal operation, a ‘pretzeled’ orbit is used so that multiple bunches of electrons and positrons can share the same beam pipe and it is useful to be able to measure the Twiss parameters for the orbit used in operation.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

[1] Matthew Sands, *The Physics of Electron Storage Rings, An Introduction*, SLAC-121 Addendum, 1970.