# OPTIMIZATION OF AN AXIALLY SYMMETRICAL ELECTROSTATIC FOCUSING SYSTEM

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## 1 ABSTRACT

A method for the optimization of an axially symmetrical electrostatic system is described. An accurate version of the Boundary Element Method is used to solve Laplace's equation to obtain the potential along the optical axis. We use this field when we reformulate the nonlinear equation of particle motion in phase space as a linear equation in phase moment space. A continuous generalized analogue of Gauss brackets is used to calculate the matrizant for the motion equation with the field coefficient matrix. In this method there is a rigorous conservation of the phase volume of the beam at each stage of the calculation.

The above matrizant for the given geometries is used and the sizes of the object (first) and aperture (second) diaphragms are varied to obtain the minimum spot size at the specimen for a fixed emittance. Some synthesized systems are presented.

## 2 INTRODUCTION

In design of charge particle focusing systems the crucial question always is: what is the best system for the application at hand? Sometimes it is difficult to determine what is the optimal system because we have some controversial demands. In this paper we formulate exactly what is the optimal system in our case. The electrostatic round lenses are widely used for forming and transportation beams themselves as well as a part of more complicated systems [1]. We consider the focusing system which consists of two round diaphragms and three cylinders, having equal diameter with thin walls and rotational symmetry about the central axis $z$. The middle cylinder has the potential $\pm V$. The potential of the remaining cylinders and diaphragms is zero. There is a charged particle beam with initial energy $E_0$ and with a given emittance which is determined by two diaphragms separated by a distance $l_{12}$. The first diaphragm, which is placed at a distance $l_d$ from the central plane of the middle cylinder, is the object diaphragm with radius $r_1$ and the second one is the aperture diaphragm with radius $r_2$. For a given brightness the emittance defines the beam current. We consider the differential equation of motion of the charged particles accurate to terms of third order inclusive. The following problem is solved: what are the values of $V$, $r_1$, $l_{12}$ and the geometry of the system which provide the minimum beam spot size on the target situated at the distance $l_t$ from the central plane of the middle cylinder? We are also investigating the influence of the geometry of the focusing system on the minimum spot size and on its demagnification. This system can be used in microprobe devices if $l_d$ is much bigger than $l_t$.

## 3 THE METHOD OF EMBEDDING IN PHASE-MOMENT SPACE FOR SOLVING THE NONLINEAR EQUATION OF MOTION

It is convenient to choose a set of variables, for which the phase volume remains unchanged during the beam motion. For the electrostatic field these variables have the following form:

$$x_1 = x,\ \ x_2 = \frac{p(z)}{p(0)}x',\ \ y_1 = y,\ \ y_2 = \frac{p(z)}{p(0)}y',$$

where $p(z) = \sqrt{\gamma^2(z) - 1}$,

$$\gamma(z) = \gamma(0) + \varphi(0) - \varphi(z), \quad \gamma(0) = 1 + \frac{E_0}{W_0}.$$

Here $p(z)$ is the dimensionless momentum of the axial particle, $\gamma(z)$ is its relative total energy, $W_0$ is the rest energy of an axial particle and $\varphi(z) = V(z)q/W_0$ is the dimensionless axial potential. In our study we take $E_0 = 30$ kV.

The analysis and calculation of the nonlinear systems of equations for monochromatic beam formation in the static field are considerably simplified by transforming from the nonlinear differential equations of motion in the phase space ($x_1, x_2, y_1, y_2$) to the system of linear equations in extended phase space - the phase-moment space. This is the essence of the method of embedding in phase-moment space [2]. For the differential equation of motion of the particles accurate to terms of k-order we have the phase-moment space of k-order. In the paraxial case (the equation of the first order) we usually obtain two linear equations for two phase-moment vectors of the first order $\tilde{x}[1] = \{x_1, x_2\}$ and $\tilde{y}[1] = \{y_1, y_2\}$. If the motion of the monochromatic beam is described by the third order equation, we have two linear equations for two phase-moment vectors of the third order:

$$\tilde{x}[3] = \{x_1, x_2, x_1^3, x_1^2 x_2, x_1 x_2^2, x_2^3, x_1 y_1^2, x_1 y_1 y_2, x_1 y_2^2, x_2 y_1^2, x_2 y_1 y_2, x_2 y_2^2\}$$

and

$$\tilde{y}[3] = \{y_1, y_2, y_1^3, y_1^2 y_2, y_1 y_2^2, y_2^3, y_1 x_1^2, y_1 x_1 x_2, y_1 x_2^2, y_2 x_1^2, y_2 x_1 x_2, y_2 x_2^2\}.$$

The writing of the nonlinear equation in a linearized form makes it possible to construct its solution using a matrizant, which is independent of the initial vector $\{x_{10}, x_{20}, y_{10}, y_{20}\}$, whereas the solution of the nonlinear equation is sought for each value $\{x_{10}, x_{20}, y_{10}, y_{20}\}$.

To the nonlinear equation of the third order we can associate a linear equation for the phase moments: $dx[3]/dz = P(z)x[3]$, where $P(z)$ depends on the axial potential $\varphi(z)$ and its first four derivatives. For the electrostatic axisymmetric field, the equation in $y$ - plane is obtained from the equation in $x$ - plane if $x \to y$, $y \to x$. The solution of this equation is written in terms of the matrizant $R(z/z_0)$ in the form:

$$x[3] = R(z/z_0)x_0[3], \quad R(z_0/z_0) = I.$$

A continuous generalized analogue of Gauss brackets [3] is used to calculate the matrizant for the motion equation with the field coefficient matrix. In this method there is a rigorous conservation of the phase volume of the beam at each stage of the calculation.

## 4 THE ANALYTICAL MODEL OF THE AXIAL POTENTIAL

We choose the analytical model of the axial potential in the form of piecewise-continuous function:

$\varphi(z) = \varphi_{2j-1}(z)$, if $z_{2j-2} \le z \le z_{2j-1}$, and $\varphi(z) = \varphi_{2j}(z)$, if $z_{2j-1} \le z \le z_{2j}$; where $\varphi_{2j-1}(z) = \text{const} = U_{2j-1}$, $j = 1,2,\ldots$, $\varphi_{2j}(z)$ is changed from $U_{2j-1}$ to $U_{2j}$, while $z$ is changed from $z_{2j-1}$ to $z_{2j}$ and the four first derivatives of $\varphi_{2j}(z)$ are zero for $z = z_{2j-1}$ and for $z = z_{2j}$. With these conditions the function $\varphi_{2j}(z)$ has the following form:

$$\varphi_{2j}(z) = U_{2j-1} + \left(z - z_{2j-1}\right)^5 \Big(\frac{\Delta U_j}{\Delta z_j^5} + \left(z - z_{2j}\right)\Big(-5\frac{\Delta U_j}{\Delta z_j^6} + \left(z - z_{2j}\right)\Big(15\frac{\Delta U_j}{\Delta z_j^7} + 35(2z - 3\Delta z_j - 2z_{2j-1})(z - z_{2j})\frac{\Delta U_j}{\Delta z_j^9}\Big)\Big)\Big),$$

where $U = \dfrac{qV}{W_0}$, $\Delta U_j = U_{2j+1} - U_{2j-1}$, $\Delta z_j = z_{2j} - z_{2j-1}$.

## 5 THE METHOD OF THE MOMENTS OF THE PARTICLE DISTRIBUTION FUNCTION OVER WHOLE TOTALITY OF THE PHASE COORDINATES

It is known that the information about the averaged characteristics of a beam can be obtained by calculating the moments of the particle distribution function in phase space [4]. We consider the beam motion as a motion of the closed phase set. This allows us to introduce the matrix of the moments $M$ of the distribution function over whole totality of the phase coordinates, where

$$M(z) = \int_\Omega f(x_1, x_2, y_1, y_2)x[3]\tilde{x}[3]dx_1 dx_2 dy_1 dy_2.$$

The integration is performed over the apertures of two diaphragms. The averaged radius $r(z)$ of the beam is determined by the matrix element $M_{11}(z)$. We suppose that $f(x_{10}, x_{20}, y_{10}, y_{20}) = 1$ for $(x_{20} + x_{10}/l_{12})^2 + (y_{20} + y_{10}/l_{12})^2 \le (r_2/l_{12})^2$, $x_{10}^2 + y_{10}^2 \le r_1^2$, and $f(x_{10}, x_{20}, y_{10}, y_{20}) = 0$ for $(x_{20} + x_{10}/l_{12})^2 + (y_{20} + y_{10}/l_{12})^2 > (r_2/l_{12})^2$, $x_{10}^2 + y_{10}^2 > r_1^2$ and $r(0) = r_1$. In this case we obtain

$$r(z) = \sqrt{M_{11}(z)}, \quad M(z) = R(z/z_0)M(0)\tilde{R}(z/z_0),$$

$$M(0) = \int_\Omega x_0[3]\tilde{x}_0[3]dx_{10}dx_{20}dy_{10}dy_{20}$$

$M(0)$ is a function of $r_1$, of emittance $em = r_1 r_2 / l_{12}$, and of $l_{12}$.

For our system we use the analytical model of the axial potential , where the first (object) diaphragm is located at the position $z_0 = 0$, the target is placed at the position $z = z_5 = l_{tot} = \sum_1^5 \Delta z_j$ and $U_1 = U_5 = 0$, $U_3 = \pm U$. We use the following notations for the geometry of the system: $\Delta z_2 = \Delta z_4 = l_g$, $\Delta z_3 = l_c$, $\Delta z_5 = g$.

## 6 THE OPTIMIZATION PROCEDURE

To apply a numerical optimization to our system a merit function has to be defined. We choose merit function as $\rho = r(l_{tot})$ for a given emittance. Before the merit function can be evaluated the third order matrizant $R(z/0)$ has to be calculated. Since this matrizant depends on the particle trajectory, the first-order stigmatic property, which is described by the equation $R_{12}(l_{tot}) = 0$, must be satisfied before calculating this matrizant. Our program calculates the elements of the matrizant of the third order, which is used for further calculations with each particular geometry. For the given geometry and initial energy $E_0$, we find from the equation $R_{12}(l_{tot}) = 0$ the value of the potential V which provides the stigmatic property of the system.

The merit function is a function of $r_1$ and of $l_{12}$. All remaining parameters are fixed when we are seeking for the minimum value of $\rho$, and we find this minimum for different parameters. The radius of the object diaphragm has the strongest influence on the beam spot size for the given emittance. It is very important to use the optimal $r_1$ for obtaining the smallest beam spot size. The result of such calculations for different emittances and for one chosen geometry is shown in Table 1 (g = 20 cm). The results for the chosen emittance ($10^{-9}$ m) and for different geometries are given in Table 2 (g = 4 cm).

Table 1

| em ($\mu$m) | $\rho$ ($\mu$m) | $r_1$ ($\mu$m) | $l_{12}$ (cm) |
|---|---|---|---|
| $10^{-5}$ | 0.0126 | 0.263 | 1.0 |
| $10^{-4}$ | 0.0711 | 1.350 | 3.8 |
| $10^{-3}$ | 0.399 | 8.31 | 10. |
| $10^{-2}$ | 2.25 | 46.8 | 31. |
| $10^{-1}$ | 12.63 | 263. | 91. |

Table 2

| $l_c$ (cm) | $\rho$ ($\mu$m) | $r_1$ ($\mu$m) | V (kV) | d |
|---|---|---|---|---|
| 33 | 0.83<br>0.79 | 19.0<br>17.4 | 12.4<br>-21.1 | -20.2<br>-20.0 |
| 25 | 0.55<br>0.52 | 13.5<br>12.4 | 14.0<br>-34.3 | -22.3<br>-21.7 |
| 17 | 0.42<br>0.40 | 11.4<br>9.8 | 18.3<br>-47.0 | -24.5<br>-23.3 |
| 14 | 0.39<br>0.37 | 10.6<br>9.5 | 19.0<br>-51.7 | -25.3<br>-23.9 |
| 11 | 0.36<br>0.34 | 10.5<br>9.2 | 19.6<br>-56.4 | -26.1<br>-24.5 |
| 0.1 | 0.29<br>0.27 | 9.4<br>7.7 | 20.6<br>-71.4 | -28.7<br>-26.5 |

## 7 THE CONNECTION BETWEEN THE PARAMETERS OF THE ANALYTICAL MODEL OF THE AXIAL POTENTIAL AND THE PARAMETERS OF A THREE CYLINDERS LENS

An accurate version of the boundary element method was used to solve Laplace's equation for the given lens geometry and polarizations [5]. We can obtain the potential and the field at any point of the lens. By this way we find the axial potential distribution and its derivatives. Once the field distribution is known, the equations of motion are integrated by the Dormand-Prince method. We have found the correspondence between the parameters of the analytical model and the following parameters of the real system: the radius of cylinders, $r_{cyl}$,, the gap between cylinders, $l_{gap}$, and their lengths, $l_1 = l_3$ and $l_2$. The result of this synthesis is shown in Table 3. In the first row, upper (lower) values have been obtained for positive (negative) potential V.

Table 3

| $l_c$ (cm) | $r_{cyl}$ (cm) | $l_1$ (cm) | $l_2$ (cm) | $l_{gap}$ (cm) |
|---|---|---|---|---|
| 25 | 1.25<br>1.90 | 7.9<br>7.9 | 32.0<br>32.0 | 0.5<br>0.5 |
| 17 | 2.74 | 9.6 | 27.6 | 1.0 |
| 14 | 3.10 | 10.2 | 26.5 | 1.0 |
| 11 | 3.5 | 11.2 | 24.4 | 1.0 |

## 8 CONCLUSION

A new optimization method is proposed, described and illustrated. This method allows to determine the real parameters of focusing electrostatic systems with rotational symmetry, which gives the minimum beam spot size on the target for a given emittance. For some systems this minimum has been found together with the appropriate values of radii of two diaphragms for a set of emittances.

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