A MATRIX METHOD FOR THE OPTIMIZATION OF NONLINEAR QUADRUPOLE FOCUSING SYSTEM

Alexander D. Dymnikov and Genoveva Martínez
Dept. Física Aplicada III, Fac. de Física
Universidad Complutense, E-28040, Madrid, Spain

1 ABSTRACT

The optimization matrix method is proposed and described for studying focusing systems which produce for a given beam current the smallest beam spot size on the target. The use of this method is illustrated in the optimization of the nonlinear quadrupole focusing system which is an essential part of a microprobe and determines the microprobe resolution.

We consider the differential equation of motion of the particles accurate to terms of third order inclusive. Before the investigation of the nonlinear equation of beam motion, we are solving the linear equation. For each geometry of the system we find the excitation of the lenses and the demagnification. For solving the nonlinear equation we use the matrix method of embedding in the space of phase moments. In this method the initial approximate differential equations are replaced by the linear equations in the space of phase moments with the same accuracy. The lower limit of a spot size and the appropriate initial beam radius for different emittances are found.

2 INTRODUCTION

The purpose of a focusing system is to obtain the minimum of a beam spot size ρ for the initially diverging beam. In this paper we deal only with a focusing system such that it forms $\rho < r_1$, where r_1 is the initial beam spot size. Such a system is used in nuclear microprobe where the beam is focused to strike the small area of the specimen that is to be analysed. The beam spot size ρ defines the microprobe resolution. What is the lower limit of $\rho = \rho_m$ for a given emittance which is possible to obtain using different focusing systems? To solve this problem it is not enough to consider the linear approximation of the equation of a beam motion. We must take into the consideration the nonlinear terms. This paper describes the analytical and numerical methods which allow us to find ρ_m. Some numerical results are presented.

3 THE MICROPROBE FOCUSING SYSTEM

The focusing system consists of the lens system and two diaphragms or two collimating slits, placed in front of the lens system, and separated by the distance l_{12}. The core of the focused accelerator beam is selected with a first diaphragm which then acts as an object (the object diaphragm with radius r_1) to be demagnified by the lens system. A second diaphragm controls the aperture (the aperture diaphragm with radius r_2) and hence the aberrations of the lens system.

The following are assumed given: the total length l_{tot} of the focusing system (the distance from the position of the object to the position of the Gaussian image), the working distance g (the distance from the exit plane of the last lens to the position of the Gaussian image) and the beam emittance $em = r_1 r_2 / l_{12}$. The distance between the j-th lens and the $j+1$-th lens is denoted by s_j, the effective length of the j-th lens is l_j, and the dimensionless excitation of the j-th lens is κ_j.

Focusing of particle beams is usually accomplished by quadrupole lenses. A number of probe forming combinations based on quadrupole doublets, triplets and quadruplets have been employed. The *Russian Quadruplet* as used on the first focused probe at Harwell is the most popular configuration [1], partly because of its symmetry and its orthomorphic character which permits the use of circular object diaphragms.

We restrict here our study to the *Russian Quadruplet*, which consists of a set of four quadrupoles (magnetic or electrostatic), with alternating polarities. The two outer ones are coupled together, with the length l_1 and the excitation $\kappa_1 = \kappa_4$, as are the central ones, with the length l_2 and the excitation $\kappa_2 = \kappa_3$. The separation between the firstand the second lenses and the third and the fourth ones is s_1 and the separation between the middle lenses is s_2. We use the rectangular model for the distribution of the axial magnetic induction gradient or electric field gradient. In the present paper we consider the differential equations of motion of the particles accurate to terms of third order inclusive. That means we take into consideration all geometrical aberrations of the third order. Two types of quadruplets have been studied: systems with negative demagnification and with no crossover inside the quadruplet (the first excitation modes) ; systems with positive demagnification and with one crossover in each plane inside the lenses (the second excitation modes).

4 THE MATRIX METHODS OF ANALYTICAL AND NUMERICAL INVESTIGATION

The calculation of the optimal focusing system involves the solution of a nonlinear inverse multiparameter problem. The entire solution of this problem includes: the selection of the coordinate system, in our case a rectangular (Cartesian) system attached to a particle moving along the longitudinal axis z; writing out the equations of motion and the electromagnetic field equations in the selected coordinate system; the expansion of the equations of motion and field equations in Taylor series in powers of the deviation from the axial particle (in our case, to terms of the third order inclusive); the technique of solving the nonlinear problem in configuration space by reformulating it as a linear problem in phase moments space (the method of embedding in the space of phase moments [2]).

Using the last method, we obtain two linear equations for two phase moment vectors of the third order

$$dx[3]/dz = P_x(z)x[3],$$
$$\tilde{x}[3] = \{x_1, x_2, x_1^3, x_1^2x_2, x_1x_2^2, x_2^3, x_1y_1^2, x_1y_1y_2, x_1y_2^2, x_2y_1^2, x_2y_1y_2, x_2y_2^2\}$$

and

$$dy[3]/dz = P_y(z)y[3],$$
$$\tilde{y}[3] = \{y_1, y_2, y_1^3, y_1^2y_2, y_1y_2^2, y_2^3, y_1x_1^2, y_1x_1x_2, y_1x_2^2, y_2x_1^2, y_2x_1x_2, y_2x_2^2\},$$

where $x_1 = x,\ x_2 = x',\ y_1 = y,\ y_2 = y'$.

The tilde denotes transpose. The writing of the nonlinear equation in a linearized form makes it possible to construct its solution using a matrizant, which is independent of the initial vector $\{x_{10}, x_{20}, y_{10}, y_{20}\}$, whereas the solution of the nonlinear equation is sought for each value $\{x_{10}, x_{20}, y_{10}, y_{20}\}$.

The solution of these equations is written in terms of the matrizant $X(z/z_0)$ and $Y(z/z_0)$ in the form:

$$x[3] = X(z/z_0)x_0[3],\quad X(z_0/z_0) = I,$$
$$y[3] = Y(z/z_0)y_0[3],\quad Y(z_0/z_0) = I.$$

Using the rectangular model for the distribution of the axial magnetic induction gradient and electric field gradient we have obtained in the relativistic case the analytical solution for $X(z/z_0)$ and $Y(z/z_0)$.

For finding the averaged radius of the beam we use the matrices of the moments M_x and M_y of the distribution function over whole totality of the phase coordinates [3], where

$$M_x(z) = \int_\Omega f(x_1, x_2, y_1, y_2)x[3]\tilde{x}[3]dx_1dx_2dy_1dy_2,$$
$$M_y(z) = \int_\Omega f(x_1, x_2, y_1, y_2)y[3]\tilde{y}[3]dx_1dx_2dy_1dy_2.$$

The integration is performed over the apertures of two diaphragms. We define the square of the averaged radius $r(z)$ of the beam as the maximum value from the matrix elements $M_{x11}(z)$ and $M_{y11}(z)$, where

$$M_x(z) = X(z/z_0)M(0)\tilde{X}(z/z_0),$$
$$M_y(z) = Y(z/z_0)M(0)\tilde{Y}(z/z_0).$$
$$M(0) = \int_\Omega f(x_{10}, x_{20}, y_{10}, y_{20})x_0[3]\tilde{x}_0[3]dx_{10}dx_{20}dy_{10}dy_{20} = \int_\Omega f(x_{10}, x_{20}, y_{10}, y_{20})y_0[3]\tilde{y}_0[3]dx_{10}dx_{20}dy_{10}dy_{20}.$$

We consider the case of two round diaphragms and we suppose that $f(x_{10}, x_{20}, y_{10}, y_{20}) = 1$ for $(x_{20} + x_{10}/l_{12})^2 + (y_{20} + y_{10}/l_{12})^2 \le (r_2/l_{12})^2$, $x_{10}^2 + y_{10}^2 \le r_1^2$, and $f(x_{10}, x_{20}, y_{10}, y_{20}) = 0$ for $(x_{20} + x_{10}/l_{12})^2 + (y_{20} + y_{10}/l_{12})^2 > (r_2/l_{12})^2$, $x_{10}^2 + y_{10}^2 > r_1^2$, and $r(0) = r_1$. In this case we have found the analytical expression for the (12×12) matrix $M(0)$, which is a function of r_1, of emittance em, and of l_{12}.

5 THE METHOD OF THE NUMERICAL INVESTIGATION

We choose merit function as $\rho = r(l_{tot} - z^*)$ for a given emittance, where $l_{tot} - z^*$ is the position of the circle of least confusion. Before the merit function can be evaluated our program calculates the elements of the matrizant of the third order, which is used for further calculations with each particular geometry. For the given geometry from the first-order stigmatic equations $X_{12}(l_{tot}) = 0$ and $Y_{12}(l_{tot}) = 0$ we find the values of the excitations κ_1 and κ_2.

The merit function is a function of r_1 and of l_{12}. All remaining parameters are fixed when we are seeking for the minimum value of ρ. The radius of the object diaphragm has the strongest influence on the beam spot size for the given emittance. It is very important to use the optimal r_1 for obtaining the smallest beam spot size.

6 THE RESULT OF CALCULATIONS

The optimal parameters (the low limit of minimum spot size ρ and appropriate values of r_1, r_2, l_{12} and demagnification d) as the result of numerical optimization are shown in Tables 1 - 3, where the upper line is for the magnetic quadruplet and the lower line is for the electrostatic system. The total length l_{tot} in all our calculations is 8 m. The systems with the negative demagnification have a minimum spot size if all lenses of these systems are grouped together [4]. All remaining lengths are fixed. Tables 1 and 2 list the values for the first and second excitation modes.

Table 1

ρ (nm)	r_1 (μm)	r_2 (μm)	g (cm)	d
362 454	7.96 10.2	19.3 23.9	5	-21.5
394 466	7.84 9.2	19.5 26.3	10	-19.2
425 498	7.55 9.0	20.1 26.7	15	-17.2
460 506	7.35 8.0	20.5 29.7	20	-15.5
483 535	7.08 7.7	13.6 30.7	25	-14.0

Table 2

ρ (nm)	r_1 (μm)	r_2 (μm)	g (cm)	d
326 442	254 339	14 11	5	813
343 460	144 194	22 17	10	417
386 488	95 120	33 27	15	250
408 543	69 92	37 30	20	165
448 584	53 69	45 37	25	116

Table 3 shows how ρ, r_1, and r_2 depend on the emittance for one chosen geometry ($s_1 = 0.04$ m, $s_2 = 4$ m, $g = 0.05$ m). It is possible to describe this dependence for ρ by the following expression: $\rho = k(g, l_1, l_2, s_1, s_2, l_{tot})\,em^{3/4}$, where for the chosen geometry $k \approx 1.8$ for the magnetic systems and $k \approx 2.4$ for the electrostatic quadruplet.

Table 3

em (nm)	ρ (nm)	r_1 (μm)	r_2 (μm)	k ($m^{1/4}$)
10	1.860 2.340	1492 1904	23.7 18.5	1.86 2.34
1	326 442	254 339	13.9 10.5	1.83 2.49
0.1	57 80	45 60	7.2 6.0	1.80 2.53
0.01	10 13	8.4 10.7	3.6 3.1	1.79 2.37
0.001	1.8 2.3	1.4 1.9	1.4 1.4	1.79 2.34

7 CONCLUSION

The described matrix method is very efficient for solving different problems connected with the optimal particle beam motion. In this paper using this method we have found the lower limit of the spot size for the microprobe focusing system, consisting of the magnetic or electrostatic *Russian Quadruplet*. This limit depends mainly on the value of the emittance of the beam. Nonseparated quadruplet (with negative demagnification and with no crossover inside) gives a smallest spot size 10-15% larger than separated systems (with positive demagnification and withone crossover inside in each plane). The separated Russian quadruplet has an advantage over the nonseparated system because it has a 5-20 times bigger demagnification. The latter configuration allows the use of an object diaphragm which is 5-20 times bigger. The electrostatic system gives a smallest ρ $\approx$30% larger than magnetic quadruplet.

REFERENCES

[1] G.J.F. Legge, Nucl. Instr. and Meth. 197 (1982) 243.

[2] A.D. Dymnikov and G.M. Osetinskij, Fiz. Elem. Chastits At. Yadra 20 (1989) 694; and Sov. J. Part. Nucl. 20 (1989) 293.

[3] A.D. Dymnikov and E.A. Perelshtein, Nucl. Instr. and Meth., 148(1978),567.

[4] A.D. Dymnikov, D.N. Jamieson and G.J.F.Legge, Nucl. Instr. And Meth. B 104 (1995) 64.

*Work supported by DGICYT, Ref.: SAB95-0208.