EVOLUTION OF AN OFF-AXIS COHERENT STATE FOR PARTICLE BEAMS IN THE PRESENCE OF SMALL ABERRATIONS

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Abstract

The evolution of a charged particle beam in an infinite 1-D quadrupole-like device with small sextupole and octupole aberrations is analitically and numerically described in the framework of the Thermal Wave Model. The transverse density distribution of the beam, whose initial configuration is represented by an arbitrary coherent state (Gaussian-like off-axis centered distribution), is described in terms of a Schrödinger-like equation where Planck's constant is replaced with the transverse emittance. In this *quantum-like* description of the transverse beam dynamics the distruption of the beam coherence is analytically and numerically investigated.

1 INTRODUCTION

Recently charged particle beam transport through a generic optical device has been described by means of the so-called *Thermal Wave Model* (TWM) for particle beam dynamics [1]. In this approach the transverse (longitudinal) dynamics of a particle beam is described in terms of a complex function, called *beam wave function* (BWF), whose squared modulus gives the transverse (longitudinal) density profile of the beam [1]-[3], [6] ([4,5,7,8]). The BWF satisfies a Schrödinger-like equation where Planck's constant is replaced by the transverse (longitudinal) beam emittance [1]-[3], [6] ([4,5,7,8]). In this paper, we study the particle beam evolution of a generic off-axis coherent state through an infinite 1-D quadrupole-like in the presence of small sextupole and octupole deviations . We show that , due to the aberrations, during the evolution, the particle beam profile does not correspond to a coherent state anymore. Numerical extimes for this effect have been given.

2 SEXTUPOLE AND OCTUPOLE DEVIATIONS THROUGH AN INFINITE QUADRUPOLE

In this section we describe the transverse dynamics of the charge particle beam while it is travelling through an infinite 1-D quadrupole of focusing strength $k_1 > 0$ with small *sextupole* and *octupole* deviations (*aberrations*). It has been shown in Ref. [6,9] that if we assume an initial $(z=0)$ transverse Gaussian-like off-axis centered distribution

$$\Theta_0(x,0)=\left(1/2\pi\sigma_0^2\right)^{1/4}\exp\left[-\frac{\left(x-x_0(0)\right)^2}{4\sigma_0^2}+\frac{i}{\varepsilon}p_0(0)x-i\delta_0(0)\right] \quad , \qquad (1)$$

the beam evolution in absence of aberrations at any $z>0$ is described by a particular coherent state which is the ground-like state

$$\Theta_0(x,z)=\frac{1}{\left[2\pi\sigma_0^2\right]^{1/4}}\exp\left[-\frac{\left(x-x_0(z)\right)^2}{4\sigma_0^2}+\frac{i}{\varepsilon}p_0(z)x-i\delta_0(z)\right] \quad . \qquad (2)$$

In Eq. (2) the beam transverse size σ_0 satisfies the equilibrium condition $\sigma_0^2=\varepsilon/2\sqrt{k_1}$ (zero-divergence of the beam), where ε is the beam emittance. The functions $x_0(z)$ and $p_0(z)$ represent the space-coordinate and the momentum coordinate shift, respectively, and they
satisfy the following differential equations

$$x_0'' + k_1 x_0 = 0 \quad , \qquad (3)$$

$$p_0'' + k_1 p_0 = 0 \quad . \qquad (4)$$

Finally, the quantity $\delta_0(z)$ is determined by

$$\delta_0' = \frac{p_0^2}{2\varepsilon} - \sqrt{k_1}\,\frac{x_0^2}{4\sigma_0^2} + \frac{\sqrt{k_1}}{2} \quad . \qquad (5)$$

In presence of small sextupole and octupole aberrations, the BWf satisfies the following equation

$$i\varepsilon\frac{\partial}{\partial z}\Psi(x,z) = -\frac{\varepsilon^2}{2}\frac{\partial^2}{\partial x^2}\Psi(x,z)+\left(\frac{1}{2}k_1x^2+\hat{V}\right)\Psi(x,z), \qquad (6)$$

with the initial condition (1), where the perturbation $\hat{V}(x)=(1/3!)k_2x^3+(1/4!)k_3x^4$ accounts for the aberrations. In particular $(1/3!)k_2x^3$ $\left((1/4!)k_3x^4\right)$ corresponds to the 1-D sextupole (octupole) potential term. Note that $\hat{V}(x)$ can be considered a small perturbation if the conditions $k_2\sigma_0/3k_1 << 1$ and $k_3\sigma_0^2/12k_1 << 1$ are satisfied.

An approximate solution of the problem (6) with the initial condition (1), and with $x_0(z)=A_0 sin\left[\sqrt{k_1}z+\vartheta_0\right]$ can be obtained by using the standard perturbation approach. We write the following expansion of the $\Psi(x,z)$ in terms of the eigenstates Ψ_m^0, namely

$$\Psi(x,z)=\sum_{m=0}^{+\infty} c_m(z)\Psi_m^0(x,z) \quad , \tag{7}$$

where the eigenstates Ψ_m^0 are defined by

$$\Psi_m^0(x,z)=\frac{\Theta_0(x,z)}{\sqrt{2^n n!}} H_n\left(\frac{x-x_0(z)}{\sqrt{2}\sigma(z)}\right)\exp[i2n\phi(z)] \quad . \tag{8}$$

and the function is given by $\phi_0(z)=-\sqrt{k_1}\,z/2$. The substitution of Eq. (8) into (6) yields the infinite set of equations

$$i\varepsilon\frac{dc_n(z)}{dz}=\sum_{m=0}^{+\infty} c_m(z)\langle n|\hat{V}|m\rangle \quad , \tag{9}$$

where $|n\rangle\equiv\Psi_n^0$. To solve the set of equation (9) we consider the first order correction to the BWF in the case in which $c_m(0)=\delta_{m,0}$, according to the initial condition (1). If we write $c_m(z)=\delta_{m,0}+c_m^1(z)$ Eqs. (9) give

$$i\varepsilon\frac{dc_n^1(z)}{dz}=\langle n|\hat{V}|0\rangle \quad , \tag{10}$$

$$\langle n|\hat{V}|0\rangle=\int_{-\infty}^{+\infty}\Psi_n^{0^*}(x,z)\hat{V}\,\Psi_0^0(x,z)dx \quad . \tag{11}$$

Hence, the coefficients $c_n^1(z)$ are given by the following expressions

$$c_0^1(z)=-\frac{i}{\varepsilon}\left[\rho_{0,0}(z)+\frac{1}{2}\rho_{2,0}(z)+\frac{3}{4}\rho_{4,0}(z)\right] \quad , \tag{12}$$

$$c_1^1(z)=-\frac{i}{\varepsilon}\left[\frac{1}{2}\rho_{1,1}(z)+\frac{3}{4}\rho_{3,1}(z)\right] \quad , \tag{13}$$

$$c_2^1(z)=-\frac{i}{\varepsilon}\left[\frac{1}{4}\rho_{2,2}(z)+\frac{3}{4}\rho_{4,2}(z)\right] \quad , \tag{14}$$

$$c_3^1(z)=-\frac{i}{\varepsilon}\left[\frac{1}{8}\rho_{3,3}(z)\right] \quad , \tag{15}$$

$$c_4^1(z)=-\frac{i}{\varepsilon}\left[\frac{1}{16}\rho_{4,4}(z)\right] \quad . \tag{16}$$

In the above expressions the functions $\rho_{m,n}(z)$ are defined as

$$\rho_{0,0}(z)=\frac{\varepsilon\tilde{A}_0^3}{4}\int_0^{\sqrt{k_1}z} sin^3(u+\vartheta_0)du+\frac{\varepsilon\tilde{A}_0^4}{4}\mu\int_0^{\sqrt{k_1}z} sin^4(u+\vartheta_0)du$$

$$\rho_{2,0}(z)=\frac{3\varepsilon\tilde{A}_0}{2}\nu\int_0^{\sqrt{k_1}z} sin(u+\vartheta_0)du+3\varepsilon\tilde{A}_0\mu\int_0^{\sqrt{k_1}z} sin^2(u+\vartheta_0)du$$

$$\rho_{4,0}(z)=\varepsilon\mu\sqrt{k_1}z$$

$$\rho_{1,1}(z)=\frac{3\sqrt{2}\varepsilon\tilde{A}_0^3}{4}\nu\int_0^{\sqrt{k_1}z}\exp[iu]sin^2(u+\vartheta_0)du+\frac{\varepsilon\tilde{A}_0}{4}\mu\int_0^{\sqrt{k_1}z}\exp[iu]sin^3(u+\vartheta_0)du$$

$$\rho_{3,1}(z)=\frac{\sqrt{2}\varepsilon}{2}\nu\int_0^{\sqrt{k_1}z}\exp[iu]du+2\sqrt{2}\varepsilon\tilde{A}_0\mu\int_0^{\sqrt{k_1}z}\exp[iu]sin(u+\vartheta_0)du$$

$$\rho_{2,2}(z)=\frac{3\varepsilon\tilde{A}_0}{2}\nu\int_0^{\sqrt{k_1}z}\exp[i2u]sin(u+\vartheta_0)du+12\varepsilon\tilde{A}_0\mu\int_0^{\sqrt{k_1}z}\exp[i2u]sin^2(u+\vartheta_0)du$$

$$\rho_{4,2}(z)=\varepsilon\mu\int_0^{\sqrt{k_1}z}\exp[i2u]du$$

$$\rho_{3,3}(z)=\frac{\sqrt{2}\varepsilon}{2}\nu\int_0^{\sqrt{k_1}z}\exp[i3u]du+2\sqrt{2}\varepsilon\tilde{A}_0\mu\int_0^{\sqrt{k_1}z}\exp[i3u]sin(u+\vartheta_0)du$$

$$\rho_{4,4}(z)=\varepsilon\mu\int_0^{\sqrt{k_1}z}\exp[i4u]du$$

3 NUMERICAL RESULTS

In the equations for $c_n^1(z)$ we have introduced the dimensionless parameters $\nu=k_2\sigma_0/3k_1$, $\mu=k_3\sigma_0^2/12k_1$ and $\tilde{A}_0=A_0/\sigma_0$. The non-normalized perturbed BWF is then obtained by the superposition (7) for five terms only,

$$\begin{aligned}\Psi(x,z)&=\left(1+c_0^1(z)\right)\Psi_0^0(x,z)+c_1^1(z)\Psi_1^0(x,z)\\&+c_2^1(z)\Psi_2^0(x,z)+c_3^1(z)\Psi_3^0(x,z)+c_4^1(z)\Psi_4^0(x,z)\end{aligned} \tag{17}$$

Note that now the BWF $\Psi(x,z)$ given by (17) is not Gaussian anymore and, consequently, does not represent a coherent state anymore. In fact, the aberrations introduce a defocosing of the particles which produces a distortion of the particle beam distribution with respect the unperturbed gasussian profile. The parameters μ and ν represent a measure of the distortion due to sextupole and octupole, respectively, once the multipole distortion is defined as the ratio of the sextupole (octupole) aberration strength to the quadrupole strength. Provided that $\mu << 1$ and $\nu << 1$, the distribution in configuration space is proportional to $|\Psi|^2$, where Ψ is given by (17). In Fig. 1 we have plotted the normalized transverse density profile versus the dimensionless transverse coordinate $x/\sqrt{2}\sigma_0$ for increasing values of the dimensionless length $\xi = \sqrt{k_1} z$ and for $\mu = 0.005$, $\nu = 0.05$, $\tilde{A}_0 = 0.4$, $\vartheta_0 = \pi/2$ [see Fig. 1(a)-1(f)]. The dashed lines give the starting distribution of the off-axis coherent state.The parameter $\tilde{A}_0$ is a measure of the shift of the starting beam distribution center from the optical axis and the condition $\vartheta_0 = \pi/2$ means that we are considering a beam distribution whose initial velocity is zero. From Fig. 1(a)-1(f) we can clearly see that the center of the particle distribution oscillates as the beam propagates throughout the optical device. The solid lines represent the distorted and shifted particle distributions for values of the dimensionless length ξ increasing from $\xi = 23$ to $\xi = 28$.

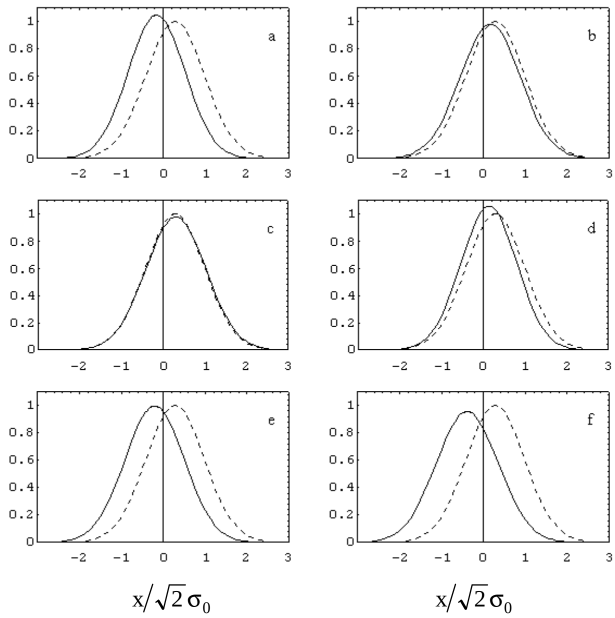

Fig.1 Normalized transverse profile of the beam density $\sqrt{2\pi}\sigma_0|\Psi|^2$ vs. $x/\sqrt{2}\sigma_0$ for increasing value of the dimensionless length ξ and for $\tilde{A}_0 = 0.4$, $\nu = 0.05$ and $\mu = 0.005$. The dashed line is the starting distribution of the off-axis coherent state; the solid line is the distorted distribution. (a) $\xi = 23$, (b) $\xi = 24$, (c) $\xi = 25$, (d) $\xi = 26$, (e) $\xi = 27$, (f) $\xi = 28$

REFERENCES

[1] R. Fedele and G. Miele, Nuovo Cimento D **13**, 1527 (1991).

[2] R. Fedele and P.K. Shukla, Phys. Rev. A **44**, 4045 (1992) .

[3] R. Fedele and G. Miele, Phys. Rev. A **46**, 6634 (1992).

[4] R. Fedele, L. Palumbo, and V.G. Vaccaro,Proc. EPAC 92 (Berlin, 24-28 March 1992) H. Henke, HHomeyer and Ch.Petit-Jean-Genaz Ed.s, Editions Frontieres 762 (1992) .

[5] R. Fedele, G. Miele, L. Palumbo, and V.G. Vaccaro, Phys. Lett. A **179**, 407 (1993) .

[6] R. Fedele, F. Galluccio, and G. Miele, Phys. Lett. A **185**, 93 (1994) .

[7] R. Fedele and V.G. Vaccaro, Physica Scripta **T52**,36 (1994).

[8] R. Fedele, G. Miele and L. Palumbo, Phys. Lett. A **194**, 113 (1994).

[9] S. De Nicola, R. Fedele, V.I. Man'ko and G. Miele, Phys. Scripta **52,** 191 (1995).