OPTIMIZATION OF THE PARAMETERS OF THE LINAC BUNCHER

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1 INTRODUCTION

There are different types of bunchers for changing dc beams into pulsed beams or into bunch succession. These bunchers consist of cavities and cavities spacing that resonating at the fundamental linac frequency or one of the harmonics. The main problem rising before creating buncher system of any type is a choice of algorithm to optimize such parameters as RF amplitudes and cavity and cavity-accelerator spacing to obtain minimum phase extent of a bunch coming out of a beam buncher system.

At present work the criterion is proposed to compare the buncher efficiency based on the correlation between start phase shape of bunches and the same one at the entrance of buncher. An analytical method has been developed to investigate any types of bunchers and to define their optimized parameters. The method also can be extended to include nonlinear effects and beam and HF system instabilities.

2 THEORY

We consider a buncher that translates initial phase θ of particle tracing through the bunching system into its final phase ϕ. The last one depends on set of parameters of buncher and beam x_i as

$\phi = \phi(\theta, x_1, x_2, \dots x_n)$, where i=1,2,...,n. Let $\Delta\phi_{av}$ being a middle-square phase dimension of bunch after passing the buncher with the initial phase θ satisfies the condition $-\alpha \le \theta \le \alpha$ with $\alpha \le \pi$ or $\alpha = P\pi/100$, in which P is percent of particles under those optimization is undertaken. It can be written general definition:

$$\Delta\phi_{av} = \left\{ \frac{1}{2\alpha} \int [\phi(\theta, x_1, x \ , \dots , x_n) - \phi_0]^2 d\theta \right\}^{-1/2} \qquad (1)$$

where $\phi_0 = \frac{1}{2\alpha} \int \phi(\theta, x_1, x_2, \dots , x_n) d\theta -$ is the average phase for the particles.

Another convenient parameter of buncher is the initial decrease factor K_{av} that can be defined as

$$K_{av} = \frac{\Delta\theta_{av}}{\Delta\phi_{av}} = \frac{\alpha}{\sqrt{3}\Delta\phi_{av}} \qquad (2)$$

It will be a optimized parameters buncher when for given α (or percent of bunching particles P) value of $\Delta\phi_{av}$ will have a minimum (or K_{av} will have maximum). The optimized values of x_i (for i=1,2...n) can be found from following relations:

$$\frac{\partial \Delta\phi_{av}}{\partial x_i} = 0 \qquad (3)$$

For the obtaining of optimized parameters x_i a set of equations (3) must be solved.

3 SINGLE GAP BUNCHER

3.1 Linear application

Let V is the voltage on the electrode when the particle crosses the gap and L is bunching distance, eV_0 is the energy of electrons or ions, where V_0- injection voltage. If the space change have been neglected we can write following expression for final phase :

$$\phi = \theta + \frac{2\pi L}{\lambda\beta_0} [1 + \eta T(d) F(r)]^{-1/2} , \qquad (4)$$

where λ is the wavelength, $\beta_0 = v_0/c$- the initial velocity of particle, $\eta = V/V_0$, T(d) - is the transit angle function, d - acceleration gap length, $F(r) = I_0(\xi r)/I_0(\xi a)$, r - certain particle radius into the beam, a - buncher electrode radius, $\xi = 2\pi/\lambda\beta_0$. If $\eta \ll 1$ we can derive from (4) neglecting other orders of η :

$$\phi = \theta - \mu \sin(\theta) + \phi_0 , \qquad (5)$$

where $\mu = \frac{\pi L}{\lambda\beta_0} \eta T(d) F(r)$, $\phi_0 = \frac{2\pi L}{\lambda\beta_0}$ .Therefore, linear approximation gives us the final phases of particle as a function of single parameter μ , and we can obtain the optimized value of μ by substitution eq. (5) into (1) and then when integrated into (3). After taking transformations we shall arrive:

$$\mu_0(\alpha) = \frac{2(\sin(\alpha) - \alpha \cdot \cos(\alpha))}{\alpha - \sin(\alpha) \cdot \cos(\alpha)} \equiv \frac{2C_1}{C_2} \qquad (6)$$

Following substitution of eq. (6) into (2) yields a next equation :

$$K_{0av}(\alpha) = \left[1 - \frac{3C^2}{\alpha^3 C_2} \right]^{-1/2} . \qquad (7)$$

The value of μ and K_{0av} as a function of P is shown in fig.3.

3.2 Nonlinear approximation

Let $\eta < 1$ and then the expression for ϕ would be obtained as the expression :

$\phi_\eta=\phi_{0av}+\theta-\mu\sin(\theta)-(3/4)\mu\eta\sin^2(\theta)-$
$-(5/8)\mu\eta^2\sin^3(\theta)+\ldots$, (8)

where $\phi_{0av}=\phi_0+\frac{3C_2}{8\alpha}\mu\eta+\frac{35C_3}{128\alpha}\mu\eta^3+\ldots$,

$C_3=(3/4)\alpha+\sin(2\alpha)/2+\sin(4\alpha)/16$.

Combining eqs. (8) , (1) and (3) we can easily show that the expression for optimized parameter of μ without terms of higher exponents is given by:

$$\mu_\eta(\alpha)=\mu_0(\alpha)[1-D_1(\alpha)\eta^2] \quad , \qquad (9)$$

where $D_1(\alpha)>0$ for all α–values. Combining eqs (9) and (1) it can be derived the next correlation of middle square bunch dimensions and $\Delta\phi_{0av}$ from eq. (7);

$$\Delta\phi_{\eta av}(\alpha)=\Delta\phi_{0av}(1)+D_2(\alpha)\eta^2 \quad , \qquad (10)$$

where $D_2(\alpha)>0$. If follows from (9) and (10) that optimized value of μ decreases with increasing voltage V and phase shape of bunches is increasing at the same time as much more as initial phase interval narrows.

3.3 Effect of beam and buncher parameter unstabilities

If the buncher and beam parameters such as bunching distance, fundamental frequency and voltage in the cavity are adjusted the final bunches have a defined shape. At practice, always there are several deviations of above parameters and fundamental frequency, initial energy and voltage on cavity can be represented in the following form:

$$\omega_d=\omega(1+\delta_1) \quad , \qquad (11a)$$

$$V_{0d}=V_0(1+\delta_2) \quad , \qquad (11b)$$

$$V_d=V(1+\delta_3) \quad , \qquad (11c)$$

where $\delta_i<<1$ are the relative deviation (i =1,2,3) . It must be emphasized that changing of frequency and energy is effected both on μ and ϕ_0 as at the same time voltage on cavity effects μ only. We obtain the relative deviation of decrease factor in a manner above:

$$\delta K_i(\delta_i)\equiv K_{iav}/K_{oav}=[1+(K_0^2-1)B_i\delta_i^2]^{-1/2} \quad , \qquad (12)$$

where $B_1=\frac{8\alpha}{\alpha-\sin(\alpha)\cos(\alpha)}\cdot\frac{1}{\eta^2}+1,$ (12a)

$$B_2=\frac{2\alpha}{\alpha-\sin(\alpha)\cos(\alpha)}\cdot\frac{1}{\eta^2}+\frac{9}{4} \quad , \qquad (12b)$$

$$B_3=1 \qquad (12c)$$

As $1/\eta^2>>1$ then $B_1>B_2>B_3$, so for the same value of δ_i the biggest effect has parameters ω (fundamental frequency), the next is eV_0 - the injection energy and the last one is a voltage on cavity V.

As the values of deviations δK_i are not fixed but are given by some, other distribution functions, it can be found the average deviation of decrease factor by integration eq.(12):

$$\delta K_i^{av}=a\cdot\int_{-\delta_{0i}}^{\delta_{0i}}\delta K_i(\delta_i)\rho(\delta_i)\,d\delta_i \quad , \qquad (14)$$

where a is constant and δ_{0i} is a maximum value of i - parameter, $\rho(\delta_i)$ is distribution density of i - parameter. The convenient example for estimation is the uniform distribution: $\rho_g(\delta)=$*const* for $|\delta|<\delta_0$, $\rho_g(\delta)=0$ for $|\delta|>\delta_0$. In such a case we have :

$$\delta K_{ig}^{av}=\frac{1}{\chi_i}\cdot\ln\left|\chi_i+\sqrt{(1+\chi_i^2)}\right| \quad , \qquad (15)$$

where $\chi_i=\delta_{0i}/[B_i(K_0^2-1)]^{1/2}$. If we in such a manner fix the maximum deviation of δK_{ig}^{av} then it can be shown the maximum admitted deviation of δ_{0i} is :

$$\delta_{0i}\le\chi_i(\delta K_{ig}^{av})/\sqrt{B_i(K_0^2-1)} \quad . \qquad (16)$$

As one can see from eq.(15) the value of the admitted deviation is inversely proportional to decrease factor.

4 DOUBLE GAP BUNCHER

Usually double gap buncher consist of two cavities closely arranged and added with one bunching distance or two cavities separated with one bunching distance or two cavities separated with bunching spacing and followed by buncher-accelerator distance. Under such circumstance a different combinations of waveforms are used (fundamental and higher harmonics) for 1 and 2 cavities.

4.1 Double gap buncher excited by fundamental frequency with two bunching spacing

Let the particle eV_0 energy is injected the first cavity with voltage on electrode V_1 and crosses through bunching spacing L_1 to the second cavity with voltage on electrode V_2 and again passes spacing L_2 and finally enters to accelerator. If $V_1<<V_0$ and $V_2<<V_0$ the linear approximation gives us the following expression for a final phase as a result of combining eq.(16) and (1) :

$$\phi_2=\theta-(\mu_1+g\mu_2)\sin(\theta)-\mu_2\sin(\theta-\mu_1\sin(\theta))+\phi_{02} \quad , \qquad (16)$$

where $\mu_j=\pi L_jV_0/\lambda\beta_0V_0$, $g=V_1/V_2$, $\phi_{02}=2\pi(L_1+L_2)/\lambda\beta_0$, j=1,2 and then substituting these into eq.(3).

Contrary single gap buncher case here it is not possible find an analytical relations between g, μ_1,μ_2 and α We have developed a numerical method for solving the system of equations (3). As a result it has been found that there are the optimized parameters μ_1^o and μ_2^o that insure minimum value of final bunches under given P and g. Absolute minimum of $\Delta\phi_{02av}$ corresponds to g=0 the hypothetical case with $L_1\to\infty$. It means that a first cavity and bunching spacing role is to provide the needed phases of particles coming the second cavity when its voltage on electrode passes the linear part of sinusoid. When g value or L_1 decreases above condition breaks and phase dimension of bunches begins to increase. Phase decrease factor as function of g is shown in fig.1.

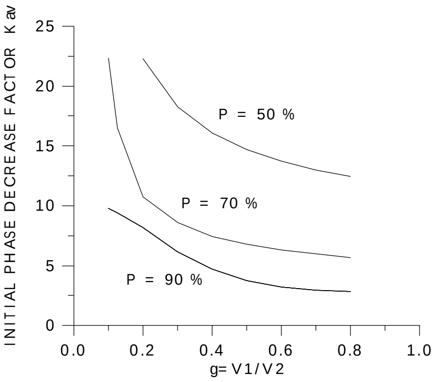

Figure 1.

K_{av} is a function of two parameters μ_1 and μ_2 is been a surface in coordinates (μ_1,μ_2). Maximum of K_{av} will be P-dependent, such as for P>70% only one maximum exists for $\mu>1$. For lower value of P $\mu<1$. In such case the optimized values of μ for the first maximum are decreased.

4.2 Double gap buncher excited by second and fundamental harmonics with a neared cavities

If the voltage on electrode of fundamental harmonic gap is V_{11} and on electrode of second harmonic gap is V_{12} then particle energy passing the construction can be written as ;

$W=eV_0+eV_{11}\sin(\theta)-eV_{12}\sin(2\theta)$. (17)

In a case $V_{11}<<V_0$ and $V_{12}<<V_0$ we obtain the following relation for the final phase of buncher:

$\phi_{21}=\theta-\mu_{12}(\sin(\theta)-g_{12}\sin(2\theta)+\phi_{021}$, (18)

where $g_{12}=V_{12}/V_{11}$, $\mu_{12}=\pi L V_{11}/\lambda\beta_0 V_0$, $\phi_0=2\pi L/\lambda\beta_0$. The expressions for μ_{12} and g_{12} optimized values can be found by combining eq. (18) and (1) and then with (3):

$g_{12}^{o}=(C_1C_4-C_2C_3)/(C_1C_5-C_3C_4)$, (19a)

$\mu_{12}^{o}=(C_1C_5-C_3C_4)/(C_2C_5-C_4^{2})$, (19b)

where C_1 and C_2 are defined in part 2.1 and then with (3):$C_3=(\sin(2\alpha)-2\alpha\sin(2\alpha))/4$, $C_4=2\sin^3(\alpha)/3$, $C_5=(4\alpha-\sin(4\alpha))/8$. The values of g_{12}^{o} and μ_{12}^{o} as a function of P are shown in fig. 2 and corresponds value K_{21av}^{o} are shown in fig. 3.

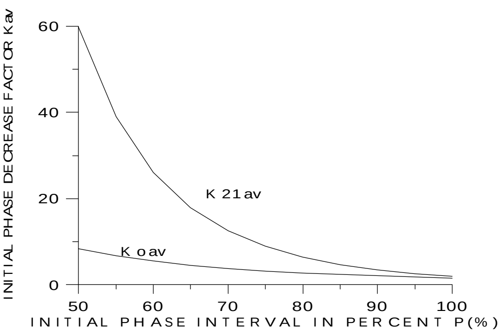

Figure 2.

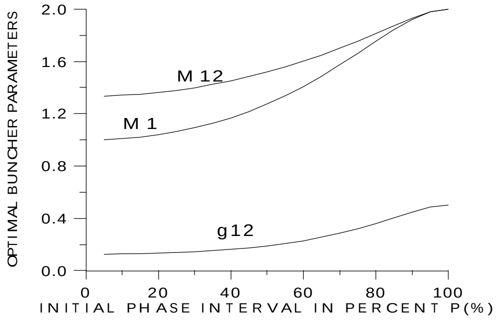

Figure 3.

5 DISCUSSION AND CONCLUSION

The proposed method of calculation of optimized parameters of buncher take it possible to analyze the effect different factors on final shape of bunchers.

It is shown for the case of single gap buncher that: - the initial phase limits are determining the optimized values of buncher parameters for which the initial phase decrease factor reaches its maximum ;

- the increase of voltage on electrode yields the increase of phase shape of buncher;
- a deviation of optimized parameters values yields the increase of phase shape of buncher and the appropriate values of deviations are inversely proportional to decrease factor;
- the biggest effect on the value of relative deviation of decrease factor has parameter ω (fundamental frequency), the next one is eV_0- the injection energy and the last one is a voltage on cavity V.

For double gap buncher excited by fundamental frequency there are exist a optimized values of parameters those depend on the initial phase interval and relation voltages in first and second cavities. Also phase shape of buncher decrease under the decreasing of the V_1/V_2 relation.

The double gap buncher excited by fundamental frequency and second harmonic all optimized parameters of a buncher are uniquely determined by initial phase interval.