

# COLLIMATOR WAKE FIELDS IN THE SLC FINAL FOCUS \*

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## Abstract

The SLC final-focus system accommodates 29 fixed or adjustable collimators for machine protection and background reduction. By amplifying pulse-to-pulse orbit variations and by generating emittance growth, collimator wake fields may degrade the beam quality at the interaction point (IP). In the SLC final focus, collimator half-apertures are larger than the bunch length, so that the standard collimator-wake formula of Bane and Morton does not apply. Numerical wake-field calculations for SLC parameters agree quite well with the high-frequency impedance of a step-out transition, derived by Gianfelice and Palumbo. Due to the nature of a final-focus system, the wake-field kicks from all collimators add coherently, and the overall impact on luminosity can be significant. This paper suggests that collimator wake fields in the final focus provide a possible explanation for the 30% discrepancy between expected and measured luminosity in the 1994/95 SLC run.

## 1 INTRODUCTION

In the 1994/95 SLC run, the measured luminosity was roughly 30% lower than that expected from emittance-measurements at the end of the linac or at the entrance to the final focus [1]. A possible explanation for this discrepancy are collimator wake fields in the final-focus system. In addition to diluting the beam emittance, collimator wake fields amplify the pulse-to-pulse orbit jitter, which also was a major concern during the last SLC run. Though wake fields originating from collimators at the end of the linac and from unsleeved bellows and beam-position monitors in the SLC arcs have equally been suspected as potential amplifiers of orbit jitter and sources of spot-size increase, these latter wake fields cannot explain the discrepancies found between the actual luminosity and that predicted from emittance measurements at the entrance to the final focus.

The SLC final focus contains a total of 29 vertical collimators, 6 of which are adjustable. The fixed collimators are round, the adjustable ones consist of two flat jaws (top and bottom) which are positioned so as to minimize the background in the SLD detector or Compton polarimeter. The collimators are made either from Titanium, Tungsten or Copper. None of them is tapered. For more details on the SLC collimators the reader may consult Ref. [2].

## 2 GEOMETRIC COLLIMATOR WAKES

Consider a bunch with a Gaussian longitudinal distribution passing close to the axis of an untapered, round collimator of radius  $a$  in a beam tube of radius  $b$ . In the standard collimator-wake model by Bane and Morton [3], the dipole wake kick is proportional to the local beam density, and the momentum change of a particle at the center of the bunch is given by

$$\Delta y'(0) \approx \frac{4Nr_e}{\sqrt{2\pi}\sigma_z a \gamma} y \quad (\sigma_z \gtrsim a, b \gg a) \quad (1)$$

where  $N$  denotes the number of particles in the bunch,  $r_e$  the classical electron radius,  $\gamma$  the relativistic Lorentz factor,  $\sigma_z$  the rms bunch length, and  $y$  the offset of the beam centroid with respect to the collimator center. The average kick over the entire bunch is  $0.71 \Delta y'(0)$  and the rms kick is  $0.28 \Delta y'(0)$ . Here, and in the following, we assume that  $y \ll a$  so that only the dipole wake field is important and the kick is proportional to the offset  $y$ .

In the SLC final focus, the rms bunch length is 0.7–1.1 mm, a typical beam-pipe radius  $b$  is 2 cm, and the collimator half-apertures  $a$  vary between 3 mm and the beam-pipe radius, depending on location. Hence, the bunch length is short compared with the other dimensions. Under such conditions the Bane-Morton model is not strictly applicable, though it is still expected to provide an upper bound for the actual wake.

To attain a better approximation to the real wake field, one may extend the diffraction model of a cavity [4, 5] to a step-out transition or a collimator. In Ref. [6], the high-frequency impedance in the limit of small step sizes ( $b - a \ll b$ ) is found to be  $\text{Re}Z_{\perp}(\omega) \approx 4Z_0(b - a)c/(\pi a^3 \omega)$  where  $Z_0$  denotes the impedance of free space ( $377 \Omega$ ), and  $c$  the velocity of light. Under these conditions, and assuming a Gaussian distribution, the variation of the wake kick along the bunch is described by an error function, and the kick at the beam center is

$$\Delta y'(0) \approx \frac{8(b - a)Nr_e}{a^3 \gamma} y \quad (\sigma_z \ll a, b - a \ll b). \quad (2)$$

In this case, the average kick is equal to the kick at the center and the rms kick is  $\Delta y'(0)/\sqrt{3}$ .

A third approach is to take the high-frequency limit of the impedance for a semi-infinite pipe of radius  $a$  inside an infinite pipe of larger radius  $b$ , which was derived by Gianfelice and Palumbo in Ref. [7],  $Z_{\perp}(\omega) \approx Z_0 c(1/a^2 - 1/b^2)/(\pi \omega)$ . Gianfelice and Palumbo arrive at this impedance using an intricate Wiener-Hopf technique. It is interesting to note that one can obtain the same result,

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and also extend it to higher azimuthal mode number  $m$ , in the following manner: Consider a tube of radius  $a$  making a transition to radius  $b > a$  at longitudinal position  $s = 0$ , and suppose a ring driving charge with charge  $q$ , radius  $r_d$ , and multipole moment  $m > 0$  moves from  $s = -\infty$  to  $s = \infty$  at the speed of light; it passes the collimator at time  $t = 0$ . We assume that, just as in the  $m = 0$  case [8], the *longitudinal* wake field radiated at high frequencies is nearly identical to the difference of stored energy between the final and the initial field configurations. The wake field for a symmetric collimator is twice as large, since the same amount of energy is radiated when going from a tube of radius  $b$  to one of radius  $a$ . Using the Panofsky-Wenzel theorem [9] we then obtain the transverse wake field from the longitudinal one, which we can Fourier transform to obtain the impedance. The multipole fields in a tube of radius  $b$  are [5]

$$E_r = 2qr_d^m \delta(z) \cos m\theta \left\{ \begin{array}{l} -\left(\frac{1}{r_d^{2m}} - \frac{1}{b^{2m}}\right) r^{m-1} \\ \frac{1}{r^{m+1}} + \frac{r^{m-1}}{b^{2m}} \end{array} \right. \quad (3)$$

$$E_\theta = 2qr_d^m \delta(z) \sin m\theta \left\{ \begin{array}{l} \left(\frac{1}{r_d^{2m}} - \frac{1}{b^{2m}}\right) r^{m-1} \\ \frac{1}{r^{m+1}} - \frac{r^{m-1}}{b^{2m}} \end{array} \right. \quad (4)$$

where the upper and lower rows refer to  $r < r_d$  and  $r > r_d$ , respectively, and  $B_\theta = E_r$ ,  $B_r = -E_\theta$ ; with  $z = s - ct$ . Using these fields to obtain the stored energy, and following the procedure sketched above, we obtain for large  $\omega$

$$Z_\perp^{(m)}(\omega) = \frac{Z_0 c}{\pi \omega m} \left( \frac{1}{a^{2m}} - \frac{1}{b^{2m}} \right) \quad (m > 0), \quad (5)$$

where the definition of  $Z_\perp^{(m)}$  follows the convention of Ref. [5]. For the dipole wake ( $m=1$ ) this formula is identical to the Gianfelicce-Palumbo result, and, for a Gaussian bunch, it predicts a center kick equal to

$$\Delta y'(0) \approx \frac{2Nr_e}{\gamma} \left( \frac{1}{a^2} - \frac{1}{b^2} \right) y \quad (\sigma_z \ll a); \quad (6)$$

here, as for the above diffraction model, the wake kick is given by an error function, the average kick equals  $\Delta y'(0)$  and the rms kick is  $\Delta y'(0)/\sqrt{3}$ . Note that the wake kicks for both these models are independent of the bunch length.

In order to compare the three analytical estimates and to determine their accuracy, numerical calculations were performed for the SLC bunch length ( $\sigma_z \approx 0.7$  mm), a beam-pipe radius  $b$  of 2 cm, and different collimator apertures, using the 2-D time-domain module of MAFIA [10]. Figure 1 shows typical wake functions obtained by MAFIA for different collimator gaps. In Fig. 2, the numerically computed transverse kick factor  $k_\perp$  (the average kick is given by  $k_\perp N e^2 y / E$ , where  $e$  is the electron charge and  $E$  the beam energy) is depicted as a function of collimator half aperture  $a$  and compared with the analytical estimates, Eqs. (1), (2), and (6). Figure 2 shows that the Palumbo-Gianfelicce formula, Eq. (6), agrees almost perfectly with the numerical results; it will, therefore, be used in the following calculations. We note that, for the collimators considered here, the resistive-wall wake fields are about 200

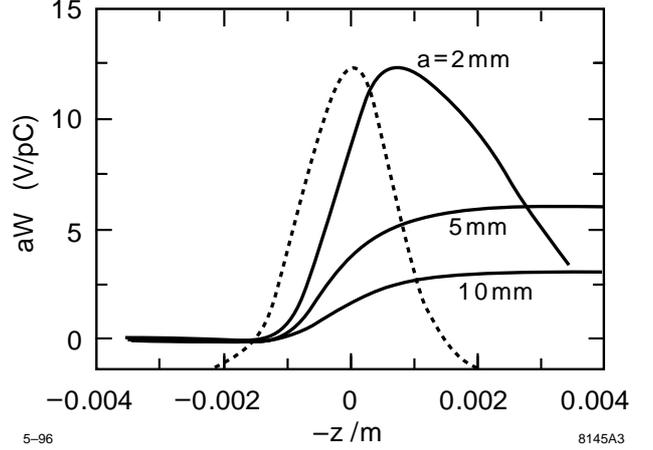


Figure 1: Dipole-wake function of a Gaussian bunch with  $\sigma_z = 0.7$  mm, as computed by MAFIA, for three different collimator half apertures  $a$ . The beam-pipe radius is  $b = 2$  cm; the beam profile is given by the dotted curve, with the head to the left.

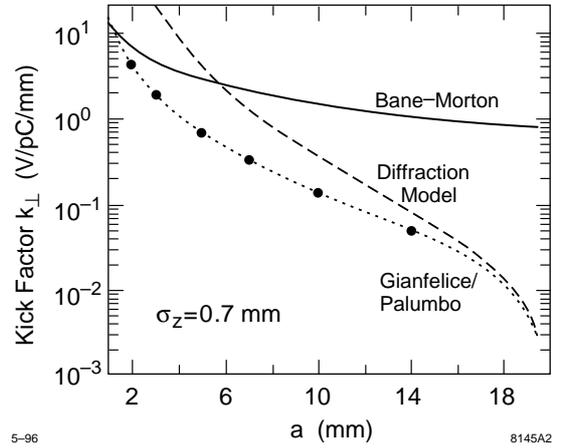


Figure 2: Transverse kick factor as a function of collimator half aperture  $a$ . The three curves correspond to three different analytical estimates, while the plotting symbols refer to the numerical calculations. A beam-pipe radius  $b$  of 2 cm is assumed.

times weaker than the geometric wakes and may safely be ignored. In the following, we will also approximate the dipole-wake kick of flat collimators by Eq. (6), with  $a$  representing the half width of the collimator gap.

### 3 ORBIT JITTER AND SPOT SIZE

Collimator wakes affect the performance of the final focus in two ways: by increasing the projected emittance of the beam, and by amplifying any incoming pulse-to-pulse orbit variation ('jitter'). Suppose all collimators are perfectly aligned, and the beam enters the final focus executing a betatron oscillation. Then the beam particles experience a wake-field kick at each collimator. For an incoming betatron oscillation the effect of all collimator wakes add coherently. To first order, the change of the vertical centroid

position at the IP is

$$\Delta y_{IP} = \sum_i R_{34,i} \Delta y'_i = \sum_i -R_{34,i}^2 \mu_i y'_{IP} \quad (7)$$

where  $\Delta y'_i$  denotes the wake kick from the  $i$ th collimator,  $R_{34}$  the (3,4) R-matrix element (in TRANSPORT notation [11]) from the collimator to the IP,  $y'_{IP}$  the IP orbit-angle corresponding to the incoming oscillation and the coefficient  $\mu_i$  the wake-field strength of collimator  $i$ . For the Palumbo-Gianfelice model we have

$$\mu_i \equiv 2 \left( \frac{1}{a_i^2} - \frac{1}{b_i^2} \right) \frac{Nr_e}{\gamma} \quad (8)$$

If the incoming orbit is perfect, but the collimators are randomly misaligned, the different contributions have to be added not coherently, but in quadrature. It is noteworthy that, in the SLC final-focus system, all collimators (except for one) are separated from the IP by an odd multiple of  $\pi/2$  in betatron phase, and, thus, additional position jitter caused by the collimators will be visible only in the IP phase, where it also does the most damage. Most other jitter sources, e.g., wake fields in the SLC arcs, will contribute equally to both betatron phases. This difference can be used for estimating the fraction of the IP position jitter which originates in the final focus.

Wake fields do not only affect the beam centroid, but the differential wake kick across the bunch also causes a spot-size increase at the IP. The latter can be described by an equation identical to (7), if one replaces  $\mu_i$  with an appropriate multiplier for the rms kick. In the case considered, the rms kick is just the average kick divided by  $\sqrt{3}$ .

If the IP position change is expressed in units of the linear IP spot size  $\sigma_{y0} \equiv \sqrt{\epsilon_y \beta_y^*}$  and the orbit angle at the IP in units of the divergence  $\sigma_{y'} \equiv \sqrt{\epsilon_y / \beta_y^*}$ , one obtains

$$\frac{\Delta y_{IP}}{\sigma_{y0}} = \left( - \sum_i R_{34,i}^2 \mu_i \frac{1}{\beta_y^*} \right) \frac{y'_{IP}}{\sigma_{y'}} \quad (9)$$

and a similar expression, with slightly different values for  $\mu_i$ , applies to the spot-size increase. The dimensionless multipliers  $\Pi_i \equiv - \sum_i R_{34,i}^2 \mu_i / \beta_y^*$  of both jitter and spot-size growth are compiled in Table 1 for seven particularly detrimental collimators (all except for PC18.5 are flat), along with the actual collimator half gaps  $a$  of the 94/95 SLC run and the corresponding collimation depths  $a/\sigma_y$ .

The overall spot-size increase for  $N = 4 \times 10^{10}$  particles per bunch,  $\sigma_z \approx 0.7$ – $1.1$  mm, energy  $E \approx 46$  GeV, and the 94/95 parameters is  $\Delta \sigma_y / \sigma_{y0} \approx 0.96 (y' / \sigma_{y'})_{IP}$ , to be added in quadrature, which implies that a 0.5 (1.8)  $\sigma_{y'}$  orbit variation would cause a 10% (100%) spot-size growth. The jitter amplification is of similar magnitude:  $\Delta y / \sigma_{y0} \approx 1.69 (y' / \sigma_{y'})_{IP}$ : if a beam enters the final focus with equal jitter amplitude in both phases, the position jitter in the IP phase is amplified by about 100%.

Six of the collimators in Table 1 are adjustable. Retracting these collimators would reduce the overall jitter amplification and the overall spot-size increase by about a factor of 3. It remains to be seen if this is possible without compromising the detector background.

coll.	$a$ (mm)	$a/\sigma_y$	$\Pi_{\Delta y}$	$\Pi_{\Delta \sigma}$	$\Delta y$ (mm)
PC185	2.0	40	0.024	0.013	0.76
PC125	9.0	17	0.113	0.064	1.72
PC75	7.0	15	0.136	0.077	1.21
PC3	7.0	14	0.165	0.093	1.10
PC12	3.0	13	0.301	0.170	0.27
C1X	2.1	10	0.364	0.207	0.20
C1Y	2.1	10	0.375	0.212	0.20

Table 1: Parameters of six important collimators for the 1994/95 SLC run: collimation depths  $a/\sigma_y$ , their respective normalized jitter-amplification and emittance-growth multipliers,  $\Pi_{\Delta y}$  and  $\Pi_{\Delta \sigma}$ , and alignment tolerances  $\Delta y$  corresponding to a 2% spot-size increase each.

When the beam orbit is not centered inside a collimator, the resulting wake field causes a (static) spot-size increase, which translates into a tolerance on the relative alignment of beam and collimator. Exemplary alignment tolerances are also listed in Table 1. The numbers quoted correspond to a 2% spot-size increase from each collimator. If we consider the combined effect of all 29 collimators, the alignment may need to be better than these numbers by a factor 3–6, to achieve a total spot-size increase below 10%.

## 4 CONCLUSIONS AND THANKS

Collimator wakefields in the SLC final-focus system have been identified as a possible source of the measured emittance increase originating between the entrance of the final focus and the IP. The wake-field effect is aggravated by the large number of final-focus collimators. In future SLC runs, more care will be devoted to precision-alignment of the adjustable collimators. Only those collimators whose impact on detector and polarimeter background is proven will be moved close to the beam. The centering of the beam inside the collimator jaws needs to be carefully maintained. Some of the fixed collimators with particularly tight alignment tolerances may require additional surveying. We thank G. Stupakov who first pointed us to Ref. [7].

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