THE RELATIVISTIC PARTICLE IN AN FLUCTUATING ELECTROMAGNETIC FIELD

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Abstract

The goal of this paper is to compute the Eulerian correlations for the electromagnetic field tensor in the case when some type of fluctuations occur. The starting point is the covariant form of the relativistic equation of motion of a charged particle in an electromagnetic field and the correlations are effectively written for the case of a vector potential satisfying the Coulomb gauge. The possibilities of computing the various moments for the random velocity and the running diffusion coefficient are pointed out.

1 INTRODUCTION

There are two more important possibilities for the analyze of the behavior of a relativistic charged particle introduced into a fluctuating field: the use of a relativistic kinetic (Boltzmann) equation and the use of a covariant Langevin-type equation obtained by adding one or two other terms to the relativistic equation of motion of a particle in an electromagnetic field. The first of these two methods was presented in [3] for a plasma and it mainly consists in the introduction of a random acceleration in the kinetic equation of the system rather than using a non-hamiltonian but deterministic collision term [1, 2]. Using this procedure, R.Balescu, J.H.Misguish and H.Wang have introduced in [3] the concept of **hybrid kinetic equation.** We do not discuss here in detail this method, but we intend to compare its results with those given by the second method mentioned above. This second method will be presented in the next section of our paper. Finally, in the last section we shall effectively compute the correlation functions in the particular case when only the vector potential of the field will present some gaussian white noise-type fluctuations.

2 THE COVARIANT EQUATION FOR THE MOTION IN THE ELECTROMAGNETIC FIELD

Let us consider a flow of relativistic particles moving in an external electromagnetic field. Each particle, with the rest mass m_0 and the electric charge q, is described by an equation of the type

$$m_0 c \frac{du_\alpha}{d\tau} = qF_{\alpha\beta}\ u^\beta \tag{1}$$

where c is the light velocity, $F_{\alpha\beta}$ is the electromagnetic field strength tensor, and u_α is the four-dimensional velocity attached to the particle in the Minkowski space-time. The metric of the space is given by the tensor

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In order to obtain a Langevin-like equation and to get a correct description of the fluctuations due to the collisions between the particles [3], we shall add two other new terms in (1): a term proportional with u_α describing the collisional friction forces and a term η_α which is a four dimensional random force. The equation (1) becomes in this case:

$$m_0 c \frac{du_\alpha}{dt} = qF_{\alpha\beta}\ u^\beta + k\, u_\alpha + \eta_\alpha \ ; \quad \alpha = 0, 1, 2, 3 \tag{2}$$

If A_α stands for the cuadripotential of the electromagnetic field, the expression for $F_{\alpha\beta}$ will be:

$$F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha \tag{3}$$

In S.I. units and using the metric tensor of the Minkowski space that we have choose above, one could consider $A_\alpha = \{(1/c)\phi, -\overrightarrow{A}\}$ where ϕ is the scalar potential and $\overrightarrow{A}$ stands for the vector potential.

Because of the gauge invariance of (2), we may fix the potentials A_α imposing for example the Lorentz gauge $\partial A_\alpha \ / \ \partial x_\alpha = 0$.

The field strengths $\overrightarrow{E}$ and $\overrightarrow{B}$ in terms of the potentials are given by the formula:

$$\begin{cases} B_i = \frac{1}{2}\epsilon_{ijk}\left(\partial^k A^j - \partial^j A^k\right) \\ E_i = -\partial_i \phi - \partial_0 A_i \\ i, j, k = 1, 2, 3 \end{cases} \tag{4}$$

One can observe that, if the components B_i and E_i are fluctuating, we must consider also the fluctuations of the quantities A_α and we must know the correlation functions of the components of the four-potential.

We note that, if all the fluctuations that can appear will be taken into account, the equation (2) will be studied as a multiple-random terms equation. Concretely, the fluctuations generated by the random force η_α, the fluctuations of the electromagnetic field and of the velocity u_α due to the collisions between the particle are to be considered. Choosing explicit forms for the statistical correlations of the velocities u_α and of the four dimensional force η_α, we may calculate the various moments for the four dimensional velocity and, finally, the running diffusion coefficient. In the next section we shall consider the case when only the fluctuations for the electromagnetic field will occur.

3 THE CORRELATIONS FOR THE ELECTROMAGNETIC FIELD TENSOR

Let us consider the case of an electromagnetic field described by a constant scalar potential. In this case the Lorentz gauge condition becomes the Coulomb one:

$$\begin{gathered} \nabla \overrightarrow{A} = 0 \Rightarrow \\ (\exists)\overrightarrow{M}(\mathbf{r}) \;\; s.t. \;\; A_i = \frac{1}{2}\epsilon_{ijk}\partial^j M^k \end{gathered} \tag{5}$$

If we choose

$$\overrightarrow{M}(\mathbf{r}) = \Psi(\mathbf{r})\overrightarrow{e}_3 \tag{6}$$

and if we pass by the Fourier transformations to the momenta space, we obtain

$$\begin{gathered} A_1(\mathbf{k}) = -i\ k_2\Psi(\mathbf{k}) \\ A_2(\mathbf{k}) = i\ k_1\ \Psi(\mathbf{k}). \end{gathered} \tag{7}$$

The translational invariance allows us to define the following statistical assumption about the scalar function $\Psi(\mathbf{k})$

$$< \Psi(\mathbf{k})\Psi(\mathbf{k}') >= \Psi(\mathbf{k})\delta(\mathbf{k}+\mathbf{k}') \tag{8}$$

Let us consider that in the momentum space we note $k_\perp^2 = k_x^2 + k_y^2$ and $k_\parallel = k_z$. Then the following relation holds:

$$\begin{gathered} < A_i(\mathbf{k})A_j(\mathbf{k}') >= \\ = (k_\perp^2\delta_{ij} - k_ik_j)\Psi(\mathbf{k})\delta(\mathbf{k}+\mathbf{k}') \equiv \\ \equiv A_{ij}(\mathbf{k})\delta(\mathbf{k}+\mathbf{k}') \end{gathered} \tag{9}$$

By direct computation one can verify that the quantities A_{ij} could be expressed in the following form:

$$\begin{gathered} A_{ij}(\mathbf{r}) = \\ = \begin{vmatrix} -\frac{2\pi}{r_\perp^4}(I_1 + 2\frac{r_y^2}{r_\perp^2}I_2) & 4\pi\frac{r_xr_y}{r_\perp^6}I_2 \\ 4\pi\frac{r_xr_y}{r_\perp^6}I_2 & -\frac{2\pi}{r_\perp^4}(I_1 + 2\frac{r_x^2}{r_\perp^2}I_2) \end{vmatrix} \end{gathered}$$

where

$$\begin{gathered} I_p = \int_0^\infty dy\ y^p\Psi(r_\perp^{-2}y, r_z)\frac{\partial^p}{\partial y^p}(I_0(\sqrt{y})) \\ p = 1,2\ ;\ \ y = k_\perp^2 r_\perp^2 \end{gathered}$$

The integral I_p and the correlations A_{ij} could be calculated only if we make a specific choice for $\Psi(\mathbf{k})$. A possible choice is to consider that the field presents some gaussian white noise fluctuations [3] :

$$\Psi(\mathbf{k}) = \beta^2\Psi_0\exp\{-\frac{1}{2}(\lambda_\perp^2k_\perp^2 + \lambda_\parallel^2k_\parallel^2)\} \tag{10}$$

In this case the explicit expressions for the correlations of the components of the vector potential are:

$$\begin{gathered} A_{mn}(\mathbf{r}) = \beta^2\{(1 - \frac{r_\perp^2}{\lambda_\perp^2})\delta_{mn} + \\ + \frac{r_mr_n}{\lambda_\perp^2}\}\exp\{-\frac{r_z^2}{2\lambda_\parallel^2} - \frac{r_\perp^2}{2\lambda_\perp^2}\} \end{gathered}$$

It is possible to obtain similar expressions for Eulerian correlations of gradients of the potential vector. If we abreviate $A_{i,\,m} \equiv \nabla_mA_i(\mathbf{r}),\ \ i,m = 1,2 \equiv x,y$ we could define the following correlation tensor:

$$\begin{gathered} A_{ij}^{mn}(\mathbf{r}) =< A_{i,m}(\mathbf{r}+\mathbf{x})A_{j,n}(\mathbf{x}) >= \\ =< A_{i,m}(\mathbf{r})A_{j,n}(\mathbf{0}) > \end{gathered} \tag{11}$$

If the stochastic process $A_{i,\,m}(\mathbf{r})$ is spatially homogeneous, as $\Psi(\mathbf{r})$ is, and if we observe the definition (11), the following symmetries are obvious:

$$\begin{gathered} A_{ij}^{mn}(\mathbf{r}) = A_{ij}^{nm}(\mathbf{r}) \\ A_{ij}^{mn}(\mathbf{r}) = A_{ji}^{mn}(\mathbf{r}) \end{gathered}$$

As in the case of the correlations for the components of the vector potential, we pass to the momenta space and we write the Fourier representation of $A_{ij}^{mn}(\mathbf{r})$ in the form:

$$A_{ij}^{mn}(\mathbf{r}) = \int d\mathbf{k}\ e^{i\mathbf{kr}}k_mk_nA_{ij}(\mathbf{k})$$

Using the expression (10) for the spectrum of $\Psi(\mathbf{k})$ one can compute $A_{ij}(\mathbf{k})$ and then one can obtain the explicit form of $A_{mn}^{ij}(\mathbf{r})$.

We can use now the last relation in order to compute the correlations corresponding to the electromagnetic tensor. These correlations will be define on the basis of the relation (3). Using the notations (11) we obtain:

$$\begin{gathered} < F_{\mu\nu}F_{\alpha\beta} >= \\ A_{\nu\beta}^{\mu\alpha} + A_{\mu\alpha}^{\nu\beta} - A_{\nu\alpha}^{\mu\beta} - A_{\mu\beta}^{\nu\alpha} \end{gathered} \tag{12}$$

We shall obtain in this manner a very interesting connection between the fluctuations of the observable quantities $\overrightarrow{E}$, $\overrightarrow{B}$ and the fundamental quantity in the relativistic description of the electromagnetic field: the strength tensor $F_{\alpha\beta}$. The correlations (12) could be used in (2) in order to compute the correlations and the mean square displacements for the relativistic motion of a charged particle in the field. These problems will be considered in a forthcoming paper [6].

4 REFERENCES

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