SELF-CONSISTENT THREE-DIMENSIONAL SASE X-RAY RADIATION VIA LIENARD-WIECHERT FIELDS

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1 ABSTRACT

We present three-dimensional numerical results of self-amplified X-ray wiggler radiation from a relativistic electron beam using Lienard-Wiechert forces to drive self-consistently the motion of electrons. A major advantage of this approach is that except for collective radiation reaction forces, all three dimensional classical interaction forces between electrons, including velocity (i.e., space charge forces) and acceleration forces, are included in the electron motion. The exact three dimensional self-consistent motion of a beam of electrons gives rise to exact three-dimensional radiation fields which can be calculated with very few approximations. As an example of this approach we report here results of three dimensional X-ray coherent wiggler radiation calculations from a short relativistic electron pulse.

2 INTRODUCTION

The usual technique applied to study the free-electron laser amplification process entails solving a paraxial version of the wave equation at one or more signal frequencies [1]. Although this approach has been quite successful in explaining most features of the free-electron laser (FEL) it has limited usefulness when dealing with self amplified stimulated emission of radiation (SASE) or when dealing with radiation from short pulses for which many frequency components must be included in the analysis.

As an alternative approach we propose using Lienard-Wiechert fields, which are exact three-dimensional spatial and temporal solutions of the wave equation for point charges, to study the collective radiation fields generated by beams of relativistic electrons interacting with undulators, wigglers and electromagnetic waves. The calculations are self-consistent because all Lienard-Wiechert field interactions between electrons, including velocity field (i.e. space charge) forces, acceleration field (i.e. radiation) forces, and external electromagnetic field forces are taken into account to calculate the motion of each electron in the beam. The nature of these particular solutions allows us to derive in a straightforward manner differential equations governing the self-consistent motion of all electrons in the beam and as a result we can explore exactly the three dimensional nature of their collective radiated fields. This approach becomes particularly useful in dealing with the FEL start-up problem. Unlike the approach used by others, our scheme requires no initial artificial electromagnetic seed to start the numerical solution of the problem. Furthermore because Lienard-Wiechert fields are time-domain solutions of the wave equation, we can study non-periodic electron beam systems. In particular we can deal satisfactorily with three-dimensional effects of very long and very short electron bunches. A limited non-self-consistent approach was used by us [2] to study the FEL stimulated emission process. The major disadvantage in our approach is the requirement imposed by the retardation condition on field calculations. That is, the calculated fields must be related to the motion of electrons at earlier (retarded) times. We have reduced the complexity of this calculations in the ultra-relativistic region with the assumption that during an integration step the longitudinal velocity of each electron remains constant. This basic approximation will be discussed in more detail in section 4.

3 BASIC EQUATIONS

In what follows we assume that the electron beam is moving through a circularly polarized static magnetic wiggler characterized by a z-dependent vector potential, $\boldsymbol{A}_u = (A_u \cos k_u z, A_u \sin k_u z, 0)$ which is independent, near the wiggler axis, of transverse coordinates (x,y). Its amplitude $\boldsymbol{A}_u$ and period $2\pi/k_u$ are assumed constant. We will use k_u to define convenient dimensionless parameters such as electron position, velocity, acceleration

$$\vec{\chi} = k_u \vec{r}, \quad \vec{\beta} = \frac{d}{d\tau}\vec{\chi}, \quad \vec{\alpha} = \frac{d}{d\tau}\vec{\beta}$$

respectively and time $\tau = k_u ct$. In addition we use the dimensionless relativistic parameter γ to describe the electron's normalized kinetic energy $\gamma = (1-\beta^2)^{-1/2}$. Since we have assumed here that near the undulator axis $\boldsymbol{A}_u$ is independent of transverse coordinates (x,y), then the corresponding electron's transverse generalized momenta

$$(\pi_x, \pi_y) = (\gamma mc\beta_x + qA_u \cos\chi_z, \gamma mc\beta_x + qA_u \cos\chi_z)$$

is conserved. From this last relation an electron's transverse position, velocity and acceleration can be obtained immediately in terms of its electron longitudinal position χ_z. The time dependence of χ_z is obtained from its longitudinal velocity β_z

$$\chi_z(\tau) = \chi_z(0) + \int_0^{\tau} \beta_z d\tau .$$

The time dependence of β_z is in turn obtained from the integration of the electron's equation of motion which is driven by the Lorentz force generated by all other electrons. Since only electric fields do work we choose to derive an energy equation from which one can then calculate the longitudinal velocity

$$\beta_z = \sqrt{\beta^2 - \beta^2_\perp} = \sqrt{1 - \frac{1 + a^2_u}{\gamma^2}}.$$

The dot product between the resultant electric force exerted by all other electrons with an electron's velocity vector yields a dimensionless energy equation of motion for the i^{th}electron

$$\frac{d\gamma_i}{d\tau} = k_u r_0 \vec{\beta}_i \cdot \sum_{j \neq i} \left[\frac{\hat{n}_{ij} - \vec{\beta}_j}{\gamma_j^2 s_{ij}^3 \rho_{ij}^2} + \frac{\hat{n}_{ij} \times (\hat{n}_{ij} - \vec{\beta}_j) \times \vec{\alpha}}{s_{ij}^3 \rho_{ij}} \right]_{\tau_{ij}}$$

which is written in terms of the electric component of the linear superposition of Lienard-Wiechert field generated by the other electrons. $\rho_{ij} = k_u R_{ij}$ is the retarded normalized electron-electron distance, the vector $\boldsymbol{n}_{ij}$ is the retarded unit vector connecting electron j with electron i, s_{ij} = 1-$\Sigma n_{ij}\beta_j$, and r_0 is the classical electron radius.

4 RETARDATION

All dynamical variables included between square brackets in the energy equation must be evaluated at a time τ_{ij} satisfying the dimensionless retardation condition $\tau = \tau_{ij} + \rho_{ij}$, which is in general a difficult equation to solve because ρ_{ij} depends on retarded time. The approximation that we make in our analysis is that since the longitudinal velocity of highly relativistic electrons does not change significantly during radiation in an undulator, even when strong radiation signals are involved, electron's longitudinal velocity remains constant during a numerical integration step. With this approximation the retarded longitudinal position of each electron $\chi_{zj}(\tau_{ij})$ can thus be derived from its present position $\chi_{zj}(\tau)$ using the relation $\chi_{zj}(\tau) = \chi_{zj}(\tau_{ij}) + \beta_{zj}(\tau - \tau_{ij}) = \chi_{zj}(\tau_{ij}) + \beta_{zj}\rho_{ij}$. Integrating numerically this simplified retardation condition with the energy equation yields the time dependence of $\vec{\chi}(\tau), \vec{\beta}(\tau)$ *and* $\vec{\alpha}(\tau)$, which are respectively an electron's position, velocity and acceleration.

5 RADIATION

The far-field angular and spectral distribution of radiation can be obtained in a straight forward manner once the self-consistent motion of each electron is found. For a multi-electron system the amount of energy radiated per unit angular frequency per unit solid angle in the direction of the unit vector $\mathbf{n}(\theta,\phi)$ is given by the well known formula [3]

$$\frac{1}{mc^2} \frac{d^2 W}{d\Omega d\omega} = \frac{r_0 \omega^2}{4c\pi^2} \left| \int_0^{t_f} \hat{n} \times \sum_j \vec{\beta}_j e^{i\omega\left(t - \frac{\hat{n} \cdot \vec{r}_j}{c}\right)} dt \right|^2 .$$

6 3DLW CODE

We have developed a three-dimensional PC code (3DLW) that solves self-consistently for the motion of a three-dimensional pulsed beam of N electrons in a magnetic wiggler taking into account all classical inter-electron forces, including Coulomb and radiation forces, with arbitrary 6N initial conditions. The program plots input and output phase space diagram, calculates the space charge spatial spectrum, evaluates and makes 3D plots of the angular and spectral distribution of radiated energy and integrates the distribution to calculate total radiated energy. At this time the largest number of electrons that are practical to run in a PC simulation is twenty thousand.

7 X-RAY SASE RADIATION

We present here three-dimensional results of Self Amplified Stimulated Emission (SASE) of wiggler x-ray radiation that can be achieved with a high energy electron accelerator such as a SLAC machine. The physical parameters of the radiator are listed in Table 1:.

Table 1: X-ray SASE parameters.

Electron Energy	7	GeV
Beam Current	2500	A
Pulse Length	0.1	fs
Wiggler Period	0.1	m
Wiggler Peak Field	0.15	T
# of Electrons	5000	
Normalized beam emittance	30	π mm-mrad
Initial Energy Spread	0.01	%
Wiggler Length	10	m

The electron beam was prepared with a set of *6N* initial conditions $\chi_{xi}, \chi_{yi}, \chi_{zi}, \beta_{xi}, \beta_{yi}, \gamma_i, \quad i = 1, N\}$, using a double precision random number generator. For example Fig.1 describes the initial x-β_x component of random phase space for five thousand particles.

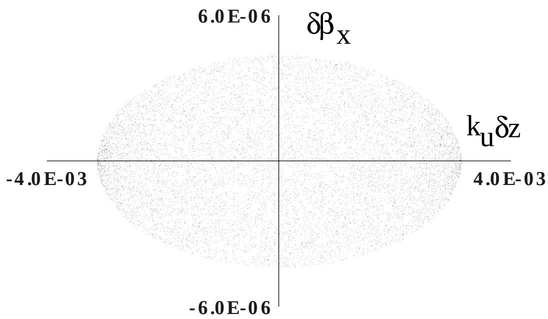

Fig.1 Initial x-β_x phase space of 5000 particles

Similar initial distributions were generated for the y-β_y and z-γ phase space components. To achieve the desired current level we assume that each particle represents the motion of about 310 electrons. Fig.2 shows the output longitudinal phase space of the electron beam after moving through ten meters along the wiggler. The most salient features of this plot is the clear level of longitudinal electron beam bunching that has occurred. Since the electron pulse length is approximately 30 nm there are nearly 38 distinct optical bunches in the pulse. A Fourier spatial analysis of the longitudinal charge distribution (see Fig.4) indicates that strong bunching has occurred at the fundamental radiation wavelength (0.8 nm) and less bunching at the first harmonic (0.4 nm) or at any higher harmonics.

Also, because of relativistic effects, there is less bunching at the trailing edge of the pulse than at its leading edge. That is, the radiation force is more intense along the forward direction than along any other direction. In fact the amplitude of the backward wave is at least a factor $2\gamma^2$ smaller than that of the forward wave. Furthermore electrons located near the leading edge of the pulse will experience a larger radiation force from electrons behind it than those located near the trailing edge of the pulse. In addition to an increase energy spread from 0.01% to 0.2%, the mean kinetic energy of the electron beam was reduced by 0.015%.

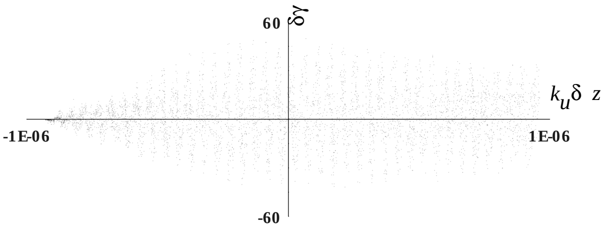

Fig.2 Longitudinal phase space of 5000 particles.

A far field angular and spectral distribution ($\phi = 0$) of radiated energy is shown in Fig.3.

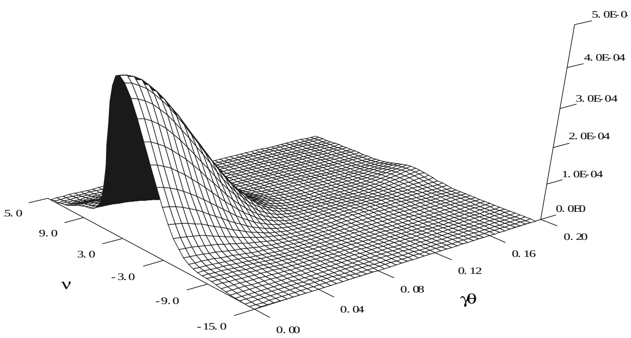

Fig.3 Angular and Spectral distribution of radiation in joules per steradian.

Instead of angular frequency we have chosen to plot the distribution in terms of the detuning parameter ν. The angular radiation frequency can be obtained from

$$\omega = k_u c \frac{\left(\beta_{z0} + \frac{\nu}{k_u L}\right)}{1 - \hat{n} \cdot \vec{\beta}}$$

where L is the undulator length. That is, the angular frequency decreases with increasing angle of observation θ.

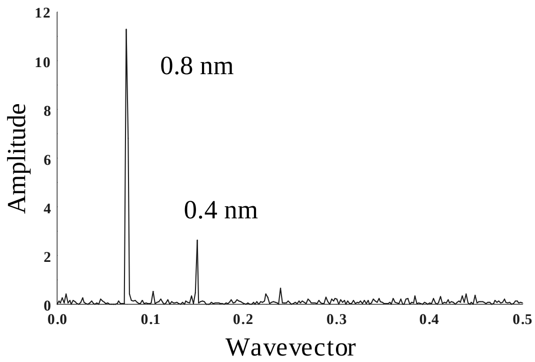

Fig.4 Longitudinal charge density Fourier analysis

Compared to the radiation distribution produced by independently radiating electrons (synchrotron radiation), the main effect of electron beam bunching has been to increase radiated energy and to decrease the angular distribution from $\theta\gamma = 1$ to $\gamma\theta = 0.06$ due to constructive interference effects, i.e., coherent radiation. The spectral distribution is as narrow as that for a single electron ($2\nu = 5.2$) because the electron beam pulse length is shorter than the slippage length of the wave. For a longer electron pulse the spectrum will be narrower but the peak power will remain unchanged. Consequently the final spectrum for a long electron pulse will be determined not by the number of periods in the undulator but by the number of electron bunches in the electron pulse. Integrating the angular and spectral distribution yields a peak radiated power of 320 kW in a 10 m wiggler. Our results indicate that for a 20 m wiggler SASE power saturates yielding a maxim power of 400 kW. The actual power levels will be higher because we assumed that electrons within a computation particle (310 electrons) contribute randomly to the total power. If they had contributed in phase with the macro particle the power would be 310 times greater (90 MW and 125 MW respectively). By no means has the problem been focused to optimize radiation energy in the X-ray region. This is one possible radiation configuration that may make possible to generate intense tunable coherent X-rays.

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