THE EFFECT OF USING GAPPED BEAMS ON A STORAGE RING RF SYSTEM

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Abstract

The use in electron storage rings of a circulating beam with a significant beam-free gap is growing in popularity. Obviously such a beam induces a voltage in the accelerating RF cavities which is different to that induced by a continuous beam. This paper analyses the cavity voltage seen by the circulating beam in the presence of a gap and the resultant longitudinal motion of the electrons. The importance of gapped beam effects on the RF system for storage ring operation is necessarily a function of the specific accelerator parameters. The Daresbury SRS, which makes increasing use of gapped beams, is presented as an example to assess this importance.

1 THE BEHAVIOUR OF AN RF CAVITY WITH A GAPPED BEAM

An accelerating RF cavity may be regarded as a parallel resonant circuit whose admittance may be written in terms of Laplace transforms as

$$\mathbf{Y_c}(s) = sC + \frac{1}{Z} + \frac{1}{sL}$$

where: C is the cavity capacitance
Z is the cavity shunt impedance
L is the cavity inductance
s is the Laplace operator

This may be re-written as

$$\mathbf{Y_c}(\mathrm{s}) = \mathrm{C}(\mathrm{s}+\alpha + \mathrm{j}\omega_0)(\mathrm{s}+\alpha - \mathrm{j}\omega_0)$$

where: $1/\alpha = 1/(2ZC)$ is the cavity time constant
$\omega_0 = 1/\sqrt{LC}$ is the cavity resonant frequency
$j = \sqrt{-1}$

Bold symbols are vectors

The cavity voltage $\mathbf{V}_c(s)$ is determined by the cavity current $\mathbf{I}_c(s)$ as

$$\mathbf{V_c}(s) = \frac{\mathbf{I_c}(s)}{\mathbf{Y_c}(s)}$$

The cavity response to a step change in alternating cavity current $I_o e^{j\omega t}$ at time t=0 is therefore

$$\mathbf{V_c}(s) = \frac{s}{s - j\omega C}\frac{I_0}{(s+\alpha+j\omega_0)(s+\alpha-j\omega_0)}$$

which transforms to

$$\mathbf{V_c}(t) = \frac{I_0}{\mathbf{Y_c}} e^{j\omega t}\left[1 - e^{-\alpha t} e^{j\left(\omega_0 - \omega\right)}\right]$$

where now

$$\mathbf{Y_c} = \frac{1 + j\tan\varphi}{Z} = \frac{1 + 2jQ\dfrac{\delta\omega}{\omega_0}}{Z}$$

$\varphi = \arctan(2Q\,\delta\omega/\omega_0)$ is the cavity detuning angle

$\delta\omega = \omega - \omega_0$

$Q = \omega_0 ZC$

$\alpha = \omega_0/(2Q)$

The $e^{j\omega t}$ term may be suppressed by using rotating axes and so we may write

$$\mathbf{V_c}(s) = \frac{Z\alpha}{s + \alpha + j\alpha\tan\varphi}\mathbf{I_c}(s)$$

where $\mathbf{I_c} = \mathbf{I_g} + \mathbf{I_b}$ is total cavity current
$\mathbf{I_g}$ is cavity current due to the generator
$\mathbf{I_b}$ is cavity current due to the beam

Rotating axes are used henceforth whenever time frame expressions are quoted.

In an electron storage ring $\mathbf{I}_c$ is the vector sum of the generator current $\mathbf{I}_g$ and the beam current $\mathbf{I}_b$. A gapped beam, orbital period T and gap length T_g, may be regarded as two opposing step functions T_g apart occurring at time intervals of T. By Heaviside's superposition theorem the beam current then becomes

$$\mathbf{I_b}(s) = \frac{\mathbf{I_b}^{peak}(s)}{s}\frac{1 - e^{-s(T-T_g)}}{1 - e^{-st}}$$

Thus the cavity behaviour with a gapped beam may be written

$$\mathbf{V_c}(s) = \frac{Z\alpha}{s+\alpha+j\tan\varphi}\left[\mathbf{I_g}(s)+\mathbf{I_b}^{peak}(s)\frac{1-e^{-s(T-T_g)}}{1-e^{-sT}}\right]$$

2 THE LONGITUDINAL ELECTRON MOTION

It will be indicated later that, in general, variations in cavity voltage due to gapped beams will be small. For small changes in $\mathbf{V_c}$ one may write

$$\Delta\mathbf{V_c} = \Delta V_c + jV_c\Delta\phi_c$$

where $\mathbf{V_c}$ has modulus and argument V_c and ϕ_c respectively. Then the standard longitudinal equations of motion of a relativistic electron beam may be written

$$\frac{d\Delta E}{dt} = f_0\left(\cos\phi_s \;\; V_c\Delta\phi_s - \cos\phi_s \;\; V_c\Delta\phi_c + \sin\phi_s \;\; \Delta V_c\right)$$

$$\frac{d\Delta\phi_s}{dt} = -\frac{2\pi f_0 h\alpha'\Delta E}{E}$$

where: ϕ_s is the accelerating phase angle
E is the electron energy
h is the harmonic number
f_0 is the orbital frequency
α' is the momentum compaction factor

These equations may be written in terms of Laplace transforms as

$$\left(s^2+\Omega^2\right)\Delta\phi_s = \frac{\Omega^2}{V_c\cos\phi_s}\left[-\sin\phi_s\Delta V_c + \cos\phi_s V_c\Delta\phi_v\right]$$

where

$$\Omega = \sqrt{\frac{2\pi h\alpha' V_c\cos\phi_s f_0^2}{E}}$$

is the synchrotron oscillation frequency.

If it is assumed that the RF generator current $\mathbf{I_g}$ remains constant as does the magnitude of the beam current $\mathbf{I_b}$ then it is fairly simple to write a transfer function which fully describes the behaviour of the beam-cavity system. However the resultant transform is a large, complicated expression whose full analysis is beyond the scope of a short paper such as this. Here attention will be restricted to the "steady state" behaviour of $|\mathbf{V_c}(t)|$ with a circulating gapped beam and the resultant effect on the synchrotron oscillation frequency. "Steady state" here means the state of the beam and cavity after initial transients have decayed.

3 STEADY STATE CAVITY BEHAVIOUR

The previously derived expression for $\mathbf{V_c}(s)$ becomes, in the time frame, after n orbits of the beam

$$\mathbf{V_c}(t) = \frac{Z}{1+j\tan\varphi}\Big[\;\mathbf{I_g}(t) + \mathbf{I_b}^{peak}(t)e^{-\alpha t}\left(e^{-\alpha T_g}-1\right)\frac{e^{\alpha T}\left(1-e^{n\alpha T}\right)}{1-e^{\alpha T}}\;\Big]$$

For a storage ring to operate efficiently it is necessary for the cavity tuning to be adjusted so that under beam-loaded conditions the cavity appears to be on tune or nearly so. Then

$$Arg\left(\mathbf{V_c}\right) \approx Arg\left(\mathbf{I_c}\right)$$

and φ may be regarded as zero. If the cavity voltage is used as a reference and is defined to be always real then

$$V_c = Z\left(I_g\cos\phi_g + jI_g\sin\phi_g - I_b\sin\phi_s + j\cos\phi_s\right)$$

$$I_g\sin\phi_g = -I_b\cos\phi_s$$

$$V_c = Z\left(I_g\cos\phi_g - I_b\sin\phi_s\right)$$

If V_R is the required energy gain per orbit then

$$I_g\cos\phi_g = \frac{V_R}{Z\sin\phi_s} + I_b\sin\phi_s$$

and so

$$I_g^2 = \left[I_b^2\cos^2\phi_s + \left(\frac{V_R}{Z\sin\phi_s} + I_b\sin\phi_s\right)^2\right]$$

$$= I_b^2 + \frac{V_R}{Z}\left(\frac{V_R}{Z\sin^2\phi_s} + 2I_b\right)$$

Storage rings always operate with an appreciable over-voltage. This means that the second term in the above equation is less than one; often very much so. Thus I_g will always be of the order of I_b. Then, for n very large, the expression for V_c becomes

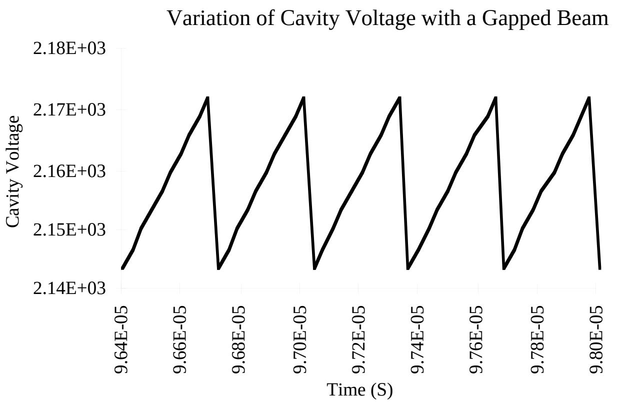
$$V_c \approx ZI_b\left[1+\frac{\left(1-e^{-\alpha T_g}\right)e^{\alpha T}e^{-\alpha t}e^{(n+1)\alpha T}}{1-e^{\alpha T}}\right]$$

αT and αT_g are normally much greater than 1 since the cavity time constant is usually many orbit periods. Then the expression for V_c tends to

$$V_c \approx ZI_b\left[1-e^{-\alpha t}e^{(n+1)\alpha T}\right]$$

The final term above represents the decay of a perturbation which is continually refreshed each beam orbit. The differences are small and so therefore are the perturbations. This will be true unless the beam currents are very large and have a gap which is comparable with the beam orbital period which itself is of the order of or greater than the cavity time constant. Even then, except in very extreme cases, it is unlikely that the effect of having a gapped beam will so degrade machine performance that the advantages of using a gapped beam will be negated. For instance the ESRF using gapped beams see typically 10% ripple in cavity voltage without any deleterious effect. In fact the effects on the beam are beneficial (see section 5).

As an example the perturbation has been computed for the Daresbury SRS at its injection energy of 0.6 GeV with a circulating 200 mA beam having a 50% gap which would be expected to give a reasonably large perturbation. The variation of V_c over a few turns is shown below. The ripple is ~1% of total cavity voltage.



4 SYNCHROTRON OSCILLATIONS WITH A GAPPED BEAM

In simple terms the variation in cavity voltage shown above is locked to the orbital frequency of the beam. Thus different bunches in the beam will experience a different cavity voltage and hence have a different synchrotron frequency to other bunches. Given the cavity voltage variation, it is a trivial matter to calculate the spread of synchrotron oscillation frequencies which will result. For the example on the Daresbury SRS the computed spread in f_s is 466 Hertz. The example situation was recently set up on the SRS and the spread in f_s was measured to be ~ 450 Hertz. The measured value of f_s was 71.2 kHz.

5 CONCLUSIONS

The effect on the RF system of having a significant gap in the circulating beam in an electron storage ring has been analysed and shown to be, in general, small but detectable provided that the orbital period and gap length do not approach or exceed the cavity time constant. The accuracy of the analysis has been tested against measurements on the Daresbury SRS.

The principal motivation for instituting gapped beams in the SRS was to provide ion clearing. The introduction of the gapped beam has, as shown, resulted in a spread in the synchrotron oscillation frequency. This spread prevents a constructive build-up of coherent longitudinal instabilities by crosstalk between successive bunches. This effect has beenobserved on the SRS as an increase in the current threshold of harmful higher order mode instabilities[1] and similar benefits have been experienced in other storage rings e.g. the ESRF[2].

REFERENCES

[1] P.A. McIntosh, “Further Cavity HOM Investigations”, SRS/APES/95/37, June 1995.
[2] A. Ropert, “Towards an Increased Brilliance in the ESRF”, EPAC 94, pp 582-584, London, 1994.