# LONGITUDINAL COUPLED–BUNCH BEAM FEEDBACK SYSTEM IN THE UNK

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*Abstract*

To damp longitudinal injection errors and ensure better stability against coupled-bunch (CB) lower-order odd multipole instabilities in the UNK proton synchrotron a band-pass CB beam feedback (FB) near RF is proposed, its bandwidth exceeding a revolution frequency. Employment of a pair of issued over-coupled RF cavities driven in quadrature to the net accelerating voltage as its acting device is foreseen. A frequency-domain impedance treatment is applied to find feasible beam stability safety margins and damping rates of injection transients. The problem of control over longitudinal emittance growth is studied in time domain with a macroparticle tracking. Being employed together with an RF FB around power amplifiers driving accelerating cavities, the beam FB proposed is shown to yield beam parameters which comply with the UNK Project's requirements.

## 1 INTRODUCTION

The UNK Project [1] foresees two band-pass longitudinal FB systems near the RF. These are:

(i) A now standard (D.Boussard) one-turn delay DC-coupled RF FB around a final power amplifier to counteract heavy pulsed beam loading of RF cavities and strong coherent instability due to their fundamental mode [2].

(ii) An AC-coupled beam FB to damp injection errors and ensure better stability against CB lower-order odd multipole instabilities [3]. The latter system is treated at length in the paper.

Both the FBs having their bandwidths $\Delta\omega^{(fb)}$ exceeding the revolution frequency $\omega_0$ while bunch in the UNK being long enough, of utmost importance is adequate understanding of the FBs' effect on CB motion of beam at dipole, quadrupole and sextupole within-bunch modes ($|m|$ =1,2,3, respectively).

To this end, a frequency-domain impedance approach to linear longitudinal CB beam FBs has been worked out [4]. Essentially, it puts on a formal basis a common intuitive notion that a FB is seen by a beam as an artificial coupling impedance controlled from the outside. Still, to account for cross-talk between various $E$-field and beam-current harmonics inflicted by frequency down- and up-mixing inside the FB circuit, an impedance matrix (with, at most, three non-trivial elements per row) must be introduced.

## 2 CHARACTERISTIC EQUATION

Let $\vartheta = \Theta - \omega_0 t$ be azimuth in a co-rotating frame, where $\Theta$ is azimuth around the ring in the laboratory frame, $\omega_0$ is angular velocity of a reference particle, $t$ is time. Let the beam with average current $J_0$ be composed of $M$ identical equispaced bunches. Denote $h$ the RF harmonic number ($h/M$ is an integer). Longitudinal field $E(\vartheta, t)$ and perturbed beam current $J(\vartheta, t)$ can be decomposed into plane waves $(E_k(\Omega), J_k(\Omega))\,\mathrm{e}^{ik\vartheta - i\Omega t}$. Frequency $\Omega$ of Fourier Transform in $t$ w.r.t. co-frame is seen as a sideband $\omega = k\omega_0 + \Omega$ in lab-frame. CB modes are labeled with $n = 0, 1, \ldots, M-1$, bunch-to-bunch phase shift of coherent motion being $2\pi n/M$.

Beam FB in question employs the same RF-band around $\pm h\omega_0$ to pick up beam signal and feed correction back to beam. Its effect can be put down in terms of coupling impedances $Z_{kk'}(\omega)$ which relate linearly $J_{k'}(\Omega)$ to $E_k^{(fb)}(\Omega)$ fed back,

$$\begin{aligned} E_k^{(fb)}(\Omega) &= -\frac{1}{L}\big(Z_{kk}(\omega)J_k(\Omega) + Z_{kk}^{(fb)}(\omega)J_k(\Omega) + \\ &+ Z_{k,k-2h\mathrm{sgn}k}^{(fb)}(\omega)J_{k-2h\mathrm{sgn}k}(\Omega)\big) \qquad (1) \end{aligned}$$

with $L$ being the orbit length, $\omega = k\omega_0 + \Omega$, $|k| \simeq h$, $|\Omega| \ll \omega_0$. Here, the first term is a conventional impedance of vacuum-chamber passive components ($\mathrm{Re}\,Z_{kk}(\omega) \geq 0$). The latter two account for an active correction imposed by the FB. These stem out of linearity of A.M. and de-A.M. procedures applied to slowly varying signals, and are free of restriction $\mathrm{Re}\,Z_{kk}(\omega) \geq 0$ which is to introduce damping into the beam coherent motion. 'Non-diagonal' impedance $Z_{k,k-2h\mathrm{sgn}k}(\omega)$ is due to an unbalanced frequency down- and up-mixing inside a FB with unequal inphase and quadrature path transfer functions. Its presence is inevitable when, say, an inphase (amplitude) control is off, as it is in the FB under study.

Now, insert Eq.1 into a conventional theory of longitudinal instabilities. Given a narrow-band FB whose $\Delta\omega^{(fb)} \ll M\omega_0/2$, it yields a characteristic equation [4]:

$$1 + \frac{\Omega_0^2\, J_0}{hV\sin\varphi_s}\left(\zeta_n(\Omega) + \zeta_n^{(fb)}(\Omega)\right) Y_{hh}(\Omega) = 0. \qquad (2)$$

Here, $\Omega_0/\omega_0$ is a small-amplitude longitudinal tune, $V$ is accelerating voltage, $\varphi_s$ is a stable phase angle ($\varphi_s < 0$ above transition, energy gain per turn is $eV\cos\varphi_s$).

$Y_{hh}(\Omega)$ is a plane-wave BTF from $E_h(\Omega)$ to $J_h(\Omega)$. Its lengthy full form can be found elsewhere [3, 4]. Generally, to study damped oscillations that are of interest in the beam FB theory, an analytical continuation of $Y_{hh}(\Omega)$ into the lower half-plane $\mathrm{Im}\Omega < 0$ is required. For reference, a bunch without incoherent tune spread exhibits

$$Y_{hh}(\Omega) = i\sum\nolimits_{m=1}^{\infty} m^2 F_{hh}^{(m)} / \left(\Omega^2 - (m\Omega_0)^2\right), \qquad (3)$$

where $F_{hh}^{(m)}$ is the bunch formfactor, $F_{hh}^{(m)} \to h^2\delta_{|m|,1}$ as bunch half-length (in RF rad) $h\Delta\vartheta_0 \to 0$.

In Eq.2, $\zeta_n$ and $\zeta_n^{(fb)}$ are the effective (or instability driving) impedances at side-bands $\Omega \simeq m\Omega_0$ of the two resonant frequency lines $k_{1,2} = n + Ml_{1,2} \simeq \pm h$ of a CB mode $n$ that occur inside $\Delta\omega^{(fb)}/\omega_0$,

$$\begin{aligned}
\zeta_n(\Omega) &= Z_{k_1k_1}(k_1\omega_0 + \Omega)/k_1 + \ldots k_1 \to k_2, && (4)\\
\zeta_n^{(fb)}(\Omega) &= Z_{k_1k_1}^{(fb)}(k_1\omega_0 + \Omega)/k_1 + && (5)\\
&+ (-1)^m Z_{k_1,k_1-2h}^{(fb)}(k_1\omega_0 + \Omega)/k_1 + \\
&+ \ldots k_1 \to k_2,\ h \to -h.
\end{aligned}$$

To apply Eq.2, one has to write down coupling ($Z$) and reduced ($\zeta$) impedances in terms the FB path transfer functions and its set-point parameters.

## 3 FB PERFORMANCE

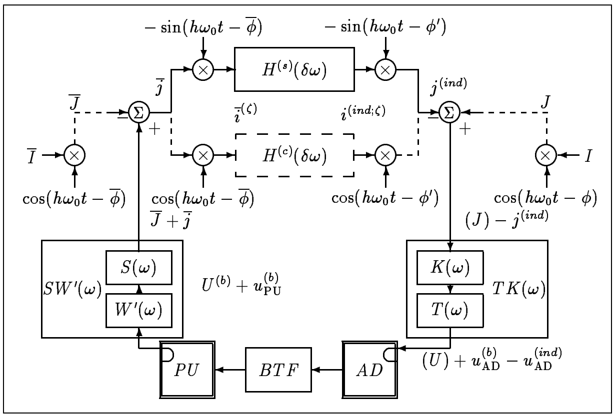


Figure 1: CB beam FB layout.

The circuit layout is shown in Fig.1. It relies on two-path inphase-quadrature filter technique. $H^{(c,s)}(\delta\omega)$ are low-pass transfer functions with $\Delta\omega_H \ll h\omega_0$. (The inphase gain (amplitude control) is off. Still, option $H^{(c)} \neq 0$ is retained in Eqs. for generality.) The rest are band-pass transfer functions: front-end electronics' admittance $S(\omega)$; current-to-current gain $K(\omega)$ through PA; $T(\omega)$ from external drive current to AD gap voltage; $-T', W'(\omega)$ from beam current to AD or PU gap voltages, respectively.

Phase $\phi'$ of up-mixing carrier is set equal to $\phi$ of a low-level drive that would have provided an inphase contribution of AD to net accelerating field. Phase $\overline{\phi}$ of down-mixing carrier is adjusted w.r.t. $\phi'$ so as to settle transit-time effects due to finite PU-AD distance. The FB is AC-coupled: it rejects strong beam loading signals at $\omega = k\omega_0$ with periodic notch filters, thus making redundant an adder unit to subtract a reference current in the l.h.s. of Fig.1.

Impedances to mediate the FB action are

$$\begin{aligned}
Z_{kk}(\omega) &= T'(\omega)|G_k^{(\mathrm{AD})}|^2, && (6)\\
Z_{kk}^{(fb)}(\omega) &= -\chi_{11}(\omega - h\omega_0) \times && (7)\\
&\times\ W'(\omega)\, G_k^{(\mathrm{AD})} G_{-k}^{(\mathrm{PU})}, \\
Z_{k,k-2h}^{(fb)}(\omega) &= -\chi_{12}(\omega - h\omega_0) \times && (8)\\
&\times\ W'(\omega - 2h\omega_0)\, G_k^{(\mathrm{AD})} G_{-k+2h}^{(\mathrm{PU})}.
\end{aligned}$$

Here $\omega = k\omega_0 + \Omega$, $k \simeq h > 0$, $|\Omega| \ll \omega_0$. The domain of $k \simeq -h < 0$ is arrived at with the reflection property $Z_{-k,-k'}(-\omega^*)^* = Z_{kk'}(\omega)$. $G_k^{(a)}$ with $a = \mathrm{PU}, \mathrm{AD}$ denote complex transit-time factors at $\omega = k\omega_0$ with $|G_k^{(a)}| \leq 1$ and $\arg G_k^{(a)} \propto \Theta^{(a)}$, the object's coordinate along the ring. These are but coefficients of Fourier series $\sum_k G_k^{(a)} \mathrm{e}^{ik\Theta}$ that decomposes function $G^{(a)}(\Theta)$, $\int_0^{2\pi} |G^{(a)}(\Theta)| d\Theta = 2\pi$ which specifies $E$-field localization. Quantities $\chi_{ij}(\delta\omega)$ are elements of the $2 \times 2$ in-out gain matrix through the open FB loop

$$\begin{aligned}
\chi_{11}(\delta\omega) &= .25\, TK(h\omega_0 + \delta\omega)\, S(h\omega_0 + \delta\omega) \times \\
&\times \left(H^{(c)}(\delta\omega) + H^{(s)}(\delta\omega)\right) \mathrm{e}^{i(\phi' - \overline{\phi})}; && (9)\\
\chi_{12}(\delta\omega) &= .25\, TK(h\omega_0 + \delta\omega)\, S(-h\omega_0 + \delta\omega) \times \\
&\times \left(H^{(c)}(\delta\omega) - H^{(s)}(\delta\omega)\right) \mathrm{e}^{i(\phi' + \overline{\phi})}; && (10)\\
\chi_{21}(\delta\omega) &= \chi_{12}(-\delta\omega^*)^*;\ \chi_{22}(\delta\omega) = \chi_{11}(-\delta\omega^*)^*.
\end{aligned}$$

The key components of the FB are: (i) an AD with

$$T(\omega) = T'(\omega) = R_T \times \left(1 - i\frac{\omega^2 - \omega_T^2}{2\omega\Delta\omega_T}\right)^{-1}, \qquad (11)$$

where $R_T$ is shunt impedance, $\omega_T = h\omega_0$ is resonant frequency, $\Delta\omega_T/\omega_0 \simeq 10$ is half-bandwidth, and (ii) a three-tap periodic FIR filter with global one-turn delay,

$$H^{(s)}(\delta\omega) = A^{(s)} \sum\nolimits_{q=0}^{2} w_q \mathrm{e}^{2\pi i\delta\omega(1 + d_1 q)/\omega_0}. \quad (12)$$

Here, $w_q$ are real weights, $d_1$ is an integer delay step measured in turns, $A^{(s)}$ is a real scale gain from beam quadrature current in PU gap to RF drive current seen inside AD gap. A low-pass filter to ensure $\Delta\omega_H \ll h\omega_0$ is not shown. Two conditions are imposed on $H^{(s)}(\delta\omega)$:

$$H^{(s)}(k\omega_0) = 0,\ \ H^{(s)}(k\omega_0 + \Omega_0) = A^{(s)}\mathrm{e}^{-i\upsilon} \qquad (13)$$

with $\upsilon$ being a prescribed phase shift at dipole side-bands (we adopt a standard $\upsilon = \pi/2$). Solving Eqs.13 yields

$$\begin{aligned}
w_0 &= .5\left(+\sin\mu - \cos\mu\cot .5\delta\mu_1\right)/\sin\delta\mu_1, && (14)\\
w_1 &= .5\cos\mu/\sin^2 .5\delta\mu_1, && (15)\\
w_2 &= .5\left(-\sin\mu - \cos\mu\cot .5\delta\mu_1\right)/\sin\delta\mu_1 && (16)
\end{aligned}$$

with $\mu = \upsilon + \delta\mu_0 + \delta\mu_1$, $\delta\mu_0 = 2\pi\Omega_0/\omega_0$ and $\delta\mu_1 = 2\pi\Omega_0 d_1/\omega_0$. To detect reliably a slow longitudinal motion ($\Omega_0 \ll \omega_0$), one has to increase $d_1$. Still, with $d_1$ growing, the circuit phase-frequency performance degrades, the FB itself tending to destabilize higher-order odd multipole modes. Of these, only sextupole ($|m| = 3$) might be of danger in practice. With these two tendencies in mind, delay $d_1\Omega_0/\omega_0$ is traded to 1/10.

Given $N_{\rm AD} = 2$ cavities, gains $A^{(s)} \gtrsim 12$ would expel dipole off-resonance CB modes $n$ beyond Landau damping threshold. Adopting a factor-of-two safety margin sets $A^{(s)} = 6$. This option yields the following gain $K$ in magnitude from beam c.o.m. phase error at 'barycentric' mode $(n, m) = (0, 1)$ to accelerating field phase correction

$$K = J_0\, N_{\rm AD}\, R_T \left| G_h^{(\rm AD)} G_{-h}^{(\rm PU)} \right| A^{(s)}/V \simeq .35. \quad (17)$$

Value of injection error treated linearly is (in units of an RF phase offset) $|h\delta\vartheta_{\rm inj}| \lesssim 2/A^{(s)} \simeq 20°$.

Fig.2 is a threshold map for the beam FB effect. Dashed lines are drawn through $\zeta_n(\Omega) + \zeta_n^{(fb)}(\Omega)$ at $\Omega = m\Omega_0$ for $m = 1, 3$, and account for action of the beam FB alone. (Quadrupole mode $m = 2$ is kept unaffected.) These lines are transformed into the solid ones by a residual destabilizing impact of the accelerating cavities controlled by RF FB [2]. Curves $A$, $B$ are threshold ones for $m = 1, 2$, respectively. Bunch half-length is $h\Delta\vartheta_0 = .54\pi$.

Injection transients show themselves up, mainly, as dipole coherent motion. Fig.3 is a $m = 1$ threshold map with contour lines of constant decrement, i.e. the images of straight lines $\Omega \simeq \Omega_1 + i\Omega_2$ plotted for $\Omega_2/\Omega_0 = -.07(.01).0$. These represent the closed-loop modes with the slowest decay which define the response time $\tau$ of beam controlled by CB FB. Damping time of injection transients depends on CB mode $n$ and falls into the range of $.015 \lesssim 1/(\Omega_0\tau) \lesssim .075$.

Feasible damping rates were verified with macroparticle tracking. Fig.4 shows tune shift $\Omega_2$ in units of incoherent spread $\Delta\Omega_s = \Omega_0 h^2 \Delta\vartheta_0^2/16$ which corresponds to a counter-clockwise detour around Fig.3 along the impedance godograph. Despite facing an ill conditioned problem of analytical continuing $Y_{hh}(\Omega)$ into ${\rm Im}\Omega < 0$, agreement between Figs.3,4 is quite satisfactory.

Fig.5 complements Fig.4 and shows amount $y$ of initial off-set squared that ultimately smears into r.m.s. emittance $\epsilon$, $y = (\delta\epsilon/\epsilon_0) \cdot (\delta\vartheta_{\rm inj}/\Delta\vartheta_0)^{-2}$. Each CB mode $n$ is crudely assumed to be injected and evolving isolatedly. On FB being off, $y \simeq 1$. Thus, FB's bandwidth w.r.t. emittance growth reduction is about $\pm 25\omega_0$.

## 4 REFERENCES


[1] Preprint IHEP 93–27, 1993.

[2] S.Ivanov, IHEP 94–43, 1994.

[3] S.Ivanov and A.Malovitsky, IHEP 96–7, 1996.

[4] S.Ivanov, IHEP 96–8, 1996.


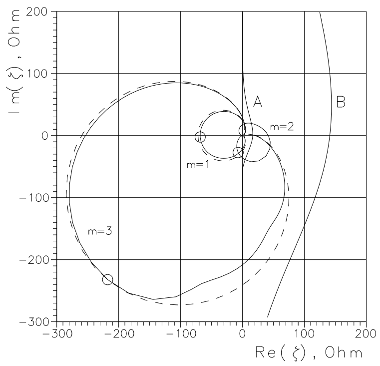

Figure 2: Stabilizing effect of CB beam FB.

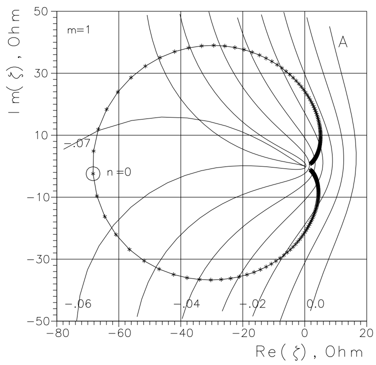

Figure 3: Threshold map of dipole oscillations.

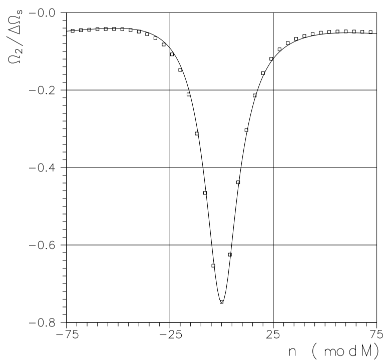

Figure 4: Damping of dipole oscillations.

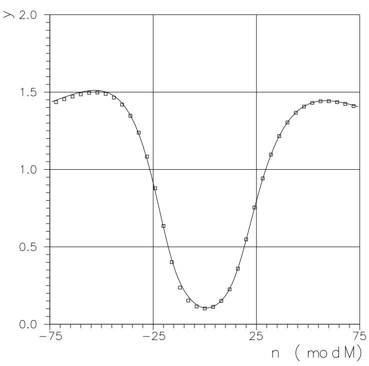

Figure 5: Control over emittance growth.