PERFORMANCE LIMITATIONS OF AN X-RAY FEL

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Abstract

In this paper we present simple design formulae for calculation of characteristics of a self amplified spontaneous emission (SASE) FEL operating in an X-ray wavelength band. It is shown also that the growth of the energy spread due to the quantum fluctuations of synchrotron radiation imposes fundamental limit on the minimal achievable value of the wavelength in the X-ray FEL.

1 INTRODUCTION

Several projects of SASE FEL (self amplified spontaneous emission free electron laser) operating in the VUV and X-ray wavelength band are developed now [1, 2, 3]. At the design stage of a SASE FEL usually the problem arises how to choose optimal parameters. As a rule, numerical simulation codes or codes based on fitting formulae are used for optimization of the FEL parameters. Nevertheless, the possibility to calculate specific numerical examples, could hardly provide a deep insight into the physics of a SASE FEL and to understand the interdependence of the FEL parameters.

In this paper we present simple design formulae for characteristics of an X-ray SASE FEL obtained with similarity techniques [4]. With increasing electron energy the effects connected with the synchrotron radiation of the electrons become to play a significant role. We have shown in this paper that the growth of the energy spread due to quantum fluctuations of synchrotron radiation imposes a limit on the minimal achievable value of the wavelength in X-ray FELs.

2 BASIC RELATIONS

We consider an FEL amplifier with helical undulator and axisymmetric electron beam. We assume the transverse phase space distribution of the particles in the beam to be Gaussian and the beam is matched to the magnetic focusing system of the undulator. The rms beam size and rms angle spread of the electrons in the beam are given by the expressions $\sigma_{\mathrm{r}} = \sqrt{\epsilon_{\mathrm{n}}\beta/\gamma}$ and $\sigma_\theta = \sqrt{\epsilon_{\mathrm{n}}/\beta\gamma}$, where β is the beta function and ϵ_{n} is the rms normalized emittance. We assume the energy spread to be Gaussian with RMS value equal to σ_{E}.

It was shown in ref. [4] that the region of parameters of proposed VUV and X-ray SASE FELs is at large values of the diffraction parameter and negligibly small influence of the space charge effects. Under these conditions it was derived criterium on the value of the shortest possible (critical) radiation wavelength which could be amplified in the FEL amplifier [4]:

$$\lambda_{\mathrm{cr}} \simeq 18\pi\epsilon\sigma_{\mathrm{E}}[\gamma I_A(1+K^2)/(IK^2)]^{1/2}/\mathcal{E}_0 . \quad (1)$$

Operation of the FEL amplifier at critical wavelength is achieved at the value of the focusing beta function

$$\beta_{\mathrm{cr}} = \epsilon\gamma^2\mathcal{E}_0/[\sqrt{2}(1+K^2)\sigma_{\mathrm{E}}] . \quad (2)$$

The critical undulator period is defined by the value of the critical wavelength (1) and FEL resonance condition: $\lambda_{\mathrm{w}}^{\mathrm{cr}} = 2\gamma^2\lambda_{\mathrm{cr}}/(1+K^2)$. Here the following notations are introduced: $K = eH_{\mathrm{w}}\lambda_{\mathrm{w}}/2\pi mc^2$ is the undulator parameter, H_{w} and λ_{w} are the amplitude of the magnetic field and the period of the undulator, respectively, $\gamma = \mathcal{E}/m_{\mathrm{e}}c^2$, $(-e)$ and m are the charge and the mass of the electron, respectively, I is the beam current, $I_A = mc^3/e$ and c is the velocity of light (we use CGS units in this paper).

When the FEL amplifier operates at the wavelength $\lambda \simeq \lambda_{\mathrm{cr}}$ we obtain the following expressions for the power gain length $L_{\mathrm{g}}^{\mathrm{cr}}$ and the FEL efficiency $\eta_{\mathrm{sat}}^{\mathrm{cr}}$:

$$\begin{aligned} L_{\mathrm{g}}^{\mathrm{cr}} &\simeq 1.6\epsilon\gamma^2\left[\gamma I_A/[IK^2(1+K^2)]\right]^{1/2} , \\ \eta_{\mathrm{sat}}^{\mathrm{cr}} &\simeq 2.3\sigma_{\mathrm{E}}/\mathcal{E}_0 . \end{aligned} \quad (3)$$

In practice there could be a situation when due to technical limitations it is impossible to design a FEL operating at the shortest possible wavelength (for instance, problems of undulator manufacturing or problems to achieve the required value of the optimal beta function). In this case, the operating wavelength has to be chosen to be larger than the minimal one and the problem arises how to optimize this general case. When $\lambda > \lambda_{\mathrm{cr}}$ that the beta function $\beta = \alpha\beta_{\mathrm{cr}}$ must be inside the limits: $\beta_{\mathrm{min}} \lesssim \beta \lesssim \beta_{\mathrm{max}}$, where the tolerable range of factor α is limited by the roots of the equation:

$$2\alpha^2 - 3\left[\lambda/\lambda_{\mathrm{cr}}\right]^{4/3}\alpha^{4/3} + 1 = 0 . \quad (4)$$

This dependency is illustrated in Fig.4. In the asymptotic case $\lambda \gtrsim 1.5\lambda_{\mathrm{cr}}$, the safety limits for the beta function are given with sufficient accuracy by:

$$(1/3)^{3/4}\beta_{\mathrm{cr}}\lambda_{\mathrm{cr}}/\lambda \lesssim \beta \lesssim (3/2)^{3/2}\beta_{\mathrm{cr}}\lambda^2/\lambda_{\mathrm{cr}}^2 . \quad (5)$$

At values of the beta function $\beta \gtrsim \beta_{\mathrm{max}}$, the operation of the FEL amplifier is destroyed due to the influence of

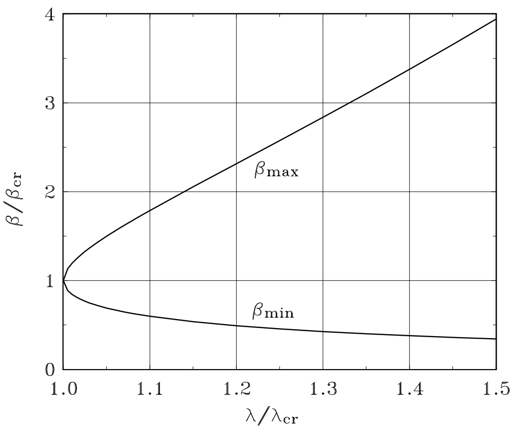

Figure 1: Safety limits for the external beta function versus the radiation wavelength.

the energy spread, and at values of the beta function $\beta \lesssim \beta_{\rm min}$ FEL operation is ruined by large longitudinal velocity spread connected with strong external focusing. Within these limits the value of the beta function should be chosen as small as possible in order to increase the field gain and the FEL efficiency, because the field gain increases with the beam current density.

For an FEL amplifier operating at $\lambda > \lambda_{\rm cr}$, we obtain the following expressions for the power gain length $L_{\rm g}$ and the FEL efficiency $\eta_{\rm sat}$:

$$
\begin{aligned}
L_{\rm g} &\simeq L_{\rm g}^{\rm cr} \left[\lambda/\lambda_{\rm cr}\right]^{1/3} \left[\beta/\beta_{\rm cr}\right]^{1/3} , \\
\eta_{\rm sat} &\simeq \eta_{\rm sat}^{\rm cr} \left[\lambda/\lambda_{\rm cr}\right]^{2/3} \left[\beta_{\rm cr}/\beta\right]^{1/3} .
\end{aligned} \quad (6)
$$

3 FUNDAMENTAL LIMITATIONS

For $K \gtrsim 1$ there is, on top of the FEL radiation process, also considerable incoherent spontaneous radiation into higher harmonics of the undulator [5]. The mean energy loss of each electron into coherent radiation is given by:

$$
d\mathcal{E}_0/dz = 2r_e^2\gamma^2 H_{\rm w}^2(z)/3 \quad , \quad (7)
$$

where $r_e = e^2/mc_e^2$. This effect impose limitation on using untapered undulator when mean energy losses become to be comparable with the bandwidth of the FEL amplifier [4].

A more fundamental limit is imposed by the growth of the uncorrelated energy spread in the electron beam due to the quantum fluctuations of synchrotron radiation. The rate of energy diffusion is given by the expression (for $K \gg 1$):

$$
< d(\delta\mathcal{E})_{\rm qf}^2/dz >= 55e\hbar\gamma^4 r_e^2 H_{\rm w}^3/24\sqrt{3}m_e c \quad . \quad (8)
$$

This effect is growing drastically with energy. When the induced energy spread becomes comparable with the initial energy spread in the beam $\sigma_{\rm E0}$:

$$
< (\delta\mathcal{E})_{\rm qf}^2 >^{1/2} \sim \sigma_{\rm E0} \ , \quad (9)
$$

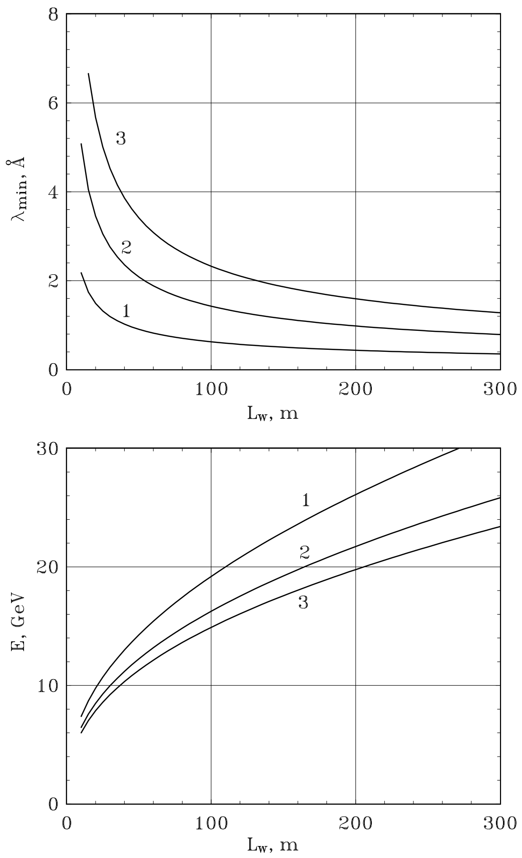

Figure 2: Minimal achievable photon wavelength in an FEL amplifier and corresponding energy of the electron beam versus the length of the undulator $L_{\rm w}$. The curves 1, 2 and 3 correspond to values of the normalized emittance 10^{-4} cm rad, 2×10^{-4} cm rad and 3×10^{-4} cm rad, respectively. The energy spread at the entrance of the undulator in all cases is equal to $\sigma_{\rm E0} = 1$ MeV.

it may dominate the amplification process. This noise effect imposes a principle limit on achieving very short wavelengths. Indeed, to achieve a shorter wavelength at specific parameters of the electron beam (i.e. at specific values of the peak current, the normalized emittance and the energy spread), the energy should be increased (see eq. (1). On the other hand, the gain length is increased drastically with increasing the energy (see eq. (3) which forces to increase the value of the undulator parameter (hence, to increase the undulator field). As a result, at some value of the energy, the energy spread caused by quantum fluctuations will stop the FEL amplifier operation.

Figs.2 and 3 present the plots of the minimal achievable wavelength and the corresponding energy of the electron beam versus the undulator length. When performing calculations we assumed that to obtain saturation of a SASE FEL, the undulator length $L_{\rm w}$ should be about 20 power gain lengths $L_{\rm g}$. The operating wavelength has been obtained using expression (1) with the energy spread given by summing up mean squared values of the initial energy

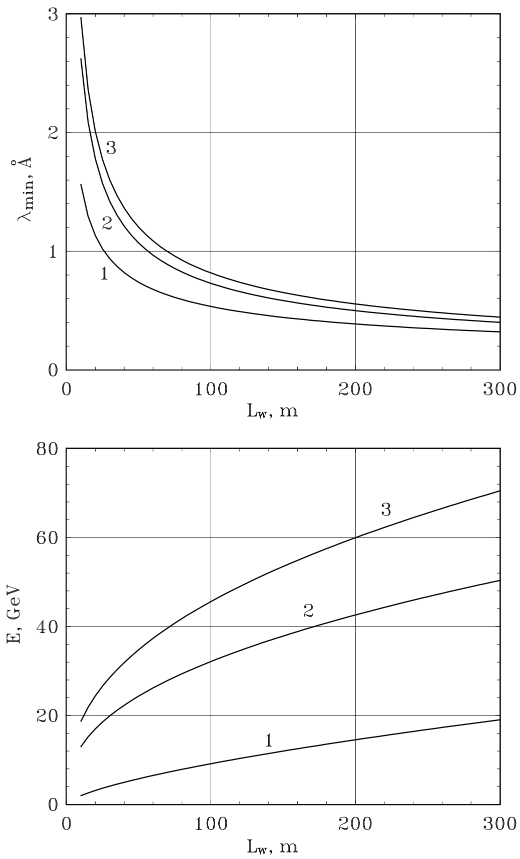

Figure 3: Minimal achievable photon wavelength in an FEL amplifier and corresponding energy of the electron beam versus the length of the undulator $L_{\rm w}$. The curves 1, 2 and 3 correspond to values of the energy spread at the entrance of the undulator of 0 MeV, 3 MeV and 6 MeV, respectively. The normalized emittance is 10^{-4} cm rad in all cases.

spread $\sigma_{\rm E0}$ and the energy spread due to the fluctuations of synchrotron radiation (9) at the undulator exit, $\sigma_{\rm E} = (\sigma^2_{\rm E0} + \sigma^2_{\rm qf})^{1/2}$. The value of optimal beta function has been calculated in accordance with eq. (2). It is seen from these plots that there is no significant decrease of the minimal wavelength for undulator lengths exceeding $L_{\rm w} \sim 100$ m.

To obtain a feeling about optimized parameters of the FEL amplifier operating at the shortest possible wavelength, we present in Table 1 two parameter sets.

It is seen from Fig.3 that after $L_{\rm w} \gtrsim 100$ m all the wavelength curves approach asymptotically the curve describing the case of zero value of the initial energy spread. This indicates that quantum fluctuations of synchrotron radiation impose a limit on the value of the minimal achievable wavelength in an X-ray FEL (which is achieved at $K = 1$)[4]:

$$
\begin{aligned}
\lambda_{\rm min} &\simeq 45\pi \left[\lambdabar_{\rm c} r_{\rm e}\right]^{1/5} L_{\rm w}^{-7/15} \left[\epsilon_n^2 I_A/I\right]^{8/15} \quad , \\
\gamma &\simeq 0.13 \left[L_{\rm w}/\epsilon_n\right]^{2/3} \left[I/I_A\right]^{1/3} \quad , \qquad (10)
\end{aligned}
$$

where $\lambdabar_{\rm c} = \hbar/mc$.

Table 1: FEL amplifier for the shortest wavelength

	# 1	# 2
Electron beam		
Energy $\mathcal{E}_0$, GeV	19.2	26
Peak current I, kA	5	5
RMS normalized emittance $\epsilon_{\rm n}$, cm rad	10^{-4}	3×10^{-4}
RMS energy spread $\sigma_{\rm E}$, MeV	1	3
Focusing beta function β, m	54	27
Undulator*		
Period $\lambda_{\rm w}$, cm	3.65	6.93
Magnetic field $H_{\rm w}$, T	0.57	0.69
Undulator parameter K	1.96	4.46
Undulator length $L_{\rm w}$, m	100	100
Radiation		
Wavelength λ, Å	0.62	2.76
Power gain length $L_{\rm g}$, m	5	5
Efficiency η, %	0.023	0.045

* Helical tapered undulator.

In conclusion we should summarize the following. In principle, quantum fluctuations impose a limit to achieving short wavelengths. The only real possibility to decrease the minimal wavelength is to decrease the value of the normalized emittance. At the present level of accelerator technology it could be possible to construct electron accelerators with a peak current of few kA, a normalized emittance of about 10^{-4} cm rad and an uncorrelated energy spread in the beam about one MeV. At these electron beam parameters the minimal achievable wavelength in an X-ray FEL will be in the range of 0.5 – 1 Å.

4 REFERENCES

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