THE SIMULATION STUDY OF THE SINGLE BUNCH INSTABILITIES IN THE SPRING-8 STORAGE RING

Takeshi Nakamura, SPring-8, Kamigori, Ako-gun, Hyogo, Japan

1. INTRODUCTION

The simulation code for single-bunch instabilities, SISR(Single-Bunch Instabilities in Storage Rings) was developed and applied to study instabilities in the SPring-8 storage ring. SISR uses complex amplitude of betatrom motion and it enable to treat both distributed broad-band impedance such as small discontinuities of a beam pipe and localized impedance like cavities.

2. EQUATIONS OF MOTION

The transverse motion of i-th electron under the force F is represented with η_i as

$$\eta_i = \frac{x_i}{\sqrt{\beta}}, \quad \theta = \frac{1}{\nu_0}\int^s \frac{ds'}{\beta} \tag{1}$$

$$\frac{d^2\eta_i}{d\theta^2} + \left(\nu_0 + \Delta\nu_i\right)^2 \eta_i = \nu_0{}^2\, \beta^{\frac{3}{2}} \frac{F_i}{E} \tag{2}$$

where ν_0, β and E is the betatron tune, the beta function and the energy, respectively.

The force F_i from wake field is

$$F_i = e \sum_{j=1}^{N_p} q_j\, x_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right)$$

$$= e\, \beta^{\frac{1}{2}} \sum_{j=1}^{N_p} q_j\, \eta_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right) \tag{3}$$

Assuming that

$$\left|\frac{d^2 a_i}{d\theta^2}\right| << \left|2\, i\nu_0 \frac{da_i}{d\theta}\right| \tag{4}$$

, which means that the typical varying time of a_i is much smaller than the betatron period $\lambda\beta$/c, and using the phasers shown below,

$$\eta_i = \mathrm{Re}\left[a_i(\theta)\, e^{i\nu_0\theta}\right] = \frac{1}{2}\left(a_i(\theta)\, e^{i\nu_0\theta} + c.c \right) \tag{5}$$

$$F_i = \mathrm{Re}\left[f_i(\theta)\, e^{i\nu_0\theta}\right] = \frac{1}{2}\left(f_i(\theta)\, e^{i\nu_0\theta} + c.c \right) \tag{6}$$

$$f_i(\theta) = e\, \beta^{\frac{1}{2}} \sum_{j=1}^{N_p} q_j\, a_j \frac{d}{ds} W^{\perp}\left(z_j - z_i, s\right), \tag{7}$$

the equation (2) become

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{\nu_0}{2iE_0}\int_{\theta-\frac{\pi}{2\nu_0}}^{\theta+\frac{\pi}{2\nu_0}} \beta^{\frac{3}{2}}\, f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)}$$

$$+ \frac{\nu_0}{2\, iE_0}\int_{\theta-\frac{\pi}{2\nu_0}}^{\theta+\frac{\pi}{2\nu_0}} \beta^{\frac{3}{2}}\, f_i\, e^{-2\, i\nu_0\theta'} \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)} \tag{8}$$

The third term of the right hand side can be set to zero in usual cases because $\beta^{3/2}f$ usually does not have the Fourier component of the frequency $2\nu_0$.

Hence the equation

$$\frac{da_i}{d\theta} = i\Delta\nu_i a_i + \frac{\nu_0}{2\, iE_0}\int_{\theta-\frac{\pi}{2\nu_0}}^{\theta+\frac{\pi}{2\nu_0}} \beta^{\frac{3}{2}}\, f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)} \tag{9}$$

is used to simulate distributed impedance.

The integral in this equation (9) can be written as

$$\int_{\theta-\frac{\pi}{2\nu_0}}^{\theta+\frac{\pi}{2\nu_0}} \beta^{\frac{3}{2}}\, f_i \frac{d\theta'}{\left(\frac{2\pi}{\nu_0}\right)} = \int_{s-\frac{\lambda}{2}}^{s+\frac{\lambda}{2}} \beta^{\frac{1}{2}}\, f_i \frac{ds'}{2\pi} = \frac{1}{2\pi}\int_{s-\frac{C}{2}}^{s+\frac{C}{2}} \beta^{\frac{1}{2}}\, f_i\, ds' \frac{\lambda_\beta}{C}$$

and from the approximation (4), we have

$$\int_{s-\frac{C}{2}}^{s+\frac{C}{2}} \beta^{\frac{1}{2}}\, f_i\, ds' = \sum_k \beta_k{}^{\frac{1}{2}} \int_{k\text{th element}} f_i\, ds'$$

$$= e\sum_k \beta_k \sum_j q_j\, a_j\, W_k^{\perp}\left(z_j - z_i\right)$$

$$= e\sum_j q_j\, a_j \sum_k \beta_k\, W_k^{\perp}\left(z_j - z_i\right)$$

and, from the definition of λ_β,, $\frac{\lambda_\beta}{C} = \frac{1}{\nu_0}$.

The final form of the equation of transverse motion of particles used in CISR is

$$\frac{da_i}{d\theta} = i\, \Delta\nu_i\, a_i + \frac{e}{4i\pi E_0} \sum_{j=1}^{N_p} q_j\, a_j \sum_{k=1}^{N_I} \beta_k\, W_k^{\perp}\left(z_j - z_i\right) \tag{10}$$

3. DIFFERENCE EQUATIONS

The difference equations used in CISR to simulate the electron motion are as follows. For longitudinal motion, $\Delta E_i = E_i - E_0$ and $z=s-ct$, where s is the co-odinate to the direction of particle, is used to describe longitudinal motion of particles. T_0, and E_0 are the revolution period and reference energy, repectively.

In the following, the symbol with superscript + and - are the value after and before the passage of each element. In the following, respectively.

3.1 Lattice with Distributed Broad-Band Impedance

$$\Delta E_i^+ = \Delta E_i^{\pm} \pm U_0 \frac{\Delta T}{T_0}\left(1 + \frac{\Delta E_i^{\pm}}{E_0}\right)^2 \tag{11}$$

$$z_i^+ = z_i^{\pm} \pm \alpha \frac{\Delta E_i^{\pm}}{E_0}\, c\, \Delta T \tag{12}$$

$$r_i^+ = r_i^{\pm} + \mathrm{Re}\left[\, f_i^{\pm}\, e^{\pm i\, \phi_{i,n}} \right] \Delta\theta \tag{13}$$

$$\phi_i^+ = \phi_i^{\pm} + \Delta\nu_i\, \Delta\theta + \frac{\mathrm{Im}\left[\, g_i^{\pm}\, e^{\pm i\, \phi_i^{\pm}} \right]}{\left[\left(r_i^+ + r_i^{\pm}\right)/2\right]} \Delta\theta \tag{14}$$

$$g_i^{\pm} = \frac{1}{4i\pi E_0} \sum_{j=1}^{N_p} q_j\, a_j^{\pm} \sum_{k=1}^{N_I} \beta_k\, W_k^{\perp}\left(z_j^{\pm} - z_i^{\pm}\right) \tag{15}$$

where $\Delta\theta = 2\pi \frac{\Delta T}{T_0}$, $r = |a|$, $a = re^{i\phi}$.

3.2 Localized Broad-Band Impedance

For the localized impedance such as cavities,

$$\Delta E_i^+ = \Delta E_i^\pm \pm e \sum_{j=1}^{N_p} q_j W^\parallel\left(z_j^\pm - z_i^\pm\right) \tag{16}$$

$$a_i^+ = a_i^\pm \pm \frac{e}{iE_0} \beta^{\frac{1}{2}} \sum_{j=1}^{N_p} q_j x_j^\pm W^\perp\left(z_j^\pm - z_i^\pm\right) \tag{17}$$

where $x = \sqrt{\beta}\eta_i = \sqrt{\beta}\mathrm{Re}\left[a_i(\theta)\, e^{i\nu_0\theta}\right]$

3.3 Acceleration

For acceleration by $eV_a(z) = eV_c \sin\left(2\pi f_{rf} \frac{z}{c} + \phi_c\right)$,

$$\Delta E_i^+ = \Delta E_i^\pm + eV_c \sin\left(2\pi f_{rf} \frac{z_i^\pm}{c} + \phi_c\right) \tag{18}$$

$$a_i^+ = a_i^\pm \pm i \frac{eV_a^\pm}{E_0} \mathrm{Im}\left[a_i^\pm e^{i\nu_0 u}\right] e^{\pm i\nu_0 u} \tag{19}$$

Eq.(25) is transverse radiation damping.

3.4 Radiation Excitation

$$\Delta E_i^+ = \Delta E_i^\pm + \sqrt{\left(4\frac{\Delta T}{\tau_E}\right)}\left(\frac{\sigma_{E,0}}{E_0}\right) u_i \tag{20}$$

$$a_i^+ = a_i^\pm + \sqrt{\left(4\frac{\Delta T}{\tau_\beta}\varepsilon_0\right)}\, v_i\, e^{i2\pi w_i}, \tag{21}$$

where u, v are the Gaussian random number and w is uniform random number. ΔT is the time difference between each excitation and τ_E and τ_b is radiation damping time for E and x, respectively. $\sigma_{E,0}$ and ε_0 are natural energy spread and emittance.

4. PARTICLE-IN-CELL METHOD

SISR is the particle code and a Particle-In-Cell(PIC) method is used to make wake field and interact the particle with the wake field. The shape function used in SISR is

$$S(x) = \begin{cases} \pm\left(\frac{x}{\Delta x}\right)^2 + \frac{3}{4} & \left(|x| \le \frac{1}{2}\Delta x\right) \\ \frac{1}{2}\left|\frac{x}{\Delta x}\right|^2 \pm \frac{2}{3} & \left(\frac{1}{2}\Delta x \le |x| \le \frac{3}{2}\Delta x\right) \\ 0 & \left(\frac{3}{2}\Delta x \le |x|\right) \end{cases} \tag{22}$$

and the distributions

$$\begin{Bmatrix} \rho(z_p) \\ a\rho(z_p) \\ x\rho(z_p) \end{Bmatrix} = \sum_{i=1}^{N} \begin{Bmatrix} q_i \\ a_i q_i \\ x_i q_i \end{Bmatrix} S(z_i \pm z_p) \tag{23}$$

are evaluated on the mesh points whose position is represented by z_p,

And wake potentials appeared in above equations are evaluated at the mesh points using these distributions as

$$\begin{aligned} \frac{E_{BB}^\perp}{e}(z_p) &= \pm \sum_{j=1}^{N_p} q_j a_j \sum_{k=1}^{N_I} \beta_k W_k^\perp(z_j - z_p) \\ &= \pm \int a\rho(z') \sum_{k=1}^{N_I} \beta_k W_k^\perp(z' - z)\, dz' \end{aligned} \tag{24}$$

$$\begin{aligned} \frac{E_{local}^\perp}{e}(z_p) &= \pm \sum_{j=1}^{N_p} q_j x_j^\pm W^\perp\left(z_j^\pm - z_i^\pm\right) \\ &= \pm \int x\rho(z') W^\perp\left(z_j^\pm - z_i^\pm\right) dz' \end{aligned} \tag{25}$$

$$\begin{aligned} \frac{E^\parallel}{e}(z_p) &= \pm \sum_{j=1}^{N_p} q_j W^\parallel(z_j - z_p) \\ &= \pm \int \rho(z') W^\parallel(z' - z_p)\, dz' \end{aligned} \tag{26}$$

The wake potentials at the particles are obtained with

$$\frac{E}{e}(z_i) = \sum_{p=1}^{N_{mesh}} \frac{E}{e}(z_p)\, S(z_p \pm z_i) \tag{27}$$

5. WAKE FUNCTIONS

The three types of impedance models listed below are used to get wake functions. For longitudinal,

$Z^\parallel$	$\pm i\omega Z_L^\parallel$	$Z_R^\parallel$	$\frac{Z_C^\parallel}{\sqrt{\omega}}(1+i)$
$W^\parallel$	$Z_L^\parallel c^2 \frac{\partial\delta(z)}{\partial z}$	$Z_R^\parallel c\,\delta(z)$	$Z_C^\parallel \sqrt{\frac{2c}{\pi}} \lvert z\rvert^{\pm\frac{1}{2}} \theta(\pm z)$

and for transverse,

$Z^\perp$	$\pm i Z_L^\perp$	$\frac{Z_R^\perp}{\omega}$	$\frac{Z_C^\perp}{\omega\sqrt{\omega}}(1+i)$
$W^\perp$	$\pm Z_L^\perp c\,\delta(z)$	$\pm Z_R^\perp \theta(\pm z)$	$\pm Z_C^\perp 2\sqrt{\frac{2}{\pi c}} \lvert z\rvert^{\frac{1}{2}} \theta(\pm z)$

6. THE SPRING-8 STORAGE RING

The CISR is applied to the study of the instabilities of the SPring-8 storage ring. The parameters of the ring is shown in Table 1.

Table 1. The parameters of the SPring-8 storage ring.

Parameter		Value	Unit
Energy	E_0	8	GeV
Revolution Frequency	T_0	208.77	kHz
Energy Loss per Turn	U_0	9.2	MV
Damping Partition Numbers	J_E / J_β	2 / 1	
Momentum Compaction Factor	α	1.41×10^{-4}	
Betatron Tune (vertical)	ν_0	16.16	
Averaged Betatron Function	β	17.3	m

The impedance of the ring is estimated in ref.[1] and is

$$Z^\parallel = -1.67\times10^5\omega i + 400 + 1.49\times10^8 \frac{1+i}{\sqrt{\omega}} \tag{28}$$

$$Z^\perp = -2.13\times10^5 i + 4.98\times10^{14} + 4.21\times10^{10} \frac{1+i}{\omega\sqrt{\omega}} \tag{29}$$

where $Z^\perp$ is vertical transverse impedance. The unit for them are Ω and Ω/m, respectively.

6.1 Longitudinal Instabilities

Figure 1 shows the dependence of the bunch length and the energy spread σ_E/E on the bunch current I_b. This ring is rather inductive compared with colliders, the potential-well distortion lengthen the bunch length and the threshold of microwave instabilities can not seen until the threshold current of the transverse instabilities mentioned later.

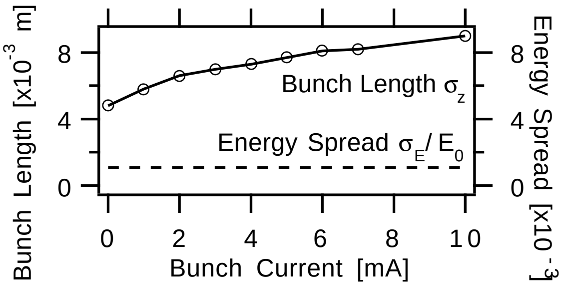

Figure 1. The bunch length and energy spread.

6.2 Transverse Instabilities

Figure 2 shows the bunch current increase vs. time used in the simulation.

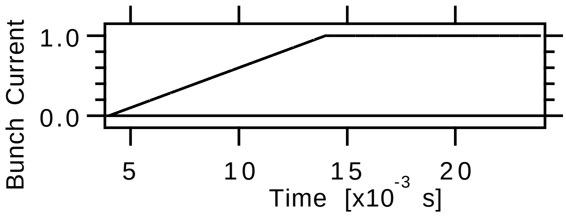

Figure 2. bunch current shape vs. time

Figure 3 and Figure 4 show the amplitude of betatron motion of the bunch vs. time for chromaticity ξ=0 and ξ=4, respectively. Instabilities occurs at I_b=3mA and I_b=7mA for ξ=0 and I_b=10mA for ξ=4.

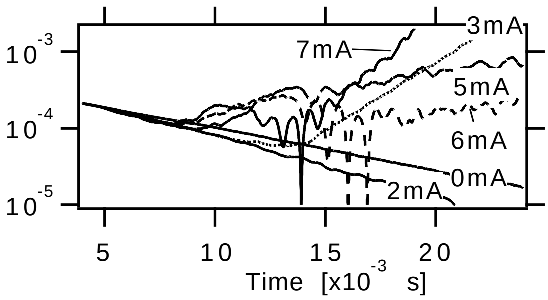

Figure 3. Amplitude of the betatron motion vs. time. ξ= 0.

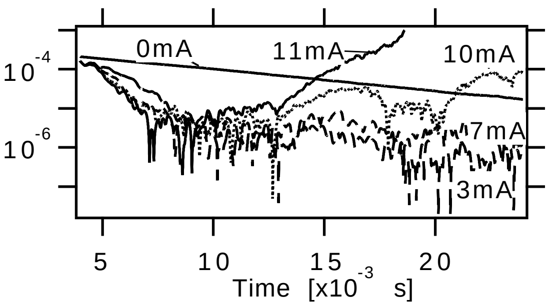

Figure 4. amplitude of the betatron motion vs. time. ξ= 4.

Figure 5 and 6 are the spectrum of the betatron motion of the bunch. (m,n)=(0,0) and (m,n)=(0,1) mode can be seen. The m=1 mode must be exist at the synchrotron frequency f_s=1.5kHz lower position, around f=32 kHz, but no signal can be seen.

From Figure 5, which is for ξ=0, the coupling of mode (m,n)=(0,0) and (m,n)=(1,0) occurs at I_b =3mA and the coupling of mode (m,n)=(0,0) and (m,n)=(2,0) occurs at I_b=7mA. Both lead to instabilities. For chromaticity ξ=4, of which data is not shown here, the signal of mode (m,n)=(0,0) can not beseen.

In Figure 4, No instabilities occurs near I_b=3mA, but the m=2 mode growths up at I_b=10mA. The difference between ξ=0 and ξ=4 seems to be from the effect of the head-tail damping, which can be seen at the beginning of the bunch current increase, where time ~ 5ms in Figure 4 and it is faster than radiation damping time which is seen for I_b=0mA.

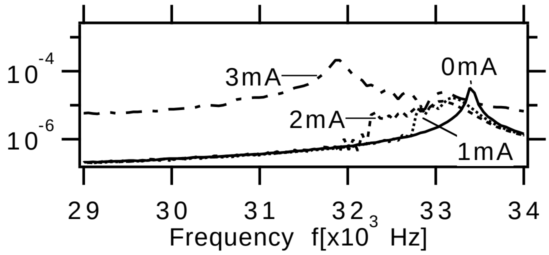

Figure 5. Spectrum of the betatron motion of the bunch ,I_b=0mA,1mA,2mA,3mA. ξ= 0.

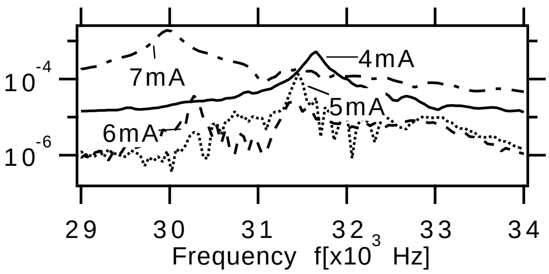

Figure 6. Spectrum of the betatron motion of the bunch ,I_b=4mA,5mA, 6mA,7mA. ξ= 0.

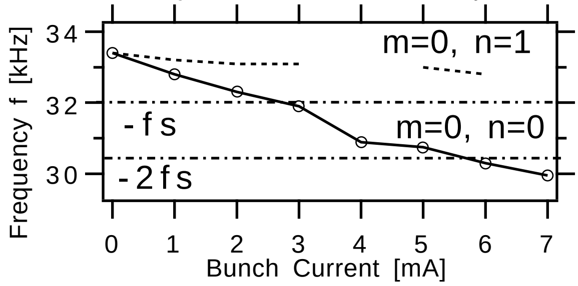

Figure 7. The position of peak of each mode of the spectrum of the betatron motion of the bunch. ξ= 0.

When chromaticity ξ=-2, the growth-rate of the head-tail mode of m=0 is so high and the threshold current is around few tenth mA.

7. CONCLUSION

The simulation code for single-bunch instabilities was developed and applied to the SPring-8 storage ring. No longtidinal microwave instabilities can not seen and the threshold of the transverse instabilities is a few mA and the positive chromaticity increase the threshold current.

8. REFERENCES

[1] T. Nakamura, "ESTIMATION OF LONGITUDINAL AND TRANSVERSE IMPEDANCE OF THE SPRING-8 STORAGE RING," this conference.