Simulated Ion Motion in the SRRC Storage Ring

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Abstract

The ions created in the electron storage ring have the possibility to be trapped by the electron beam. Beam quality is deteriorated consequently. As the ions are trapped, the electron beam and the trapped ions will mutually intact with each other and perform two beam oscillations. In this paper we will consider the ion behavior first. The problem is investigated from the simulation method with the first and the second driven force terms considered. Behavior of only the first driven force term and of two driven force terms are compared. Condition of neglecting higher order terms is discussed. Behavior of the trapped ions is also shown in the paper by taking SRRC storage ring as an example.

1 INTRODUCTION

At the beginning the author was going to simulate the beam ion interaction from mutually interaction behavior of two beam equations of motion. The equation for the ions was first studied. As the driven force from the electron bunches takes series expansion, it is found the high order terms can't be omitted. This is also true for the electron's equation of motion. While as the author knows there are many people doing the study by taking the first order term only. Therefore the author would like to demonstrate the reason of why the high order driven force terms can't be omitted by comparing the first and the first two terms with the motion of H_2^+ ion as an example.

In this paper the theoretical development for the ions equation of motion is given in section II. Two driven force terms are used for the demonstration. The electron driven force is taken as the kick force as it passing through ion cloud. The longitudinal information of the electron bunched beam is also used for the development of kick force model. By taken the SRRC storage ring as an example, the simulation results are given in the section III. Results with the first order term and the first two terms are compared and the criteria for neglecting the high order terms is shown. Ions behavior under two driven force terms is also shown in section III. Section IV is the discussion and conclusion of the author.

2 THEORETICAL DEVELOPMENT

Considering a transverse Gaussian charge distribution, the charge density is

$$\varrho(x,y) = \frac{N}{2\pi\sigma_x\sigma_y} e^{-\frac{x^2}{2\sigma_x^2}} e^{-\frac{y^2}{2\sigma_y^2}} \tag{1}$$

where N is the number of particles within charge distribution, σ_x and σ_y are horizontal and vertical beam size respectively. The potential produced by this Gaussian beam charge distribution writes

$$V(x,y) = -\frac{N}{\pi\epsilon_0}\int_0^\infty dt \frac{1-e^{-\frac{x^2}{2\sigma_x^2+4t}} e^{-\frac{y^2}{2\sigma_y^2+4t}}}{\sqrt{2\sigma_x^2+4t}\sqrt{2\sigma_y^2+4t}} \tag{2}$$

where ϵ_0 is the permittivity of vacuum. By taking the gradient of equation (2), the vertical electric field produced from this charge distribution read

$$E_y(x,y) = \frac{N}{2\pi\epsilon_0}\{\frac{y}{\sigma_x(\sigma_x+\sigma_y)} - \frac{\sigma_x+2\sigma_y}{6\sigma_y^3(\sigma_x+\sigma_y)^2}y^3+\ldots\} \tag{3}$$

The high order terms with the power higher than three are not shown in equation (3). If origin of the coordinate system is chosen to be at the ideal orbit, the vertical equation of motion for ion with atomic weight A under the attractive force of electron bunch can be written as

$$\ddot{y} = \frac{2r_p c^2 N}{A}\{\frac{\bar{y_e}-y}{\sigma_y(\sigma_x+\sigma_y)} + \frac{\sigma_x+2\sigma_y}{6\sigma_y^2(\sigma_x+\sigma_y)^2}(y-\bar{y_e})^3+\ldots\} \tag{4}$$

in which r_p is the classical proton radius, c the velocity of light and $\bar{y_e}$ the averaged vertical coordinate of the electron bunch seen by the ions. For simplicity we use coef1 and coef2 to represent the coefficient of the first and the second powers respectively in the following equations. Horizontal equation of motion for ion can be obtained in the same manner with the interchange of x and y.

While the electron bunch has longitudinal distribution. Taking the k-th bunch as an example, the charge density distribution and the following physical quantities in equations (1) to (3) need to be modified according to

$$PQ_k(s,x,y) = PQ(x,y)\frac{1}{(2\pi)^{0.5}\sigma_{kl}} e^{-\frac{(s-(nT_0-pk)/c)^2}{2\sigma_{kl}^2}} \tag{5}$$

in which PQ_k is the physical quantity obtained from the influence of k-th electron bunch, T_0 the electron beam revolution period, p spacing time between adjacent bunches and σ_{kl} longitudinal beam size for the k-th bunch. Since electron longitudinal beam size is around a few tens psec, which is much smaller than spacing time p of nsec. Therefore it is easy to deduce from equation (5) that the force is zero as s $\neq (nT_0 + pk)/c$ and need to multiply a new factor $1/(2\pi)^{0.5}\sigma_l$ as $s = (nT_0 + pk)/c$. Hence the ions will see a δ-like force from the electron bunches. It implies that the kick force model can be taken. Equation (4) is then modified to

$$\ddot{y} = \frac{\delta_{s,(nT_0+pk)/c}}{(2\pi)^{0.5}\sigma_l}\{(coef1)(\bar{y_e}-y)+(coef2)(y-\bar{y_e})^3+...\} \quad (6)$$

in which coef1 and coef2 are used to replace the coefficients of the first and the second driven terms in equation (4) respectively.

It is also worthy to notice that the ion longitudinal position is assumed to be fixed in the very short time for the following simulation study. The ions will see the kick force from the electron bunch strains and perform oscillation as time goes. Hence the longitudinal coordinate s in equation (6) corresponds to the traveling electron bunch coordinate w.r.t. the fixed ions in the very short time in our study.

3 SIMULATION RESULTS

Since the ion attracted by the kick force of the electron bunch trains, ion motion will like drift space in the force free period. For the interpolation of ion motion at the onset of kick force, we need to know $\ddot{y}$, given in equations (4) and (6), and $\dot{y}$, which is the integration of $\ddot{y}$ and given as

$$\dot{y} = \frac{1}{c}\{(coef1)(\bar{y_e} - y) + (coef2)(y - \bar{y_e})^3 + ...\} \quad (7)$$

Four more assumptions are also added: a)electron beam distribution and population are identical for all of bunches, b)no wake field and environment boundary condition seen by the ions, c)ions are assumed to be created at zero velocity and accelerated by the traveling electron bunches immediately and d)the electron path follows the ideal trajectory, i.e $\bar{y_e} = 0$.

Under above assumptions the motion of the ion at some fixed position in the ring can be simulated. The SRRC storage ring is taken as an example in this simulation. From literatures[1,2,3,4,5] and our experience[6], the ion effects will be serious as all of the RF buckets are filled. The main composition of the residual gas in the SRRC storage ring is H_2. Hence H_2^+ ion under the influence of full filled electron bunch trains is studied first. Ten percent emittance ratio and 30 psec longitudinal electron beam size are also assumed in the simulation. At the beginning H_2^+ ion created at the insertion middle with different initial position are simulated at total beam current of 200mA. Figure 1 show the periodic behavior of motion in vertical with H_2^+ ion created at 1 σ_y for only the first driven force term and the first two driven force terms considered respectively. From figure 1 it is clear that the oscillation frequency is different, around 19 MHz for one driven term and 21 MHz for two driven terms. The frequency discrepancy is also true for the ion horizontal plane motion. This behavior can be explained from equation (4). The second driven force term is the first driven term multiplied a factor as

$$(second\ term) = (first\ term)\frac{\sigma_x + 2\sigma_y}{6(\sigma_x + \sigma y)}(\frac{y}{\sigma_y})^2 \quad (8)$$

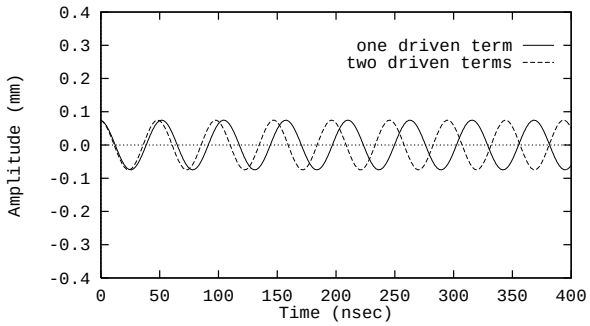

Figure 1: Comparison of H_2^+ ion vertical oscillation at one σ_y amplitude as one driven force term and two driven terms considered

Since horizontal beam size larger than vertical size, the second driven term can be neglected if oscillation amplitude y much smaller than σ_y. For example as y=0.5σ_y, ratio of the second driven force term to the first term is around 4%, which is a small value but the second term still can't be ignored. While if y equals to one sigma, then the second driven force term will give significant contribution and the frequency will be modulated. From this fact we can infer that if the frequency does not distort by the second driven term, then high order terms will be neglectable for the expansion form of high order terms are something like that in equation (8) but with the power higher than two. After some try it is found that ion oscillation frequency will not be modulated by the second driven term as oscillation amplitude small down to 0.3 sigma for the vertical and down to 0.1 sigma for the horizontal at the straight section middle. It implies the criteria for neglecting the high order terms is that the oscillation amplitude less than $0.1\sigma_x$ in horizontal and less than $0.3\sigma_y$ in vertical at the straight section middle. That means the horizontal plane is more critical. Since the oscillation amplitude is in unit of one sigma as shown in equation (8), above criteria will be more or less true for different location in the ring. While it is extended to $0.3\sigma_x$ in horizontal and $0.5\sigma_y$ in vertical to include the second driven term in the following simulations.

For the author understanding, the oscillation of the ions and electron beam is large in the ion trapping and detrapping process. Above results show it will becomes very difficult in the simulation. While since the expansion form of the high order terms similar to equation (8) but with

power higher than two, the high order term contribution will become quit small by compared with the first and the second terms as the oscillation amplitude not too big. Therefore taken two driven terms is also a good approximation as the oscillation amplitude is small. The SRRC storage ring conditions are also used in the simulation. Figure 2 shows the vertical oscillation behavior of H_2^+ at 200mA with different empty gaps. The flat line at the beginning time corresponds to force free region at the first turn. The revolution time of SRRC storage ring is 400 nsec. Force free region of the second and the third turns is easy to be identified by the inclined straight lines. From figures 2, mechanism of ion to be lost in the empty gap by the overfocusing force of electron bunches is clear shown from the amplitude growth in the next turn and propagation in the gap. It is also shown that H_2^+ ion seems easy to escape from trapping. While since the oscillation amplitude is larger than two sigma in the second turns, high order terms can't be neglected as shown in the previous. That means ion could not lost really except 140 empty gap one. For 140 empty gap case in figure 2, the oscillation amplitude goes to 30 σ_y in the gap at the second turn before the electron bunch trains coming. This indicates H_2^+ ions is really lost at 140 empty gap. The simulation also shows H_2^+ ion will be lost as the empty gap larger than 140. From figure 2, it is also interesting to find out that the ion oscillation frequency strong dependents on the number of electron bunches as well as the oscillation amplitude. The horizontal behavior of ion motion is similar to that of the vertical except the trapping ability is different.

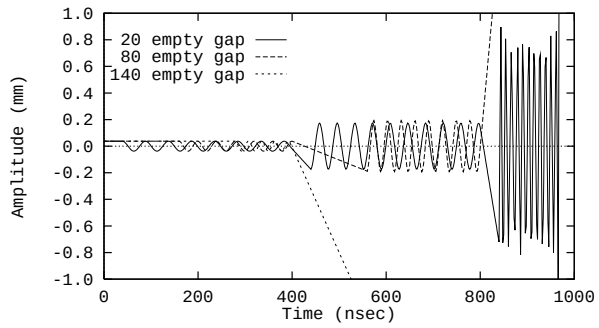

Figure 2: Motion of H_2^+ ion under the force of different number of electron bunches

Since the high order terms can't be neglected as the amplitude getting larger, it is very difficult to identify the ion is lost or still under trapped as the oscillation amplitude larger than one sigma in many cases. Therefore some of the above results are deduced from the trend of oscillation. Above results are under our expectation and consistent with our observation[6] except the number of empty gap to avoid trapping is ambiguous. The ambiguity comes from the high order terms can't be neglected at big oscillation amplitude.

Different species of ions are also studied. It is shown that they are easier to be trapped by the electron beam than H_2 and there is also the high order terms problem as the amplitude getting big.

4 CONCLUSION

The ion trapping mechanism is complicated. From above study with the SRRC storage ring as an example, it is shown all species of ions will be trapped, including H_2, as all of the RF buckets are filled at 200mA. It is also shown that the high order terms can't be neglected as oscillation amplitude larger than $0.1\sigma_x$ and $0.3\sigma_y$ at the straight section middle by taken H_2^+ ion as an ensemble. This fact will relax a little bit at different location in the ring but not too far away. By considering the first two driven force terms, the ion motion in the SRRC storage ring is investigated with relax criteria of $0.3\sigma_x$ and $0.5\sigma_y$. The de-trapping process with the help of empty gap is clear shown from the amplitude growth in the next turn and escape in the gap by the electron attractive force. It is also shown that bigger empty gap can help ions to avoiding trapping. From the fact of high order terms can't be neglected as the amplitude getting larger, there is a ambiguity on empty gap for which ion would avoid trapping. While as empty gap larger than 140, H_2^+ ion will clearly escape from trapping in the gap. It is also found the ion oscillation frequency strong dependents on its amplitude as well as the number of the electron bunches, small amplitude and less electron bunches give small oscillation frequency. Different species of ion are also studied. They show the same behavior for both planes except the trapping and de-trapping ability is different.

Since the simulation was done under some assumptions, the behavior will be slightly different from the real case. For example the force is derived from free space with no boundary conditions imposed. So if we can approach the problem with the suitable boundary conditions, a clear and definite results will be gotten. Contribution of the higher order terms at big amplitude also reveals the need of theoretical development in the future.

5 ACKNOWLEDGEMENT

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