# 'Binary Star' Instability

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*Abstract*

A single-bunch stability threshold can be found by analyzing the Vlasov equation with respect to small perturbations. In the case of proton synchrotrons the instability is always followed by turbulent bunch lengthening, but in the case of electron storage rings where radiation damping causes the particles to be confined, the behavior of the beam beyond the instability threshold may vary depending on the shape of the wake fields and the intensity. It was observed in simulations that in the case when the natural bunch length is smaller than the characteristic length of the wake field, at some intensity (which is typically much higher than the instability threshold found by solving the Vlasov equation) the bunch splits into two equal parts and these oscillate in each other's fields so that their motion in phase space resembles that of a 'binary star'. The signature of this behavior is strong quadrupole oscillations. Another feature is that as intensity increases, low-frequency sidebands begin to appear around the quadrupole frequency. This qualitatively resembles an effect observed at LEP in 1991.

## 1 INTRODUCTION

It is common to study stability of a beam from the point of view of the stability of a stationary distribution with respect to an infinitesimal perturbation. Questions remain about what happens to the beam when the threshold intensity is exceeded. In the case of protons, a turbulent bunch lengthening effect is observed, i.e. the bunch distribution alters under the self forces until the distribution becomes stable. In the case of electrons turbulent bunch lengthening is also observed, but it has different features.

At the threshold intensity the bunch becomes unstable, but radiation damping causes the particles to be confined and the instability does not necessarily cause loss of particles. The 'sawtooth effect' observed at SLAC SLC damping ring [1], anomalous quadrupole sidebands at CERN LEP [2] and the recent low-$\alpha$ experiments at LBL ALS (where bunch splitting has been observed [3]) and also an hysteresis effect at TRISTAN AR [4] indicate that the behavior beyond threshold can no longer be described in terms of solutions of the Vlasov equation. Moreover, it is becoming clear [5] that one can expect to see nonlinear phenomena such as solitons. The nonlinear regime is extremely difficult to treat analytically. We used a multiparticle tracking technique as described in this paper.

## 2 NUMERICAL SIMULATIONS

To simulate the electron's motion in a synchrotron we use a standard multi-particle tracking scheme [6].

In our simulations we used a relatively small number of macro particles – 5,000 – but we found that some of the interesting effects can already be seen even at this low number. This depends upon the wake field being fairly smooth: many times more macroparticles are required for wake fields which have many oscillations in one bunch length [6].

For the work described here, we have chosen the wake field of a broad band resonator with a quality factor $Q = 1$ and bunch length parameter $k_0 = \omega_0\sigma_z/c = 0.5$. ($\omega_0$ is the resonant frequency and $\sigma_z$ the rms bunch length.) The radiation damping time $\tau_e$ was set to 500 - 2,000 turns and the other parameters have been chosen to obtain a synchrotron period $T_s$ of 100 turns. In order to be able to relate the results of this paper with earlier work [7] we will use as intensity the dimensionless parameter $I = eN\omega_0(R/Q)/(V'_{\rm rf}\sigma_{z0})$.

### 2.1 Results

We start with a large emittance and allow the beam to damp. At low intensities the beam relaxes to a thermodynamically stationary distribution which is well described by the Haïssinski equation [8]. At higher intensity, however, it takes more time for particles to reach thermodynamic equilibrium, particularly when the distribution has a two-peak line-density profile.

In some simulations (especially with the shorter damping times) we have observed that energy spread and bunch length may oscillate in a sawtooth fashion beyond a certain threshold [7]: the bunch length decreases slowly after injection until a threshold is reached, when the length increases sharply (in less then a synchrotron period) and then the process repeats.

The region close to threshold is difficult to model because of the slow growth rate of the instability. Above approximately $I = 20$, the sawtooth instability becomes apparent. As intensity is raised, the sawtooth periodicity also increases, and at very high intensity, the behavior becomes irregular. If intensity increases further the bunch may split into a 'binary star' system.

It has been found earlier [7] that the sawtooth behavior was most clearly seen in the region $0.4 < k_0 < 0.6$. Since the 'sawtooth' instability develops in a fraction of a synchrotron period, radiation time was artificially reduced in order to reduce computation time.

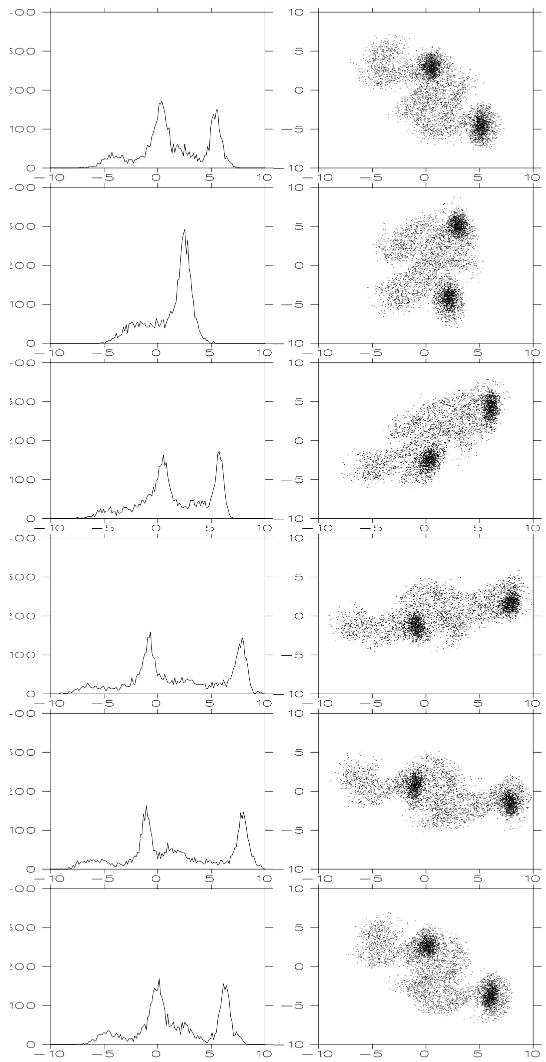

Figure 1: Snapshots of the phase-space distributions for the 'binary star' instability at 10 turn intervals ($\approx$ 1/10 of a synchrotron period).

In the case when $\tau_d = 20T_s$, the binary star behavior showed up at a lower intensity, $I = 20$. Previously, when we had $\tau_d = 5T_s$ we saw just chaotic oscillations of the bunch parameters, but now the picture is completely different: the bunch splits into two almost identical sub-bunches and they oscillate through each other (see Figs. 1), their motion in phase space resembling that of a binary star. The intensity at which this phenomenon is observed is more than twice the threshold intensity for this wake.

The spectrum of this signal has a strong line at approximately twice the synchrotron frequency (see Fig. 3), but it is interesting to note that at some intensity two sidebands of the quadrupole frequency appear.

Though the distribution of particles in phase space clearly shows that a strong turbulent process takes place all the time, this 'binary star' oscillation is reproducible. Different initial distributions evolve into the same steady state behavior, either sawtooth, or binary star, depending upon intensity. There is also a regime where a distribution can evolve into either of the two behaviours.

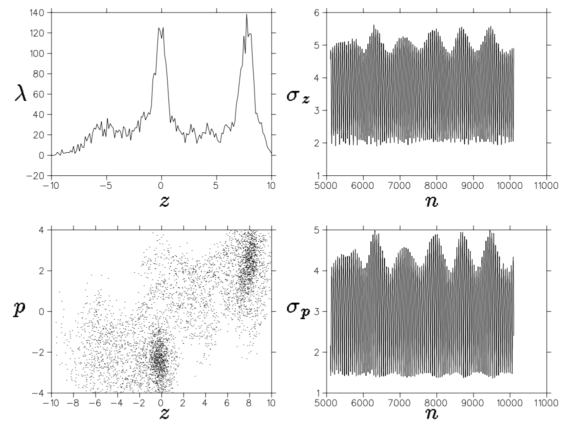


Figure 2: Modulation of the quadrupole oscillations which gives sidebands of the quadrupole frequency in the spectrum of the oscillations. Conditions are the same as for a previous figure

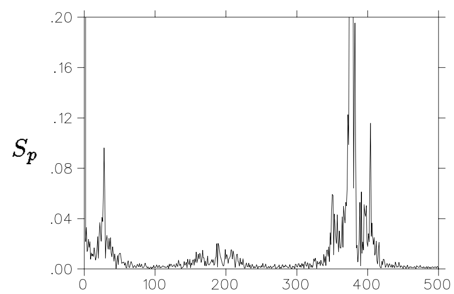


Figure 3: Spectrum $S_p$ of the rms momentum spread ($\sigma_p$) during the 'binary star' instability. The quadrupole mode has the largest strength.

It is tempting to describe the binary star case with a two-particle model. Unfortunately, such a straightforward procedure does not work for the following reason. In the case of linear external forces one can rewrite the equation of motion for separate macroparticles in two equations in new coordinates, one describing the centre of mass motion and the other the relative motion $q_1 - q_2$. The equation describing the relative motion will not depend on the centre of mass coordinate, but since there is radiation damping $q_1 - q_2 \rightarrow 0$ eventually. We therefore conclude that nonlinear phenomena, such as turbulence, play a significant role in this instability.

As one can see from the snapshots Fig. 1 the bunches are not 'rigid' and there are always some tails and filaments. These filaments do not disappear when the number of particles in the simulations is increased, but become better defined. We've tried to remove them artificially from the simulations (i.e. substituting the particles which move far from

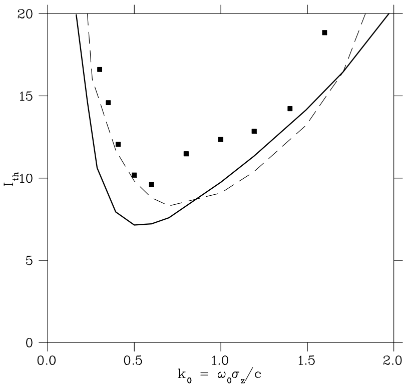


Figure 4: Threshold intensity vs. bunch length parameter $k_0$ in the case of a broad-band ($Q = 1$) resonator. The points and different curves are the results of different calculations. See [7].

the core of each bunch by the particles inside the core) and the result is that the bunches merge. This indicates that the filaments and turbulent phenomena play an essential role.

In our simulations the 'binary star' state has never been seen for $k_0 > 0.6$ One can see from Fig. 4 that $k_0 = 0.6$ corresponds to the minimum in the threshold and the slope of the threshold current vs. the bunch length is different on different sides. Our hypothesis for this behavior is as follows. The behavior of the bunch above threshold should be different depending if we approach the threshold from the left or right side of the minimum. In the case of long bunches ($k_0 > 0.6$), when the bunch reaches the threshold intensity the energy spread increases (and so does the bunch length) and the the bunch stabilizes. This is the case of 'turbulent bunch lengthening' and it is very well understood. However, 'turbulent bunch lengthening' won't stabilize the beam if $k_0 < 0.6$. The length increase will cause the bunch to be more unstable. Therefore, we conclude that the distribution of the particles ($\psi(E)$) will have to change and cannot be stationary.

## 3 CONCLUSIONS

There is no clear evidence that the 'binary star' instability as it is described here has been observed in existing accelerators. However, there are some observations of anomalous behavior of bunches beyond the stability threshold in electron synchrotrons, which have some essential features of the 'binary star' instability.

'Two peak' distributions can be easily found from the Haïssinski equation [8], and have they been observed experimentally by using a streak camera [2]. Last year 'two-peak' distributions have also been seen in the ALS low-$\alpha$ experiments [3]. At small values of $\alpha$ large bunch size oscillations were seen. The bunch apparently split into two distinctive but unequal subbunches, with the small satellite bunch orbiting the main bunch.

Large amplitude oscillations of bunch length (quadrupole oscillations) as well as low frequency sidebands around the quadrupole frequency at higher intensities have been observed at CERN in LEP [2]. It was suggested in [2] that these lines may correspond to radial modes predicted by conventional theory derived from the linearized Vlasov equation; however, the quadrupole oscillations observed at LEP had a very large amplitude (more then 30 percent of the bunch length, and therefore it is unlikely that these lines can be associated with any modes found using a perturbation formalism.

The 'binary star' effect observed in the simulations has some qualitative features of the behavior observed in LEP. However, quantitative agreement has not yet been found. Rigorous analysis of the 'binary star' instability is very difficult because of the turbulent processes which are playing an important role. This subject is, certainly, a fruitful area for future developments in instability theory.

## 4 REFERENCES

[1] P. Krejcic, K. Bane, P. Corredoura F.J. Decker et al, *High Intensity Bunch Length Instabilities in the SLC Damping Ring*, Proc. 1993 Part. Acc. Conf., Washington D.C., 3240 (IEEE, 1993).

[2] D. Brandt, K. Cornelis and A. Hoffmann, *Experimental Observations of Instabilities in the Frequency Domain at LEP*, Proc. 3rd Europ. Part. Acc. Conf., Berlin, 345 (Editions Frontières, 1992); see also CERN/LEP Report MD-35 (1990).

[3] D. Robin et al. *Low Alpha Experiments at ALS*, LBL-37759 UC-410 September, 1995.

[4] T. Ieiri, *Bunch Lengthening Observed Using Real-Time Bunch-Length Monitor in the TRISTAN AR*, Proc. 1993 Part. Acc. Conf., Washington D.C., 3333 (IEEE, 1993).

[5] J.J. Bisognano, *Solitons and Particle Beams*, AIP Conf. Proc. **253**, 42 (1992).

[6] K. Bane and K. Oide, *Simulations of the Longitudinal Instability in the SLC Damping Ring*, Proc. 1993 Part Acc. Conf., Washington D.C. 3339 (IEEE, 1993).

[7] R. Baartman, M. D'yachkov, *Simulations of 'sawtooth' instability*, Proc. 1995 Part. Acc. Conf., Dallas.

[8] J. Haïssinski, *Exact Longitudinal Equilibrium Distribution of Stored Electrons in the Presence of Self-Fields*, Nuovo Cimento **18B**, 72 (1973).