ACCELERATION WAKEFIELD MODES IN A PHOTONIC BAND GAP STRUCTURE

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1 INTRODUCTION

The propagation of electromagnetic waves in periodic, dielectric structures became a widely investigated subject. The photonic bands concept [1] or new types of localized states [2] are consequences of such phenomena.

In condensed matter systems with scatterers periodically arranged, the propagation is naturally described by a vector version of the band theory [1].

It is also well known that dimensionality plays a major role in the behavior of strongly scattering systems. The lower the dimension, the easier it is to have band gaps (a certainty in 1D). The theory of waves in a random medium suggests that all states are localized in 1D and marginally localized in 2D. For 3D it is generally believed that a mobility edge exists.

The case of microwave propagation in strongly scattering dielectric lattices is experimentally and theoretically investigated [3], [4].

However, our work solves a different problem: analyzing the electromagnetic wave propagation in a given dielectric, periodical structure which has a defect, obtaining the photonic band structure and the spatial distribution of the transmitted power.

The numerical solution of this case is obtained in two steps:

a) computing the photonic band structure by using a proper version of the band theory;

b) computing the spatial distributions of the electric and magnetic fields, as well as of the transmitted power, by using an original version of the "collocation method", for the case of coupled differential equations and an infinite number of independent variables.

2 THEORY AND SIMULATIONS

The periodical structure we studied consists of a square lattice of dielectric rods (having dielectric constant ε_1) imbedded in a medium with an ϵ_2 dielectric constant.

For a general 3D case, one obtains from Maxwell's equations, by standard calculations and using the condition that the electric field points along the "z" axis, the following scalar 2D wave equation:

$$\left(\nabla^2 + \varepsilon\left(\overrightarrow{r}\right)\omega^2\right)\psi\left(\overrightarrow{r}\right) = 0 \qquad (1)$$

The dielectric constant distribution is given in our case by:

a) $\varepsilon = \varepsilon_1$, for $\left|\left(x_m^0 - x_i^0\right)^2 + \left(y_m^0 - y_i^0\right)^2\right| \leq a^2$

b) $\varepsilon = \varepsilon_2$, for the rest of the domain,

where:

$\left\{x_i^0, y_i^0\right\}, i = \overline{1, N}$ is a primary division of the 2D-domain, defined by the centers of the dielectric rods;

$\left\{x_m^0, y_m^0\right\}, m = \overline{1, M}$ is the secondary division and the working one, defined by the "collocation" points ($M >> N$) which are not equidistant.

The direction of propagation $\overrightarrow{k}$ may be chosen in various ways, therefore being necessary to establish the connection between the coordinates of the two lattices described previously and two new lattices: $\{x_i, y_i\}, \{x_m, y_m\}$, obtained by rotating the old ones such as the y axes to point along $\overrightarrow{k}$.

With the new coordinates, the wave equation is:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \omega^2\varepsilon\left(x, y\right)\psi = 0 \qquad (2)$$

We seek its solution $\psi\left(x, y\right)$ as a linear combination over a set of suitable orthogonal functions $\phi_n\left(x\right)$:

$$\psi\left(x, y\right) = \sum_{n=1}^{M} c_n\left(y\right)\phi_n\left(x\right) \qquad (3)$$

The choice of $\phi_n\left(x\right)$ depends on the boundary conditions and the symmetry of the guiding structure. In our case, the Laguerre - Gauss functions are the most appropriate:

$$\phi_n\left(x\right) = A_{n-1}L_{n-1}\left(\alpha x\right)\cdot\exp\left(-\frac{1}{2}\alpha^2 x^2\right) \qquad (4)$$

where A_{n-1} is the normalization constant and α is a parameter which can be chosen arbitrarily, but its choice crucially influences the accuracy for a given value of M.

The expansion in equation (3) must satisfy exactly the differential equation (2) at M "collocation points". These points are chosen such that they are zeroes of ϕ_{M+1}:

$$L_M\left(\alpha x_j\right) = 0\,, j = 1, ..., M \qquad (5)$$

the solutions being well documented in literature.

Writing the Helmholtz equation (2) at each of these collocation points, we obtain a set of M total differential equations which can be written in matrix form:

$$\frac{d^2\Psi}{dy^2} + D + R\Psi = 0 \qquad (6)$$

where: $(\Psi(y))_i = \psi(x_i, y)$, $(R(y))_{ij} = \omega^2 \varepsilon(x_i, y)\,\delta_{ij}$, $D_i(y) = \left.\frac{\partial^2 \psi}{\partial x^2}\right|_{x=x_i}$.

The expression in (6) may also be written as:

$$\frac{d^2\Psi}{dy^2} + S\Psi(y) = 0 \tag{7}$$

if: $S = BQ^{-1} + R(y)$, with $B_{ij} = \left.\frac{\partial^2 \phi_j}{\partial x^2}\right|_{x_i}$ and $Q_{ij} = \phi_j(x_i)$.

We made no approximation in deriving (7) except that M is finite. The matrix total differential equation can be now solved as an initial value problem using any standard method.

The constraints imposed by the periodicity of the solution in the initial coordinates ($\{x_i^0, y_i^0\}$) give rise to the equations which define the photonic band structure $\omega\left(\overrightarrow{k}\right)$.

We must underline that we did not use average or smoothing dielectric constants as in [1], [3] and that the computing time and the efficiency are real advantages of our approach.

3 RESULTS AND DISCUSSIONS

If we create a defect (a missing rod at (x_0, y_0)) the band structure is clearly changed in comparison with the case when we have a totally periodic structure, the solution ψ and the spatial power distribution having also different shapes. This is the phenomenon which allows the application studied in our work: the acceleration wakefield modes.

The electric field has a strong variation in the neighborhood of the defect. The great values of the field allow the application of this effect in producing particle acceleration structures. The particle beam must point exactly along the axis of the missing rod from the 3-D lattice, thus interacting with the generated field.

4 REFERENCES

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