

# EQUIVALENT SCHEME AND PARAMETERS OF DISC LOADED WAVEGUIDE AT DIPOLE MODE\*

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## ABSTRACT

An equivalent scheme of disc loaded waveguide (DLW) with constant gradient in dipole mode consisting of series and parallel branches are used for obtaining of dispersion equation and for field calculation. Contrary of other works the scheme under consideration includes the coupler. The values of resonant frequencies for two modes, coupling coefficients and Q-factors for DLW cells were calculated with using of URMEL program. The demands of frequencu and Q-factor of the coupler at first dipole mode which provides withdrawal of the hybrid mode power are obtained.

## 1 EQUIVALENT SCHEME

The analysis of hybrid waves excitation in DESY collider section was carried out on the basis of an equivalent scheme of DLW with HEM<sub>11</sub>-mode. Such a scheme for the DLW exited in hybrid mode is shown at Fig.1.

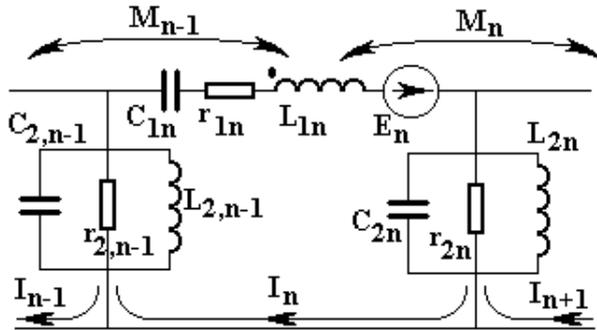


Fig.1. Equivalent scheme of a DLW exited at hybrid mode.

The series branch ( $L_{1n}$ ,  $C_{1n}$ ,  $r_{1n}$ ) represents the existence of the E<sub>11</sub>-mode electromagnetic field in this scheme. On one hand the coupling between cells is realized by magnetic field that is represented by mutual inductance  $M_n$  in the scheme. On the other hand there is resonance type coupling by means of H<sub>11</sub>-mode electromagnetic field that is represented by parallel branches ( $L_{2n}$ ,  $C_{2n}$ ,  $r_{2n}$ ). The cell excitation is modeled by introducing of complex e.m.f.  $E_n$  into the series

branch.  $I_n$  is the loop current complex amplitudes in the scheme under consideration.

It is more convenient to operate with the following parameters expressed in terms of  $L$ ,  $C$ ,  $r$ :

$$\begin{aligned} f_{1n} &= \frac{1}{2\pi\sqrt{L_{1n}C_{1n}}}; & f_{2n} &= \frac{1}{2\pi\sqrt{L_{2n}C_{2n}}}; \\ \frac{K_n}{2} &= \frac{M_n}{\sqrt{L_{1n}L_{1,n+1}}}; & \frac{K_{12n}}{2} &= \frac{\sqrt{C_{1n}C_{1,n+1}}}{C_{2n}}; \\ Q_{1n} &= \sqrt{\frac{L_{1n}}{C_{1n}}} \frac{1}{r_{1n}}; & Q_{2n} &= \sqrt{\frac{C_{2n}}{L_{2n}}} r_{2n}; \\ \dot{X}_n &= \sqrt{L_{1n}} \dot{I}_n; \end{aligned} \quad (1)$$

By using these parameters one can obtain the following equations set with respect to complex values  $\dot{X}_n$ :

$$\begin{bmatrix} A_1 & B_1 & 0 & \dots & \dots & 0 \\ B_1 & A_2 & B_2 & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & B_{N-2} & A_{N-1} & B_{N-1} \\ 0 & \dots & \dots & 0 & B_{N-1} & A_N \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dots \\ \dot{X}_{N-1} \\ \dot{X}_N \end{bmatrix} = \begin{bmatrix} \dot{E}_1 \\ j\omega\sqrt{L_{11}} \\ \dots \\ \dot{E}_N \\ j\omega\sqrt{L_{1N}} \end{bmatrix} \quad (2)$$

where  $\omega=2\pi f$  is the section excitation frequency,

$$\begin{aligned} A_n &= 1 - \frac{f_{1n}^2}{f^2} - j \frac{f_{1n}}{f} \frac{1}{Q_{1n}} - \\ &- \frac{f_{1n}^2}{f^2} \left[ \frac{\sqrt{C_{1n}}}{\sqrt{C_{1,n-1}}} \frac{K_{12,n-1}}{2} \cdot \right. \\ &\cdot \frac{1}{1 - \frac{f_{2,n-1}^2}{f^2} - j \frac{f_{2,n-1}}{f} \frac{1}{Q_{2,n-1}}} + \\ &\left. + \sqrt{\frac{C_{1n}}{C_{1,n+1}}} \frac{K_{12,n}}{2} \frac{1}{1 - \frac{f_{2n}^2}{f^2} - j \frac{f_{2n}}{f} \frac{1}{Q_{2n}}} \right]; \end{aligned}$$

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$$B_n = \frac{K_n}{2} + \frac{K_{2n} f_n f_{1,n+1}}{2 f^2} \frac{1}{1 - \frac{f_{2n}^2}{f^2} - j \frac{f_{2n}}{f} \frac{1}{Q_{2n}}}$$

At  $n=1$  the multiplier  $\sqrt{\frac{C_{1n}}{C_{1,n-1}} \frac{K_{12,n-1}}{2}}$  in  $A_1$  should be

changed by  $\frac{K_{120}}{2} = \frac{C_{11}}{C_{20}}$ , whereas at  $n=N$  the multiplier

$$\sqrt{\frac{C_{1N}}{C_{1,N+1}} \frac{K_{12N}}{2}}$$
 should be changed by  $\frac{K_{12N}}{2} = \frac{C_{1N}}{C_{2N}}$ .

Note that in the equivalent scheme the number of parallel branches is equal to  $N+1$ , whereas that of series branches equals  $N$ , where  $N$  is the number of cells in the section. Both the first and the last parallel branches (indexes 0 and  $N$ ) are introduced for modeling beam pipe in the input and output couplers.

Voltages across the capacitive element  $C_{1n}(\dot{U}_{2n})$  and  $C_{2n}(\dot{U}_{1n})$  are related to  $\dot{X}_n$  by the expressions which may be fined using the equivalent scheme.

## 2 DISPERSION EQUATION

The dispersion equation can be obtained by using the equivalent scheme for the infinite uniform lossless DLW with identical cells:

$$f = \sqrt{\frac{F(\varphi) \pm \sqrt{F^2(\varphi) - 4f_1^2 f_2^2 (1 + K \cos \varphi)}}{2(1 + K \cos \varphi)}}, \quad (3)$$

where  $F(\varphi) = f_1^2 [1 + K_{12}(1 - \cos \varphi)] + f_2^2 (1 + K \cos \varphi)$ ,  $\varphi$  is the phase shift per DLW cell,

$f$  is the frequency corresponding to  $\varphi$ .

The upper sign in (3) corresponds to the higher branch of the dispersion curve whereas the lower one corresponds to the lower branch.

If  $f_{01}$  and  $f_{\pi 1}$  are frequencies of 0 and  $\pi$  modes of the lower branch (Fig.2) and  $f_{02}$  and  $f_{\pi 2}$  are similar values for the higher branch of the dispersion curve (Fig.3) we can write down the following expressions for finding of branches frequencies  $f_1$ ,  $f_2$  and coupling coefficients  $K$ ,  $K_{12}$ :

$$f_1 = f_{01} \sqrt{1+K}, \quad f_2 = f_{02} \quad (4)$$

$$\text{or} \quad f_1 = f_{02} \sqrt{1+K}, \quad f_2 = f_{01} \quad (4')$$

$$K = \frac{2M}{L_1} = \frac{(f_{\pi 1} f_{\pi 2})^2 - (f_{01} f_{02})^2}{(f_{\pi 1} f_{\pi 2})^2 + (f_{01} f_{02})^2} \quad (5)$$

$$K_{12} = \frac{2C_1}{C_2} = \frac{1}{2} \left[ (1-K) \frac{f_{\pi 1}^2 + f_{\pi 2}^2 - f_2^2}{f_1^2} - 1 \right] \quad (6)$$

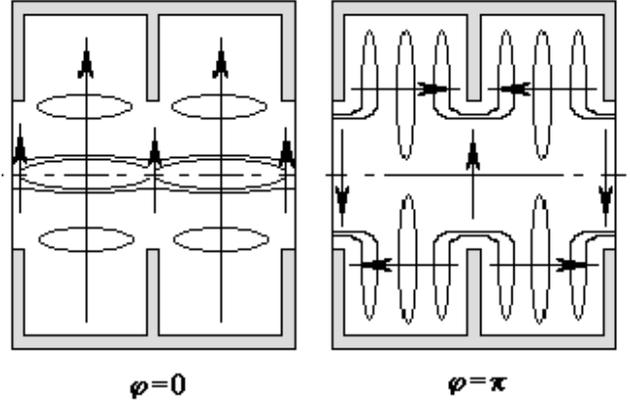


Fig.2. Field distribution at phase shifts  $\varphi=0$  and  $\varphi=\pi$  of the lower branch.

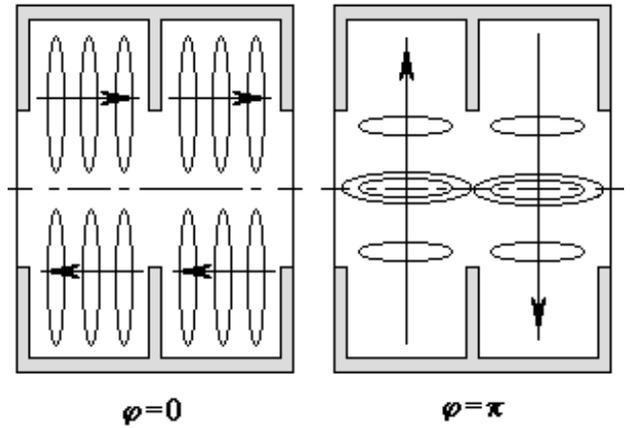


Fig.3. Field distribution at phase shifts  $\varphi=0$  and  $\varphi=\pi$  of the higher branch.

## 3 CALCULATION OF FIELD AND COUPLER PARAMETERS

Modeling of the input coupler was realized by means of addition of coupler cell coupling with first cell of DLW. Frequency  $f_{1C}$  and Q-factor  $Q_{1C}$  of this additional cell are chosen in such a way that we would have

- traveling wave regime in the DLW consisting of cells similar to the first one,
- the operational frequency of the traveling wave regime being  $f_{op}$ ,
- the phase shift per cell at this frequency being  $\varphi_{op}$ .

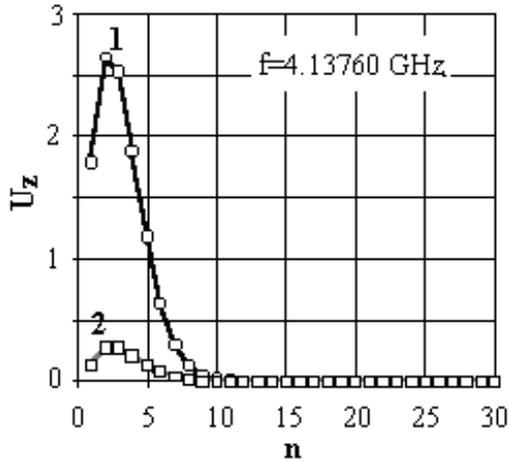


Fig.4. Cell voltages  $U_z$  versus cell number  $n$  for the first 30 cells of SBLC DLW at frequency 4.13760 GHz.

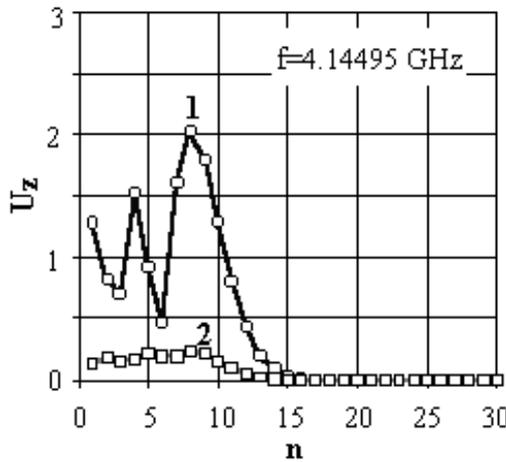


Fig.5. Cell voltages  $U_z$  versus cell number  $n$  for the first 30 cells of SBLC at frequency 4.14495 GHz.

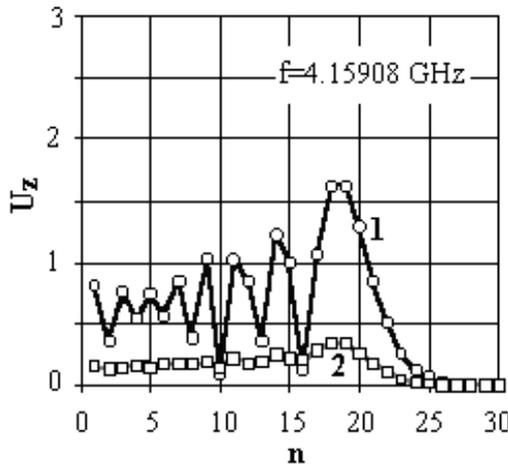


Fig.6. Cell voltages  $U_z$  versus cell number  $n$  for the first 30 cells of SBLC at frequency 4.15908 GHz.

Using the equivalent scheme shown at Fig.1 cell voltages  $U_z$  were calculated as function of the cell number at different frequencies for SBLC accelerating section with constant gradient consisting of first 30 cells (see Fig.4, 5 and 6) [1].

Curves 1 at these graphs correspond the case when input coupler is mismatched with first accelerating section cell. Curves 2 correspond the case when input coupler is matched with DLW at the frequency  $f_{op}=4.151$  GHz ( $\varphi_{op}=95^\circ$ ). Excitation of the accelerating section has been simulated by introduction of e.m.f. into the cells 2, 8 and 18 at the frequencies 4.13760, 4.14495 and 4.15908 GHz correspondingly. The series branch resonant frequency and Q-factor of this branch were chosen  $f_{ic}=4.846777$  GHz and  $Q_{ic}=14.677$  in these calculations. These values correspond to the lowest resonant frequency of coupler cell  $f_c=4.152224$  GHz and it's Q-factor  $Q_c=159.77$ .

One may see at Fig.4-6 that the matching of the coupler at the hybrid mode reduces the amplitude of electric field more than five times in broad frequency band. The analogous results for DLW with constant gradient but without a coupler was obtained in paper [2].

## CONCLUSION

The results of the experimental research of longitudinal electric field distribution in first 30 cells of DESY SBLC accelerating system with accounting of coupler in the frequency region of first dipole mode are adduced in paper [3]. These results confirm the justice of stating calculation method.

## REFERENCES

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3. S.N.Ivanov, V.E.Kaljuzhny, O.S.Milivanov, A.N.Parfenov, N.P.Sobenin, Investigation of the High Order Mode (HOM) Field Distribution in the Beginning Part (30 Cells) of the SBLC Accelerating Structure, (see in the Proc. of this conf.).