SIMULATION OF ION ACCELERATION IN A TWO-BEAM ELECTRON-ION ACCELERATOR

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Abstract

The macro-particle method for computer simulation of the two-beam electron-ion accelerator have been realized. It is found that the collective transverse instability defines the upper limit of ion current to be accelerated. For the parameters of the test accelerator EAS [1] the ion limit current about 3 A have been determined.

1. The theory of the two-beam electron-ion accelerator based on the Doppler effect was given in [2]. Here, for this accelerator, the motion of ions in the RF fields with following topography have been considered:
in the vacuum region ($r_b < r$, r_b - beam radius) -

$$E_z^{\mathbf{RF}} = E_1\, \mathbf{I}_0(k_\perp r)\,\mathbf{cos}\,\varphi\,,\quad E_r^{\mathbf{RF}} = -\frac{k_z}{k_\perp} E_1\, \mathbf{I}_1(k_\perp r)\,\mathbf{sin}\,\varphi$$

in the electron beam region ($0 < r < r_b$) - (1)

$$E_z^{\mathbf{RF}} = E_0\, \mathbf{J}_0(\kappa r)\,\mathbf{cos}\,\varphi\,,\quad E_r^{\mathbf{RF}} = \frac{\varepsilon_{||}}{\varepsilon_\perp} E_0\, \mathbf{J}_1(\kappa r)\,\mathbf{sin}\,\varphi$$

where E_0 - field amplitude on the axis; $E_1 = E_0\, \mathbf{J}_0(\kappa r_b)/\mathbf{I}_0(k_\perp r_b)$; $\varphi \equiv \omega\left(\int \frac{dz}{V_{ph}} - t\right)$ - ion phase taken from the maximum of the accelerating wave; $V_{ph} = \omega/k_z(z)$, k_z - changing wave number; J_0, J_1, I_0, I_1 - usual and modified Bessel functions, and

$$k_\perp \approx k_z\,,\qquad \kappa = \sqrt{-\varepsilon_{||}/\varepsilon_\perp}\,k_z\,,\qquad \varepsilon_{||} = 1 - \frac{\omega_{b||}^2}{(\omega - k_z V_0)^2}\,,$$

$$\varepsilon_\perp = 1 - \frac{\omega_{b\perp}^2}{(\omega - k_z V_0)^2 - \omega_H^2}\,,\qquad \omega_{b\perp}^2 = 4\pi\, n_0 e^2/m\gamma_0\,,$$

$\omega_{b||} = \omega_{b\perp}/\gamma_0$; $\omega_H = eH/mc\gamma_0$.

Choosing the appropriate value of ω_H nearby the anomalous Doppler effect (ADE) resonance, we can obtain and use the following relationships: $\left(-\varepsilon_{||}/\varepsilon_\perp\right) \approx 1$, and $\kappa \approx k_z$ Firstly, the dynamics of ions in this accelerator by the one-particle approximation was investigated. We have determined that dynamics of longitudinal motion of accelerated ions were similar to those of conventional linear accelerators. For dynamics of radial motion of accelerated ions it was taken into account both DC space charge fields of a driving electron beam, and RF fields of charge density waves of this beam. In that case, simultaneous radial and phase stability of accelerated ions was achieved.

2. Furthermore, in this work Coulomb interaction of accelerated ions will take into account by large particle simulation method [3,4]. In our studies we use main limitations adopted for linac simulation codes [5], as follows: for describing of beam average properties, including second moments of distribution function and rms emittance, about *one thousand* macro-particles are adequate for obtaining reproducible and accurate results with account of space-charge effects; ion-ion and ion-neutral collisions can not taken into consideration.

3. It is considered a collisionless model of a single axially-symmetric ion bunch driving in an electron beam under action of the RF accelerating field (1) of the wave with the phase velocity $V_{ph} = \sqrt{V_{ph0}^2 + b(z - z_0)}$. In this approximation the ion bunch is presented as a set of particles ("clouds") in a form of infinitely thin rings with variable radial and longitudinal positions; the axis of each ring is aligned with the RF structure (main) axis. The field of such bunch is determined from the Poisson equation: $\Delta\Phi = -4\pi\rho$ with the space charge density

$$\rho(r,z) = \frac{\sigma_0}{r}\sum_{i=1}^{P} r_{0i}\,\delta(r - r_i)\,\delta(z - z_i)\,,$$

and the boundary conditions: $\Phi|_{r=r_d} = 0$, $\Phi|_{z=\pm\infty} = 0$.

Here r_d is the drift tube radius, $\sigma_0 = \dfrac{I_{i0} l}{\pi\, r_b P V_{i0}}$, I_{i0} - initial ion current, r_b - beam radius, V_{i0} - initial ion velocity, coinciding with the initial phase velocity of the RF field, l - period length of the RF field; r_{0i} , r_i , z_i - initial radius, current radius and longitudinal coordinate of i-th particle, correspondingly, P - number of macro-particles, δ - Dirak s delta-function. The solution was found in form of Fourier-Bessel series:

$$\Phi(r,z) = \sum_{i=1}^{M}\sum_{j=1}^{N} A_{ij}\, \mathbf{J}_0\!\left(\frac{\mu_i}{r_d} r\right)\mathbf{sin}\,\frac{\pi j}{l}(z - V_0)\,,$$

where r_d - drift tube radius, μ_i - roots of zero order Bessel s function, z_0 - coordinate of point with phase $\varphi = -\pi/2$; coefficients A_{ij} are determined by:

$$A_{ij} = \frac{16\pi\,\sigma_0}{r_d^2 l\left[\mathbf{J}_1(\mu_i)\right]^2\left[\left(\frac{\mu_i}{r_d}\right)^2 + \left(\frac{\pi j}{l}\right)^2\right]} \times \tag{2}$$

$$\sum_{k=1}^{P} r_{0k}\, \mathbf{J}_0\!\left(\frac{\mu_i}{r_d} r_k\right)\mathbf{sin}\,\frac{\pi j}{l}(z_k - V_0)$$

The components of the electric field are:

$$
\begin{aligned}
E_r^i(r,z) &= \sum_{i=1}^{M}\sum_{j=1}^{N} A_{ij}\,\frac{\mu_i}{r_d}\,\mathbf{J}_1\!\left(\frac{\mu_i}{r_d}r\right)\mathbf{sin}\,\frac{\pi j}{\lambda}\left(z-\zeta_0\right)\\
E_z^i(r,z) &= -\sum_{i=1}^{M}\sum_{j=1}^{N} A_{ij}\,\frac{\pi j}{\lambda}\,\mathbf{J}_0\!\left(\frac{\mu_i}{r_d}r\right)\mathbf{cos}\,\frac{\pi j}{\lambda}\left(z-\zeta_0\right)
\end{aligned}
\tag{3}
$$

The motion equations for i-th ring (under transition to the variable ζ determined from the equation: $\int \frac{d\zeta}{V_{ph}(\zeta)} - t = \mathbf{0}$) are recorded as follows:

$$
\begin{aligned}
\frac{d^2 z_j}{d\zeta^2} &= \frac{1}{V_{ph}^2}\left[\frac{e}{m}\left(E_{zj}^{\mathbf{RF}} + E_{zj}^{i}\right) - \frac{\beta}{2}\frac{dz_j}{d\zeta}\right]\\
\frac{d^2 r_j}{d\zeta^2} &= \frac{1}{V_{ph}^2}\left[\frac{e}{m}\left(E_{rj}^{\mathbf{RF}} + E_{rj}^{e} + E_{rj}^{i}\right) - \frac{\beta}{2}\frac{dr_j}{d\zeta}\right]
\end{aligned}
\tag{4}
$$

where $E_{zj}^{\mathbf{RF}}, E_{rj}^{\mathbf{RF}}$ - are the RF field components (1), for which the phase φ is determined (in accordance with the expression for V_{ph}) from the equation:

$$\varphi_j = \frac{2\omega}{\beta}\left[V_{ph}\left(z_j\right) - V_{ph}(\zeta)\right];$$

E_{zj}^{i}, E_{rj}^{i} are the components of the space charge field of the ions (3); ω is the RF field frequency; $E_{rj}^{e} = \frac{2\pi I_e}{SV_e} r_j$ is the space charge field of the electron beam, I_e is the current of the electron beam, V_e is the electron beam velocity, S is the cross-section of the electron beam.

Equations (2)-(4) are solved by Adams method for the test accelerator (EAS) main parameters.

4. On the figures placed below the results of simulations are presented in which parameters of the two-beam electron-ion accelerator model (the EAS section) are used as follows: current, energy, and radius of the electron beam are 150 A, 350 keV, and r_e = 1.3 cm; initial and final energy of the proton beam are 5 MeV and 8 MeV, its initial radius $r_i = r_e$; current of the proton beam is varied from 30 mA to 10 A; the accelerator length is 161 cm; mean accelerating rate is 24 kV/cm; amplitude of accelerating wave and synchronous phase are assumed to be equal to 56 kV/cm and 65° in one case, and 27 kV/cm and 30° in another one. Radial stability of the accelerated protons is ensured by Coulomb field of the electron beam (both its constant and variable parts, see above). It is supposed that the electron beam is weak-disturbed. Longitudinal stability is ensured by usual phase stability mechanism. The problem of simulation by the macroparticles method includes determination of accelerated current values for the parameters pointed above, and features of accelerated particles dynamics.

Fig.1 represents the results of computer simulation of proton beam acceleration with current 4 A by the wave with amplitude E_a = 56 kV/cm and synchronous phase φ_s=65°. Injection interval of the particles extends from -φ_s to 2φ_s, i.e. from -π/3 to 2π/3. Particle phases (relative to the wave amplitude maximum) are plotted on the x-axis, and their radii (in centimeters) are plotted on the y-axis. The results of simulation were obtained as a plot of sequences that paint positions of the macro- protons in coordinates φ, r for every next (1/6)th part of the resonator length. In this paper, for lack of the space, only 1st and 2nd parts are presented (z_1=27cm, z_2=54 cm). As far as opposite motion of particles can be occur, so positions of the particles outstripping the synchronous one, i.e. moving to the right (in this case, top sequences 1a-1b), and falling behind it, i.e. moving to the left (bottom sequences 2a-2b) are represented separately (it is made for convenience of observation; really these pictures must be brought into coincidence). The boundary of the electron beam is shown by the horizontal dashed line, and the boundary of the drift tubes is shown by the thick line. Synchronous phase is shown by the vertical dashed line. Amount of the macroparticles: P = 930, number of series expanding terms on r : M = 10, on z: N = 30. The particles that drop onto drift tubes or go out the phase interval of [-π/2, 3π/2] were recognized as lost ones. It ought to note that unlike analytical considerations (see, e.g., [6]) longitudinal and transverse motions do not separate in this case, and one does no assumptions about form of the ion bunches (e.g., like ellipsoids, long cylinders, pulsed ellipsoids, etc.).

At a relatively small proton current (30 mA) and on the initial stage of acceleration, behaviour of the accelerated particles is like that it was observed (by simulation) for similar conditions in case of non-interacting particles acceleration (at so called single-particle approx i-mation). Furthermore, from the results of simulation (e.g., Fig.1) one can see well that some particles move in opposite directions (relative to the synchronous phase φ_s) exchanging their places and bunching near φ_s in longitudinal and radial directions at the certain moments. (It should be mentioned that the images of some particles can coincide, especially, in the place of bunching). In case of E_a = 56 kV/cm nearly one period of phase oscillation is located on the length of the EAS section. The feature of radial motion of the particles near the synchronous phase is as follows: the bunch collapses here both in radial and phase directions (it corresponds to simultaneous radial and phase focusing), and as a result, a part of the ions leave in radial direction to the drift tube. This is explained by that the concentration of the ions increase so much that the radial focusing field can not compensate the Coulomb field of bunch, and a part of the ions is thrown off. This instability of the accelerated bunch (which can be called as the collective transverse one) did not consider earlier by analytical methods (e.g., [6]). In particular, an account of this instability can be essential for high intensity accelerators intended for irradiation technologies and nuclear fuel breeding (due to very strong requirements for ion transverse losses).

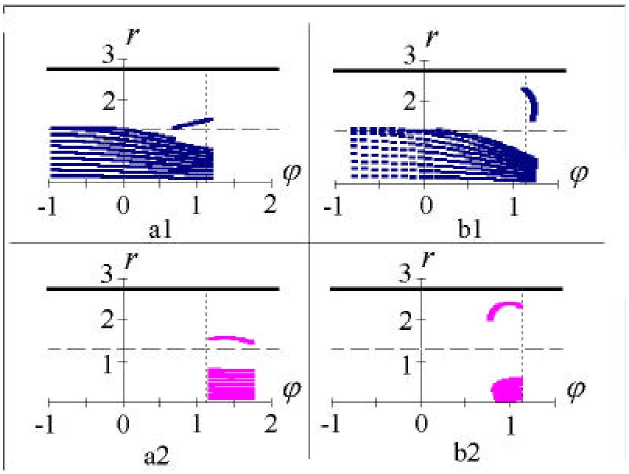

Fig. 1. Evolution of the ion bunch near the synchronous phase for a current of 4 A.

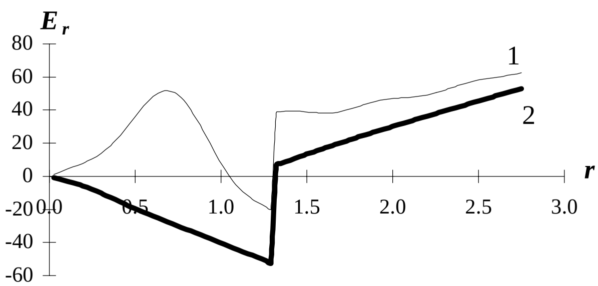

Fig. 2. Dependence of the net radial electric field on radius for an ion current of 4 A (curve 1) and 30 mA (curve 2).

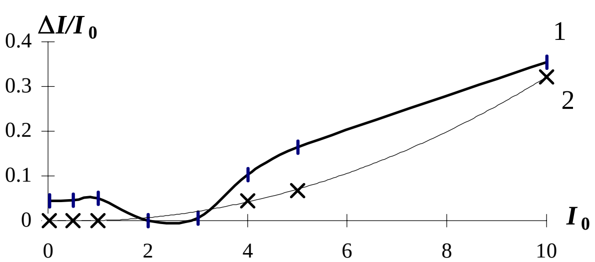

Fig. 3. The plot of relative losses of ion beam current versus input current (1 - E_z=56 kV/cm; 2 - E_z=27 kV/cm)

Our simulation shows that in case of ion current increasing the total field E_r become defocusing one in the ion bunching region, that results in transverse losses growing. So, in this case, the ion limit current can be determined as 3 A at E_z =56 kV/cm. The simulation allows to determine the net radial field $E_r(r, z)$ produced by the RF field, the electron beam, and the ion bunch. Fig. 2 (curve 1) shows the function $E_r(r)$ for the conditions corresponding to Fig.1, plot (b1), $\varphi=\varphi_s$; in this case, the net field is partially defocusing which causes radial ion losses. For comparison, curve 2 shows $E_r(r)$ for similar conditions but with the ion current 30 mA; in this case, the net field is focusing inside the electron beam and defocusing outside it. At I_i = 5 A and I_i = 10 A motion of the ions becomes strongly disturbed in the second half of the acceleration section. However, due to a short length of the acceleration section, the main part of the ions can pass through it. The plot of relative losses of the ion beam $\Delta I/I_0 = (I_0 - I)/I_0$ versus I_0 is presented in Fig. 3, curve 1 (here I_0 and I are input and output ion currents, respectively). It should be mentioned that the phase stability take place up to 10 A.

In case of E_z =27 kV/cm and $\varphi_s = 30°$ the beginning of the collective transverse instability was seen well for ion current of 1 A. For currents of 4, 5, 10 A increasing of transverse losses were observed. Besides, for the ion current of 1 A and more, it was seen a self-retardation of the back part of the ion formation that increases with ion current increasing and leads to longitudinal losses of accelerated ions. The simulations determined the ion limit current (relative to transverse and longitudinal instabilities) about of 1 A. The plot of relative losses of ion beam current ($\Delta I/I_0$) versus input current I_0 for E_a=27 kV/cm is presented in Fig.3, curve 2. It should be mentioned that in this case the radial instability grows slowly because at $r > r_e$ the radial defocusing field is twice less than in the first case but the focusing field of an electron beam is the same. Then, for rather large ion currents (I_i=5 A and 10 A), the main part of the ions have opportunity for passing through the acceleration section.

So, we found that the collective transverse instability defines the upper limit of ion current to be accelerated. For the parameters of the test accelerator EAS the ion limit current about 3 A have been determined.

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