METHODS OF ION FOCUSING AND ACCELERATION IN RF FIELD CONTAINING NONSYNCHRONOUS HARMONICS

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Abstract

In this paper the possibility of ion focusing and acceleration in radio-frequency field without synchronous harmonic is discussed. The investigation of the ion beam dynamic is executed by averaging method. The expression for three-dimensional effective potential is obtained. The study of a longitudinal and transverse dynamics is carried out by means of this potential.

1 INTRODUCTION

In a usual RF linac ion beams are accelerated by a synchronous wave. Nonsynchronous wave will be focusing the particles only [1]. Electromagnetic field consists of the space harmonics in a periodical RF structure. The synchronous and nonsynchronous space harmonics amplitudes must be chosen to provide both longitudinal and transversal stability. For the two wave approach (synchronous and one nonsynchronous space harmonics) these conditions were founded in paper [2, 3]. Another line of attack on the problem of the particle accelerating in the field of two nonsynchronous waves was discussed in the paper [4]. This idea can be realized either in a uniform RF resonator structure, where periodical undulators are used [5], or in a special periodical RF structure without undulator. A few theoretical aspects of the method of ion acceleration in the field without synchronous wave and possible technical realization for ion linac are investigated.

2 PARTICLE MOTION EQUATIONS

The motion equation of particle in the field of two waves may be written using Lagrange function

$$\frac{d\boldsymbol{P}}{dt} = e\nabla(\boldsymbol{v}\cdot\boldsymbol{A}_\Sigma), \qquad (1)$$

where $\boldsymbol{P} = \boldsymbol{p} + e\boldsymbol{A}$ is the canonical momentum, $\boldsymbol{A}_\Sigma(\boldsymbol{R}) = Re[\boldsymbol{A}_n(\boldsymbol{R}^\perp)e^{i\varphi_n} + \boldsymbol{A}_l(\boldsymbol{R}^\perp)e^{i\varphi_l}]$ is the overall vector potential of the two waves with amplitudes $\boldsymbol{A}_{n,l}$, phases $\varphi_{n,l} = k_{n,l}z - \omega_{n,l}t + \alpha_{n,l}$ and wave numbers $k_{n,l}$. When the phase velocities $v = \omega_{n,l}/k_{n,l}$ differ significantly from the average velocity of the particles v_b, the trajectories of the particles can be expressed by the summation of slowly variation $\boldsymbol{R}_c$ and rapidly oscillations $\tilde{\boldsymbol{R}}$. By averaging over rapidly oscillations from (1) we can obtain the time-averaged equation of nonrelativistic ion motion when $v_b \approx v_s \equiv (\omega_n \pm \omega_l)/(k_n \pm k_l)$:

$$\frac{d^2\boldsymbol{R}_c}{dt^2} = -\frac{e}{m}\nabla U_{eff}, \qquad (2)$$

where 3D effective potential

$$U_{eff} = U_1(\boldsymbol{R}_c^\perp) + U_2(\boldsymbol{R}_c^\perp, \psi),$$

$$U_1(R_c^\perp) = \frac{e}{4m}\left(|\boldsymbol{A}_n|^2 + |\boldsymbol{A}_l|^2\right), \qquad (3)$$

$$U_2(R_c^\perp, \psi) = \frac{e}{2m}Re\left(\boldsymbol{A}_n\cdot\left(\boldsymbol{A}_l e^{i\psi_+} + \boldsymbol{A}_l^* e^{i\psi_-}\right)\right),$$

$\psi_\pm = (k_n \pm k_l)Z - (\omega_n \pm \omega_l)t + (\alpha_n \pm \alpha_l)$ -- slowly varying phase.

The acceleration of the particles is provided by a combined wave with phase velocity v_s which is close to the average particle velocity. The choice of the function $\boldsymbol{A}_n(\boldsymbol{R}^\perp{}_c)$ and $\boldsymbol{A}_l(\boldsymbol{R}^\perp{}_c)$ is not arbitrary because simultaneously with acceleration it is necessary to keep up the transverse focusing of the beam. Equilibrium trajectories can exist for all particle phases if two conditions are valid:

$$\nabla_\perp U_1 = 0, \; \nabla_\perp U_2 = 0. \qquad (4)$$

The motion about the equilibrium trajectory is stability if the potential function $U_{1,2}$ has an absolute minimum. The obtained conditions connect values of amplitudes $\boldsymbol{A}_n(\boldsymbol{R}^\perp{}_c)$ and $\boldsymbol{A}_l(\boldsymbol{R}^\perp{}_c)$ and confine configuration of the fields.

For low energy ion accelerator it is difficult to create resonator system with two generators ($\omega_n \neq \omega_l$). It is interesting to consider two versions of the accelerator: i) $\omega_n = 2\pi c/\lambda$, $\omega_l = 0$ --a combining of RF field ($k_n = 2\pi/\lambda$) and static periodical undulator field ($k_l = k_0 = 2\pi/\lambda_0$), where λ_0 is slowly varying period of a undulator, $\lambda >> \lambda_0$; ii) $\omega_n = \omega_l = 2\pi c/\lambda$, $k_n \neq k_l$ --combining of two space nonsynchronous harmonics of RF field in the periodical structure, $k_n = nk_0$, $k_l = lk_0$, $\lambda_0 = 2\pi/k_0$ --RF structure period.

3 RF FIELD AND MAGNETOSTATIC UNDULATOR

Let's assume a resonator is installed in magnetostatic field of plane undulator [6]. The RF system must have a small transverse size to be located inside the undulator. For this purpose shielded pair of longitudinal electrodes are used. The cross-section of this system are shown in Fig. 1.

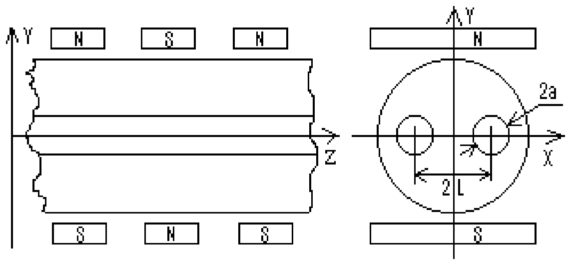

Figure 1: Cross-section of RF system and M- undulator.

For plane magnetic undulator

$$\boldsymbol{A} = A_0 \boldsymbol{e}_x \cosh(k_0 y) \sin k_0 z \tag{5}$$

and the dimensionless effective potential can be found from (3):

$$\tilde{U}_1 = \frac{1}{4}\left(\boldsymbol{b}_0{}^2 + \boldsymbol{b}_v{}^2\right),\ \tilde{U}_2 = -\frac{1}{2}\boldsymbol{b}_0 \cdot \boldsymbol{b}_v \sin\psi . \tag{6}$$

Here $\boldsymbol{b}_{0,v} = e\boldsymbol{B}_{0,v}{}^{\perp}\lambda_{0,v} /(2\pi mc)$ are dimensionless amplitudes of transverse components of the undulator field $\boldsymbol{B}_0{}^{\perp}$ and RF field $\boldsymbol{B}_v{}^{\perp}$.

The rate of energy gain for the synchronous particle is given by $dW_s / dz = eT_M E_v \cos\psi$, where $E_v = cB_v$ is nonsynchronous RF field amplitude, $\beta_s = \lambda_0 / \lambda$ is normalized velocity of synchronous particle and $T_M = \dfrac{eB_0\lambda_0}{2\pi mc}$ is the acceleration efficiency factor.

The condition (4) can be satisfied only if $x = y = 0$. The small oscillations will be stable if

$$\Omega_x{}^2 = \frac{b_v}{(l^2 - a^2)k^2}(b_v + b_0 \sin\psi) \succ 0$$
$$\Omega_y{}^2 = \frac{1}{2}b_0 \frac{k_0{}^2}{k^2}(b_0 + b_v \sin\psi) - \Omega_x{}^2 \succ 0, \tag{7}$$

where $\Omega_{x,y}$ are transversal oscillations frequencies. The focusing takes place for all ψ if

$$B_v = \beta_s B_0 \text{ and } \sqrt{2}\pi\sqrt{l^2 - a^2} > \lambda_0 . \tag{8}$$

The first condition connects B_v, B_0 and β_s. The second one restricts the distance between two electrodes. It is possible to express factor T_M through the amplitude of RF field:

$$T_M = \frac{eE_v\lambda}{2\pi mc^2\beta_s} . \tag{9}$$

4 RF FIELD AND ELECTROSTATIC UNDULATOR

For low injection energy it is advisable to replace the magnetostatic undulator by the electrostatic one. In the paper [7] the linear accelerator, in which the ribbon ion beam is accelerated in the transverse RF field and the field of a plane electrostatic undulator was suggested. A periodic undulator field is generated across the adjacent pairs of electrodes. Simultaneously RF potentials $\pm V_v \sin\omega t$ are applied to the electrodes of the upper and bottom row respectively. So the same electrodes are used to generate both field. (Fig. 2.).

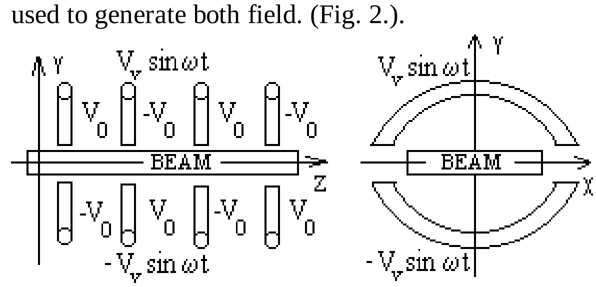

Figure 2: Cross section of RF-system and E- undulator.

In the Cartesian system of coordinates electrostatic field components $\vec{E}_0$ can be written as

$$E_x = E_{0x} \sinh(k_x x)\sinh(k_y y)\cos(k_0 z)$$
$$E_y = E_{0y} \cosh(k_x x)\cosh(k_y y)\cos(k_0 z) \tag{10}$$
$$E_z = -E_{0z} \cosh(k_x x)\cosh(k_y y)\sin(k_0 z).$$

$E_{0x}{}^2 + E_{0y}{}^2 = E_{0z}{}^2$, $k_x{}^2 + k_y{}^2 = k_0{}^2$. The RF field can be presented as $\boldsymbol{E}_v = E_v \boldsymbol{e}_y \sin\omega t$. The dimensionless effective potential

$$\tilde{U}_1 = \frac{1}{4}\left(\boldsymbol{a}_0{}^2 + \boldsymbol{a}_v{}^2\right),\ \tilde{U}_2 = -\frac{1}{2}\boldsymbol{a}_0 \cdot \boldsymbol{a}_v \sin\psi , \tag{11}$$

where $\boldsymbol{a}_{v,0} = e\boldsymbol{E}_{v,0}\lambda / (2\pi mc^2)$ are dimensionless amplitudes of RF field $\boldsymbol{E}_V$ and undulator field $\boldsymbol{E}_0$. The rate of energy gain $dW_s / dz = eT_E E_v \cos\psi$, $T_E = eE_0\lambda^2 /(2\pi mc^2\lambda_0)$.

The analysis of potential function for paraxial particles ($k_x x$ <<1, $k_y y$ <<1) shows, that

$$\Omega_x{}^2 = \frac{k_x{}^2}{2k^2}\left(a_{0y}{}^2 - a_{0y}a_v \sin\psi\right). \tag{12}$$

The oscillation frequency Ω_Y will be larger than Ω_X :

$$\Omega_y{}^2 = \frac{k_y{}^2}{k^2}\left(\frac{a_{0x}{}^2 + a_{0y}{}^2}{2}\right) + \frac{k_x{}^2}{k_y{}^2}\Omega_x{}^2 . \tag{13}$$

The simultaneous focusing along x and y directions are possible if $E_0 > E_v$. The acceleration efficiency factor $T_E = \frac{eE_v\lambda}{2\pi mc^2\beta_s}$ is equal to T_M.

5 TWO HARMONICS OF RF FIELD

The combined acceleration field can be created without using of the undulator. Let's consider the axially symmetric periodical RF structure. In the cylindrical system of coordinates space harmonic component $\boldsymbol{E}_v^{\ n}$ can be written as

$$E_{v,z}^{\ n}(\boldsymbol{r}) = E_n I_0(k_n r)\cos\varphi_n$$
$$E_{v,r}^{\ n}(\boldsymbol{r}) = E_n I_1(k_n r)\sin\varphi_n . \qquad (14)$$

For two odd harmonics $k_1 = k_0$, $k_3 = 3k_0$ and the beam velocity $\beta_b = \lambda_0 / 2\lambda$ we can obtain the motion equation in the form (2), (3) if the 2-nd harmonic is absent. The transverse oscillation frequency can be found from (3):

$$\Omega_r^{\ 2} = \frac{k_0^{\ 2}}{8k^2}\left(3a_1^{\ 2} + 27a_3^{\ 2} - 14a_1a_3\sin\psi\right) \qquad (15)$$

where $a_n = eE_n\lambda / (2\pi mc^2)$. The condition $\Omega_r^{\ 2} > 0$ is always satisfied and small transversal oscillations are stable. The transversal acceptance will be maximal if $E_1 = 3E_3$.

The acceleration gradient for synchronous particle is $dW_s / dz = eT_{rf} E_1 \cos\psi$, where

$$T_{rf} = 4\frac{e\lambda E_3}{2\pi mc^2\beta_s} . \qquad (16)$$

The ion beam is modulated in frequency 2ω.

6 SYNCHRONOUS HARMONIC AND RF FOCUSING

It is interesting to compare considered methods of ion focusing and acceleration to RF focusing in usual linac, where the synchronous wave E_s accelerates beam and the nonsynchronous wave E_n focuses the particles. In the paper [8] it was shown, that the transversal oscillations of particle will be stable if

$$E_s < \frac{3}{4}\frac{n^2}{(n-s)^2}\frac{eE_n^{\ 2}\lambda}{2\pi mc^2\beta_s} . \qquad (17)$$

Using of this condition we can find the efficiency factor $T_s = \frac{3}{4}\frac{n^2}{(n-s)^2}\frac{eE_n\lambda}{2\pi mc^2\beta_s}$. This magnitude is equal to $3T_{rf} / 4$, when s=1, n=2, and the transversal acceptance will be less.

All possible methods of focusing and acceleration in RF field containing nonsynchronous harmonic will be effective for small ion energy. Factor $T_{M,E,rf}$ decreases with growth of the velocity. The frequencies of small transversal and longitudinal particle oscillations will be commensurable and it's important to study non-linear oscillations, the coupling resonances and overlapping of separatrixs because additional restrictions on the amplitudes $E_{n,l}$ and factor T exist.

7 CONCLUSION

Three methods of ion focusing and acceleration in nonsynchronous wave of RF field were considered. The rate of acceleration is comparable with usual RF linac, but the acceptance will be large. In new type of the ion accelerators it's possible to use not only longitudinal, but also transversal RF field. For TEM wave the drift tubes are absent in RF structure and we can enlarge the beam cross-section for the intensity beams. The other important way for increasing of beam intensity is using the space charge compensation. The effective potential (3) does not depend on a sign of the particle charge and the averaged motion of positive and negative charges are identical. These ions will be inside the same separatrix (bunch) and the current limit of the ion beam can be substantially increased. It's possible to accelerate plasma bunches by offered method.

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