# EXACT PHASE ADVANCES FOR A TWO-STAGE COLLIMATION SYSTEM

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*Abstract*

We propose a specification for a two-stage collimation insertion. We compute exact correlated phase advances between primary and secondary collimators , and determine the number of jaws needed to reach an almost ultimate performance.

## 1 INTRODUCTION

An exact treatment of a two-stage collimation system considered as an optical device, i.e disregarding true scattering in collimator jaws, exist for the one-dimensional case and in the special 2D-case of an optics with equal phase advance in the two transverse dimensions [1]. The problem of a 2D-system with an arbitrary optics was solved with numerical methods in conjunction with the approximate concept of phase modulation with some success [1][2][3], but without cutting the amplitude of the secondary halo down to the ultimate limit of the aperture of the secondary collimators. In this paper, we propose an exact solution of the phase advances between collimators approaching the ultimate limit.

## 2 DEFINITION AND NOTATIONS

We use horizontal and vertical betatron coordinates as well as horizontal dispersion normalised with the transformations $\mathbf{X} = \mathbf{N_x x}$, $\mathbf{Y} = \mathbf{N_y y}$ and $\chi = \mathbf{N_x D}$ with

$$\begin{pmatrix} X \\ X' \end{pmatrix} = \frac{1}{\sigma_x} \begin{pmatrix} 1 & 0 \\ \alpha_x & \beta_x \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}. \qquad (1)$$

In normalised coordinates, noted by 4-vectors $\mathbf{A}_i = (X, X', Y, Y')$ the betatronic transfer matrix $\mathbf{M}_{ij}$ between two locations is made of two clockwise rotations, one for each proper plane, where the angles of rotation $\mu_x$ and $\mu_y$ are the betatronic phase advances, i.e.

$$\mathbf{M}_{ij} = \begin{pmatrix} \cos\mu_x & \sin\mu_x & 0 & 0 \\ -\sin\mu_x & \cos\mu_x & 0 & 0 \\ 0 & 0 & \cos\mu_y & \sin\mu_y \\ 0 & 0 & -\sin\mu_y & \cos\mu_y \end{pmatrix}. \qquad (2)$$

The normalised invariant amplitudes are $A_x = (X^2 + X'^2)^{1/2}$, $A_y = (Y^2 + Y'^2)^{1/2}$ and $A = (A_x^2 + A_y^2)^{1/2}$.

## 3 BETATRON COLLIMATION

We first consider circular collimators in normalised coordinates. The normalised aperture of the primary and secondary collimators are $n_1$ and $n_2$. These numbers are fixed in our problem, in the sense that they cannot be varied to optimise a collimation system but must rather fit to external parameters like the dynamic aperture or the effective geometrical aperture of the ring. We use the approximation of slow diffusion of the primary halo, meaning that the impact parameter at the primary collimator is small compared to $n_1$, or that the impact points are at the surface of the collimator and also that both betatronic oscillations are at their maxima, i.e. $X'_o = Y'_o = 0$. Finally we will minimise the extension of the secondary halo after it is cut by the secondary collimators treated as black absorbers. primary collimator are $\mathbf{A}_o = (n_1\cos\alpha, 0, n_1\sin\alpha, 0)$. The scattering process adds an arbitrary value to $X'_o$ and $Y'_o$, following here for simplicity an isotropic distribution. The coordinates at the primary collimator after scattering are

$$\mathbf{A_1} = (n_1\cos\alpha, K\cos\phi, n_1\sin\alpha, K\sin\phi), \qquad (3)$$

with the $X-Y$ azimuth $\alpha$ and the polar variables $K$ and $\phi$ in the $X'_o - Y'_o$ plane.

### 3.1 Phase advances

For arbitrary $\alpha$ and $\phi$, we transport the particle with (2) and (3) to a location of yet unspecified phase advances $\mu_x$ and $\mu_y$ where a secondary collimator is located and get

$$\mathbf{A_2} = \begin{pmatrix} n_1\cos\alpha\cos\mu_x + K\cos\phi\sin\mu_x \\ -n_1\cos\alpha\sin\mu_x + K\cos\phi\cos\mu_x \\ n_1\sin\alpha\cos\mu_y + K\sin\phi\sin\mu_y \\ -n_1\sin\alpha\sin\mu_y + K\sin\phi\cos\mu_x \end{pmatrix}. \qquad (4)$$

The efficiency of the secondary collimator is measured by the smallest amplitude $A_{cut}$ that it can intercept. $A_{cut}$ is minimised if $X_2$ and $Y_2$ are maximised. Using the invariance of $A_{x,2}$ and $A_{y,2}$, this condition is equivalent to asking for $X'_2 = Y'_2 = 0$. With these conditions in (4) we get

$$\tan\mu_x = \frac{K\cos\phi}{n_1\cos\alpha} \ , \ \tan\mu_y = \frac{K\sin\phi}{n_1\sin\alpha}. \qquad (5)$$

These conditions allow to compute the sole free parameters $\mu_x = \mu_x(\alpha,\phi,K)$ and $\mu_y = \mu_y(\alpha,\phi,K)$. While $\alpha$ and $\phi$ are free variables, $K$ is restricted to its maximum allowed value corresponding to the smallest possible $A_2 = A_{cut} = n_2$ (see Figure 1). This is obtained by using (5) in (4) with again $X'_2 = Y'_2 = 0$. We get

$$K = K_c = \sqrt{n_2^2 - n_1^2}, \qquad (6)$$

which is independent of both $\alpha$ and $\phi$. Writing $\tan\mu_o = K_c/n_1 = (n_2^2 - n_1^2)^{1/2}/n_1$, (5) is now

$$\tan\mu_x = \tan\mu_o\frac{\cos\phi}{\cos\alpha} \ , \ \tan\mu_y = \tan\mu_o\frac{\sin\phi}{\sin\alpha}. \qquad (7)$$

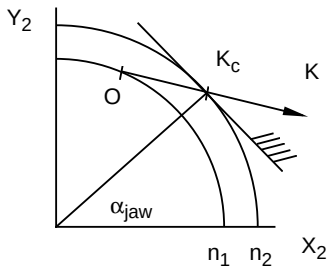


Figure 1: The line of the scattered particles at the primary collimator parametrised with $(n_1, \alpha, K, \phi)$ transforms at the location of a secondary collimator to another line which crosses the circle of radius $n_2$ when $K = K_c$ whatever $(\alpha, \phi)$. A flat jaw at azimuth $\alpha_{jaw}$ is sufficient to cut at amplitude $A = n_2$, see text.

These formulae indicate that an optimum collimation for all possible $\alpha$ and $\phi$ would need an infinity of collimators, with an optics able to offer an infinity of pairs of phase advances $(\mu_x, \mu_y)$.

Before compromising on the number of collimators, it must be noticed that for given $(\alpha,\phi)$, the secondary collimator at $(\mu_x, \mu_y)$ needs not be circular. A single flat jaw at the $X - Y$ azimuth $\alpha_{jaw} = \tan^{-1}(X_2/Y_2)$ is sufficient (see Figure 1). With (4), the azimuth of the jaw must be

$$\tan \alpha_{jaw} = \frac{\sin\alpha \cos\mu_y + \tan\mu_o \sin\phi \sin\mu_y}{\cos\alpha \cos\mu_x + \tan\mu_o \cos\phi \sin\mu_x}. \quad (8)$$

### 3.2 A finite number of collimators

To limit the number of collimators, we consider a system made of three primary collimators which delimit an octagonal primary aperture. Further we consider the scattered particles to be issued from the central point of each jaw, i.e. at azimuth $\alpha = 0, \pi/4, \pi/2$ (Figure 2). Then we compute the phase advances between a primary and the secondary collimators associated to the four scattering azimuths $\phi = \alpha, \alpha+\pi$ and $\phi = \alpha \pm \pi/2$, called respectively plane ($\parallel$) and orthogonal ($\perp$) scattering . For $\parallel$-scattering with (7) we get $\mu_x = \mu_y = \pm\mu_o + k\pi$ which is the old result found for 1D-collimation [1], and with (8) we get $\alpha_{jaw} = \alpha + k\pi$. For $\perp$-scattering , we get $\mu_x = \pm\tan^{-1}(\tan\mu_o \tan\alpha)$, $\mu_y = \pm\tan^{-1}(\tan\mu_o / \tan\alpha)$. The resulting phases in Table 1 are the smallest ones. One can add $\pi$ to any of these phases but then $\alpha_{jaw}$ must be reevaluated.

### 3.3 Simulation for realistic primary halo

To check the relevance of the 3-point approximation, we wrote a simple simulation program. Primary impacts are uniform along the inner surface of the jaws. Scattering angles are uniform in the $K - \phi$ plane. The tracking is made with transfer matrices like (2) in which $(\mu_y, \mu_x)$ are taken from Table 1. At each collimator it is verified if the particle touches a jaw. The particles surviving all secondary collimators are added to a $A_x - A_y$ plot and to a combined amplitude distribution $dN/dA$ (Figure 2). The octagonal

Table 1: Secondary collimator locations and jaw orientations for three scattering centers ($\alpha$)and for parallel and orthogonal scattering.

| $\alpha$ | $\phi$ | $\mu_x$ | $\mu_y$ | $\alpha_{jaw}$ |
|---|---|---|---|---|
| 0 | 0 | $\mu_o$ | - | 0 |
| 0 | $\pi$ | $\pi - \mu_o$ | - | 0 |
| 0 | $\pi/2$ | $\pi$ | $3\pi/2$ | -$\mu_o$ |
| 0 | $-\pi/2$ | $\pi$ | $3\pi/2$ | $\mu_o$ |
| $\pi/4$ | $\pi/4$ | $\mu_o$ | $\mu_o$ | $\pi/4$ |
| $\pi/4$ | $5\pi/4$ | $\pi - \mu_o$ | $\pi - \mu_o$ | $\pi/4$ |
| $\pi/4$ | $3\pi/4$ | $\pi - \mu_o$ | $\pi + \mu_o$ | $\pi/4$ |
| $\pi/4$ | $-\pi/4$ | $\pi + \mu_o$ | $\pi - \mu_o$ | $\pi/4$ |
| $\pi/2$ | $\pi/2$ | - | $\mu_o$ | $\pi/2$ |
| $\pi/2$ | $-\pi/2$ | - | $\pi - \mu_o$ | $\pi/2$ |
| $\pi/2$ | $\pi$ | $\pi/2$ | $\pi$ | $\pi/2 - \mu_o$ |
| $\pi/2$ | 0 | $\pi/2$ | $\pi$ | $\pi/2 + \mu_o$ |

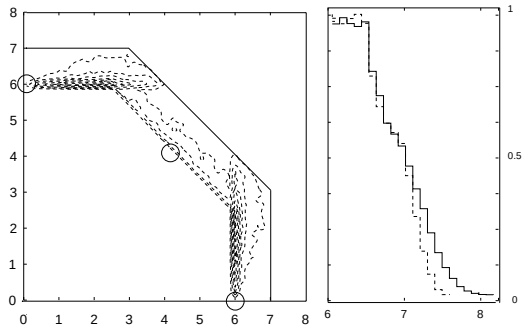

Figure 2: Left: the axis are either $X$ and $Y$ or $A_x$ and $A_y$. The circles indicate the sourcepoints used to compute the collimator location given in Table 1. The contour plot is the density of the secondary halo $d^2N/dA_x dA_y$ obtained with the numeric simulation described in Section 3.3. Right: the amplitude distribution $dN/dA$ integrated from $d^2N/dA_x dA_y$. Full line, the 12-jaw case and dotted line, the 24-jaw case. We used $n_1 = 6$ and $n_2 = 7$.

primary aperture and the flat secondary collimators generate an octagonal footprint in $A_x - A_y$, with largest secondary amplitude $A = n_2 \cos(\pi/8) = 1.08 n_2$, whereas circular collimators would sharply cut at $A = n_2$. This fact taken into account, the 3-point approximation is very good, the count rate being small above $A = 1.08 n_2$. A 24-jaw solution is explored with in addition to $\parallel -$ and $\perp -$ scattering the scattering angles $\phi = 45^o + k\pi/2$. The result is closer to the ultimate limit (see Figure (2)) but certainly not worth the additional hardware investment.

We therefore need four secondary collimators for each of the primary azimuths, i.e. twelve with three primary collimators. This result was already obtained by D. Kaltchev [3] with numerical methods.

## 4 EXISTING SOLUTIONS

With a symmetric optics ($\mu_x(s) = \mu_y(s)$) the secondary halo is cut at $A_{sec} = 1.32n_2$ [1] with a ratio $n_2/n_1 = 7/6$. The present best performance obtained with a modulated optics for the LHC collimation insertion is $A_{sec} = 1.21n_2$ [3]. It was emphasized in former studies [1][2][3] that to cut on large amplitudes associated to $\perp$-scattering large phase modulation, i.e. large $\mu_y - \mu_x$, was needed along the cleaning insertion. This argument was right but incomplete. Strict correlation of the phase advances $\mu_x$ and $\mu_y$ is mandatory and the maximum modulation $\mu_y - \mu_x = \pi/2$ is needed for some jaws (see Table 1). While it may be unfair to compare the performance of existing optics to our nearly ultimate limit $A_{sec} = 1.08n_2$ obtained with a yet virtual optics, a potential gain exists and we explain what is lacking to the existing insertions.

## 5 MOMENTUM COLLIMATION

We restrict our discussion to a momentum cleaning insertion installed in a straight section, where the dispersion function is a betatronic trajectory. In that case, the condition $D'/D = -\alpha_x/\beta_x$, or equivalently $\chi' = 0$ (see (1)), must be satisfied at the primary collimator [1][4] to ensure that the cut made on the secondary halo does not depend on the momentum offset $\delta_p$. It also strictly reduces the treatment of the momentum collimation to the betatronic case in a straight section[1], while outside the straight section $x_\beta$ and $x_{\delta_p}$ must of course be distinguished.

Contrary to the betatron halo which may drift away from the beam in all transverse directions, momentum losses are concentrated in the horizontal plane. The most demanding case occurs at ramping when off-bucket protons are lost. Most of these protons keep their initial betatronic amplitude at injection and are therefore confined in $A_{x,y} \approx 2$. It is therefore enough to use a single horizontal primary collimator, to which four secondary collimators must be associated. Their relative locations correspond to the case $\alpha = 0$ of Table 1 and they limit the components of the betatron vector after scattering to $\mathbf{A}_1 = (n_1, K_c, \approx 2, K_c)$.

In the arc of a ring, the aperture limitation for a particle with momentum offset is located near horizontally focusing quadrupoles where both $\beta_x$ and $D_x$ are at their maximum. With also $\beta_y$ small, it is thus adequate to fit the largest horizontal excursions $A_{x,\beta}$ of the secondary halo with the aperture $N_{arc} = N_{x,arc}$ at that location. The straight sections of a ring need not be considered for momentum collimation since the dispersion is usually supressed in these areas. distance $N_{arc}$.

### *5.1 Amplitude cut with momentum offset*

In the general case, a particle reaches the primary collimator with a mixing of betatron amplitude and momentum offset. With the dispersion $\chi_1$ at the primary collimator, and using the approximation of slow diffusion we write

$$n_1 = \chi_1\delta_p + X_\beta = \chi_1\delta_p + A_{x,\beta} \qquad (9)$$

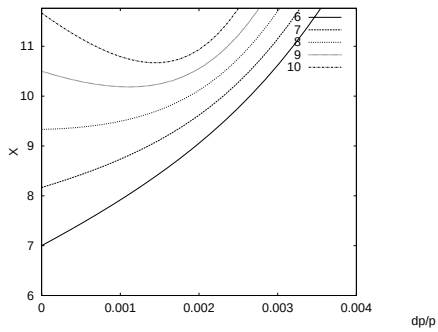


Figure 3: The maximum excursion $X_{max}$ of a particle as a function of the relative momentum offset $\delta_p$ (abscissa) and of the primary collimator aperture $n_1$ (index in the right upper corner of the figure). Each curve is ended at $\delta_p = \delta_c(n_1)$ where $X_{max} = N_{arc} = 11.8$, a case study for LHC.

and define the largest momentum offset which can pass the primary collimator as $\delta_c = n_1/\chi_1$ with $A_{x,\beta} = 0$. After scattering and the cut of the amplitude by the secondary collimators , the maximum horizontal betatronic amplitude is $A_{x,\beta} = [(n_1 - \chi_1\delta_p)^2 + K_c^2]^{1/2}$. Expanding $A_{x,\beta}$ with (6), the maximum horizontal excursion in the arc is

$$X_{max}(n_1, \chi_1, \delta) = \chi_{arc}\delta + (\chi_1^2\delta^2 - 2n_1\chi_1\delta + n_2^2)^{1/2} \qquad (10)$$

and is plotted in Figure 3. The largest allowed excursion $X_{max}(n_1, \chi_1, \delta_c) = N_{arc}$ fixes $\delta_c(n_1) = n_1/\chi_1 = [N_{arc} - (n_2^2 - n_1^2)^{1/2}]/\chi_{arc}$. Would large $n_1$ values be considered, the large $X_{max}$ excursion at small $\delta$ values would be cut at the betatron cleaning insertion. The system is completely fixed by chosing $n_1$ and computing

$$\chi_1(n_1) = \frac{n_1}{\delta_c} = \frac{n_1\chi_{arc}}{N_{arc} - (n_2^2 - n_1^2)^{1/2}}. \qquad (11)$$

As for the choice of $n_1$, a lower limit is fixed by the acceptable effective cut of the horizontal betatronic amplitude at the edge of the bucket $n_{edge} = n_1(1 - \delta_b/\delta_c)$ with $\delta_b$ the bucket width.

## 6 CONCLUSIONS

By using correlated phase advances between primary and secondary collimators in both $x$ and $y$ planes simultaneously, the amplitude of the secondary halo of a two-stage collimation system can be cut down to the aperture of the secondary collimators. The remaining difficult problem is to find an optic satisfying these correlated constraints.

## 7 REFERENCES


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