FREE ELECTRON LASER AND STORAGE RING MICROWAVE INSTABILITY

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Abstract

Recent observations and numerical simulations suggest that the Free Electron Lasers (FEL), operating on a storage ring, may provide a negative feedback for some instabilities.The explanation of this effect is related to the interplay between the FEL induced longitudinal dynamics and the conditions supporting the on set and the growth of the instability. The microwave instability (*M.I.*) is counteracted by the FEL because of the induced energy spread and because of the intensity dependent corrections to the longitudinal damping time.The first mechanism causes a shift of the *M.I.* threshold.The second provides a faster damping of the higher order modes.

In this contribution we show that simple considerations, based on the Boussard criterion,allow to derive the threshold laser power necessary to counteract the instability.

1 INTRODUCTION

The microwave instability (*M.I.*) affects almost all operational Storage Rings (S.R.) and it is recognized as one of the most serious limiting factors in S.R. longitudinal brightness[1].

The Boussard criterion [2] fixes a threshold current above which this instability grows and manifests itself through an "anomalous" increase of the energy spread and bunch lengthening.

According to refs.(1,2) we define the threshold value of the peak current as

$$\hat{I}_{th}[A]=2\pi\cdot 10^{9}\cdot\alpha_c\,\frac{E[GeV]}{\left|\frac{Z_n}{n}[\Omega]\right|}\sigma_{\varepsilon,n}^2 \qquad (1)$$

where α_c the momentum compaction, E the e-beam energy, Z_n/n is the longitudinal broad-band impedance at the nth harmonic of the revolution frequency and $\sigma_{\varepsilon,n}$ is the natural r.m.s. energy spread.

Modern S.R.s, used for the production of synchrotron radiation, exploit low emittances lattices and in particular for a Chasman-Green lattice one gets

$$\alpha_c=\frac{\pi^2}{3\cdot N_d^2}\cdot\frac{\rho_m}{C_R} \qquad (2)$$

with N_d being the number of achromats in the ring lattice, ρ_m is the bending radius and C_R is the circumference of the ring. The natural energy spread can be written as

$$\sigma_{\varepsilon,n}\cong 1.2\cdot 10^{-3}\,\frac{E[GeV]}{\sqrt{\rho_m[m]\cdot J_s}} \qquad (3)$$

with J_s being the longitudinal partition number.By combining eqs.(1-3)we find

$$\hat{I}_{th}[A]\cong 2.976\cdot 10^{4}\,\frac{E[GeV]^3}{N_d^2\cdot J_s\cdot C_R\cdot\left|\frac{Z_n}{n}[\Omega]\right|} \qquad (4)$$

to give an idea of the numbers involved in, we note that by assuming E=1GeV, N_d=20, J_s=2, C_R=100m and Z_n/n=0.5Ω we find a threshold value of $\hat{I}_{th}$=0.7A.

For current values exceeding (4), the energy spread increases and the link with the peak current is provided by

$$\sigma_\varepsilon\cong\frac{3.16}{\sqrt{2\pi}}\cdot 10^{-5}\left[\frac{\left|\frac{Z_n}{n}[\Omega]\right|\cdot\hat{I}[A]}{\alpha_c E[GeV]}\right]^{1/2} \qquad (5)$$

which, on account of eq.(2), yields

$$\sigma_\varepsilon\cong 6.95\cdot 10^{-6}\cdot N_d\cdot\left[\frac{\left|\frac{Z_n}{n}[\Omega]\right|\cdot\hat{I}[A]\cdot C_R[m]}{\rho_m[m]\cdot E[GeV]}\right]^{1/2} \qquad (6)$$

The above relations can be exploited in many flexible ways,in the following we will see that,exploited with basic formulae accounting for the S.R.-FEL physics, we can derive useful and transparent informations on the interplay between FEL and *M.I.* dynamics.

2 FEL AND *M.I.* EFFECTS

According to the discussion of the previous section,we introduce the factor

$$\delta^2=\frac{\hat{I}}{\hat{I}_{th}} \qquad (7a)$$

and note that, if Boussard criterion holds,we also have

$$\sigma_\varepsilon^2=\delta^2\cdot\sigma_{\varepsilon,n}^2 \qquad (7b)$$

As is well known the natural energy spread is due to a balance mechanism between the damping and quantum diffusion effects [3]. The energy spread induced by *M.I.* is due to a self induced single bunch force,caused by the field generated by the electrons interacting with the vacuum pipe. An energy spread, due to a diffusive effect,may combine quadratically with the natural energy spread,shifts the threshold and eventually switch off the instability. This is indeed the case of the FEL and according to eqs.(7) the threshold induced energy spread,to counteract the *M.I.* is provided by

$$\sigma_i^2 = (\delta^2 - 1) \cdot \sigma_{\varepsilon,n}^2 \quad (8)$$

The equilibrium induced energy spread in a *S.R.* is provided by [4]

$$\sigma_{i,e}^2 = \frac{7.47 \cdot 10^{-2}}{N^2} \cdot \frac{\tau_s}{T} \cdot x \quad (9)$$

where N is the number of undulator periods, τ_s is the longitudinal damping time and T the machine revolution period, finally

$$x = \frac{I}{I_s} \quad (10)$$

where I is the intracavity FEL power density and I_s the saturation power density ,which, in practical units, reads

$$I_s \left[\frac{\text{MW}}{\text{cm}^2} \right] = 6.9 \cdot 10^2 \cdot \left(\frac{\gamma}{N} \right)^4 \frac{1}{\left[\lambda_u [\text{cm}] \cdot K \cdot f_b \right]^2}$$

$$f_b = J_0(\xi) - J_1(\xi) \quad (11)$$

$$\xi = \frac{1}{4} \cdot \frac{K^2}{1 + \frac{K^2}{2}}$$

By recalling that [4]

$$I_s \cdot g_o \cdot \Sigma = \frac{1}{2 \cdot N} \cdot P_E \quad (12)$$

where g_o is the small signal gain, Σ is the e-beam cross-section assumed to be matched to the laser beam cross-section and P_E is the e-beam power and since

$$P_E \cdot \frac{T}{\tau_s} = P_s \quad (13)$$

with P_s denoting the power lost by synchrotron radiation, we can combine the previous relations to infer the threshold power necessary to shift the *M.I.* threshold, indeed we get

$$I^* \cong 1.673 \cdot \frac{\delta^2 - 1}{g_o \cdot \Sigma} \cdot \mu_\varepsilon(0)^2 \cdot \left(\frac{1}{4 \cdot N} \cdot P_s \right)$$
$$\mu_\varepsilon(0) = 4 \cdot N \cdot \sigma_{\varepsilon,n} \quad (14)$$

This value can be compared to that corresponding to the S.R.-FEL equilibrium power, namely [4]

$$I_e \cong 1.422 \cdot \frac{\tilde{a}}{g_o \cdot \Sigma} \cdot \mu_\varepsilon(0)^2 \cdot \left(\frac{1}{4 \cdot N} \cdot P_s \right) \quad (15)$$

where ã is provided by the root of the cubic equation

$$(1+\tilde{a}) \cdot \left[1 + 1.7 \cdot \mu_\varepsilon(0)^2 \cdot (1+\tilde{a}) \right]^2 = \frac{1}{r^2}$$
$$r = \frac{\eta}{1-\eta} \cdot \frac{1}{0.85 \cdot g_o} \quad (16)$$

with η being the cavity losses.

Typical values of ã are given in fig.1. It is clear that the switching off the instability requires that

$$\frac{I_e}{I^*} f 1 \quad (17)$$

i.e.

$$\tilde{a} f 1.18 \cdot (\delta^2 - 1) \quad (18)$$

It is evident that δ may be any value larger than 1. To fix a reasonable range of values we remind that, to ensure sufficient gain, the beam energy spread cannot exceed the value

$$\sigma_\varepsilon^* = \frac{0.767}{2 \cdot N} \sqrt{\frac{1-r}{r}} \quad (19)$$

thus finding that δ should range between

$$1 p \delta p \frac{0.767}{4 \cdot N \cdot \sigma_{\varepsilon,n}} \sqrt{\frac{1-r}{r}} \quad (20)$$

An idea of the values of ã is provided by fig. (1), where we have reported this quantity for different values of the cavity losses or of the e-beam energy spread. By assuming $\delta=2$, eq. (18) demands for ã=3.5 to switch off the instability and this value is largely within the S.R.-FEL capabilities.

3 CONCLUDING REMARKS

In the previous sections we have used general considerations to link *M.I.* instability threshold and FEL induced energy spread, the arguments we have given are

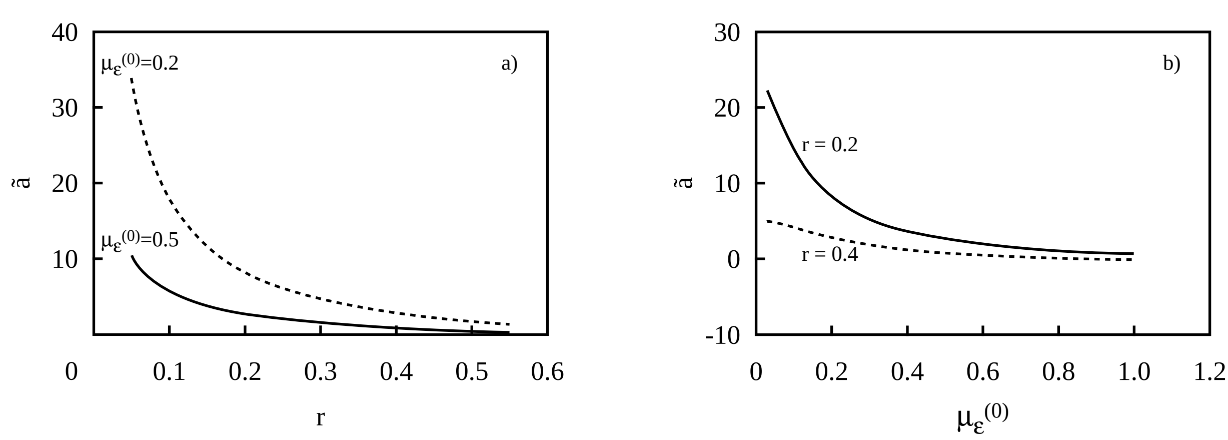

Figure 1: a) Dimensionless intracavity power ã vs r for different $\mu_\varepsilon(0)$ values, b) Dimensionless intracavity power ã vs $\mu_\varepsilon(0)$ for different r values

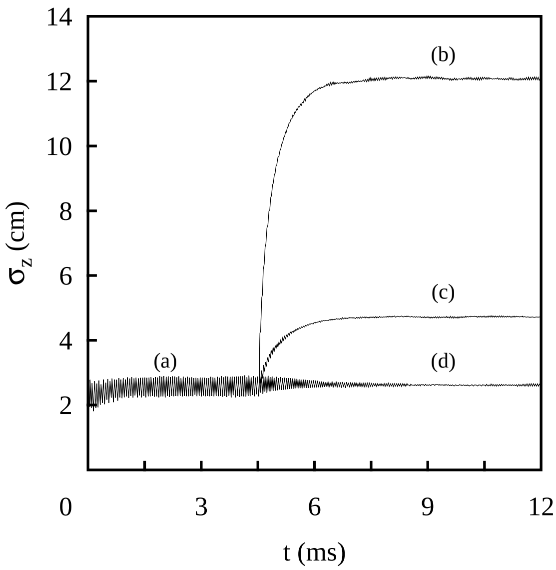

Figure 2: Bunch length evolution vs time a) evolution dominated by M.I. only; b) the FEL interaction is switched on at t=4.5 ms with x~100x*, (x*=I*/I_S see eqs. (10) and (14)); c) same as b) with x~10x*; d) same as b) with x~x*

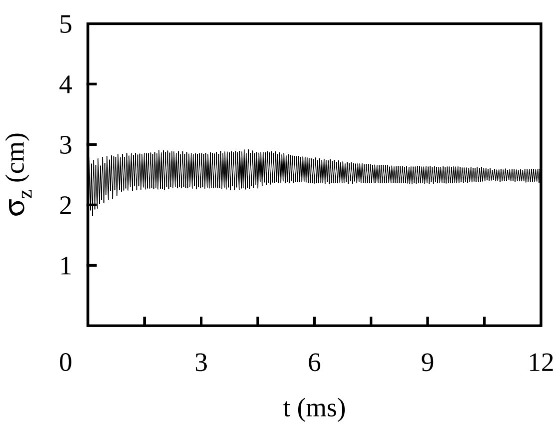

Figure 3: Same as fig. 2 but x=x*/2 (values of x=x*/10 leaves the evolution unaffected)

fairly simple and it may be argued that we may have left out elements which may significantly modify our prediction. We have therefore tested the validity of our model,by using a completely numerical procedure,and the results of the numerical experiment are summarized in fig. 2, which shows the evolution of the r.m.s. bunch length (recall that r.m.s.bunch length and r.m.s. energy spread are proportional) for a S.R. e-beam affected by *M.I.*.The evolution is very noisy and the bunch value is twice larger than the natural value.When the FEL interaction is switched on,it may happen that if the power is too large,large energy spread (and thus a corresponding bunch lengthening) is induced.The evolution becomes however more regular. The bunch length does not increase if the amount of laser power is that predicted by eq. (14) but the level of noise,characteristic of the instability ,has been completely eliminated. In figure 3) we have considered values below the power threshold and it is evident that the effect of the instability are not counteracted any more.

The results of this paper indicates that the FEL may play, within the context of S.R.,the role of machine element useful to inhibit the growth of *M.I.*; further elements supporting this point of view will be presented elsewhere

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