ANALYTICAL THEORY OF MULTIPASS CRYSTAL EXTRACTION

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Abstract

An analytical theory for the efficiency of particle extraction from an accelerator by means of a bent crystal is proposed. The theory agrees with all the measurements performed in the broad energy range of 14 to 900 GeV, where the efficiency range also spans over two decades, from ∼0.3% to ∼30%. Possibilities for crystal extraction from sub-GeV accelerators and from muon colliders are discussed.

1 INTRODUCTION

Crystal extraction experiments have greatly progressed at high energy accelerators in recent years [1,2]. The experimental data are in good agreement with predictions from the detailed Monte-Carlo simulations [3,4]. Although the transmission of particles by a bent crystal can be described analytically with good accuracy, the process of extraction involves essentially multiple encounters of circulating particles with the crystal and many turns in the accelerator, and therefore its efficiency cannot be scaled easily, for instance with energy. An analytical theory of multipass crystal extraction would be highly helpful in understanding the existing experimental results, in extrapolation to future applications, and in optimization. Below we derive an analytical formula for the crystal extraction efficiency.

2 THEORY

Suppose that a beam with divergence σ, Gaussian distribution, is aligned to the crystal planes. Then as many as $(2\theta_c/\sqrt{2\pi}\sigma)(\pi x_c/2d_p)$ particles get channeled in the initial straight part of the crystal. Here θ_c stands for the critical angle of channeling, d_p the interplanar spacing, $x_c \approx d_p/2 - a_{TF}$ the critical distance, a_{TF} being the Thomas-Fermi screening distance.

We shall first consider the more typical case, where particles first come to the crystal with nearly zero divergence, due to very small impact parameters. As experiments indicate [1,2], in this first passage the channeling is suppressed, apparently due to the poor quality of the crystal structure near surface. In our model we assume that the first passage of particle through the crystal is always "inefficient", i.e. there is no channeling, but there is scattering and possibly nuclear interactions.

After some turns in the accelerator ring, the scattered particles come to the crystal with rms divergence as defined by scattering in the first pass: $\sigma_1 = (E_s/pv)(L/L_R)^{1/2}$, where E_s=13.6 MeV, L is the crystal length, L_R the radiation length, pv the particle momentum times velocity. In a real experiment, σ_1 may be affected by betatron oscillations, and also by the fact that in the first passage the particle may enter the bent crystal quite near its surface and hence leave the crystal before crossing its full length L. These complications are to be taken into account in the detailed Monte Carlo simulations [3,4], as well as the apertures, etc. However, our objective is to derive a simple analytical theory which includes only the basic physical parameters of crystal extraction process, and to see how far it goes. We assume then that any particle always crosses the full crystal length; that pass 1 is like through an amorphous matter but any further pass is like through a crystalline matter; that there are no aperture restrictions; and that the particles interact only with the crystal not a holder.

After k passes the divergence is $\sigma_k = k^{1/2}\sigma_1$. The number of particles lost in nuclear interactions is $1-\exp(-kL/L_N)$ after k passes; L_N is the interaction length. In what follows we shall first assume that the crystal extraction efficiency is substantially smaller than 100% (which has actually been the case so far), i.e. the circulating particles are removed from the ring predominantly through the nuclear interactions, not through channeling.

That pulled together, we obtain the multipass channeling efficiency by summation over k passes:

$$F_C = \left(\frac{\pi}{2}\right)^{1/2} \frac{\theta_c x_c}{\sigma_1 d_p} \times \Sigma(L/L_N) \qquad (1)$$

where

$$\Sigma(L/L_N) = \Sigma_{k=1}^{\infty} k^{-1/2} \exp(-kL/L_N) \qquad (2)$$

may be called a "multiplicity factor" as it just tells how much the single-pass efficiency is amplified in multipasses.

A fraction of channeled particles is to be lost along the bent crystal due to scattering processes and centripetal effects. The transmission factor for the channeled particles in a bent crystal we denote as T. Then the multipass extraction efficiency is

$$F_E = F_C \times T = \left(\frac{\pi}{2}\right)^{1/2} \frac{\theta_c x_c}{\sigma_1 d_p} \times \Sigma(L/L_N) \times T \qquad (3)$$

We shall use an analytical approximation (as used also in [6]) for silicon

$$T = (1-p/3R)^2 \exp\left(-\frac{L}{L_d(1-p/3R)^2}\right), \qquad (4)$$

where p is in GeV/c, and R is in cm; L_d is dechanneling length for a straight crystal. The first factor in T describes

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a centripetal dechanneling. E.g., at pv/R=0.75 GeV/cm (which is close to the highest values used in extraction) our approximation gives $(1-p/3R)^2$=0.563 whereas Forster et al. [7] measured 0.568±0.027. The dechanneling length L_d we describe by the theoretical formula[9].

To have all formulas explicit, we give an analytical expression for the sum (2), for $L \ll L_N$:

$$\Sigma(L/L_N) \simeq (\pi L_N/L)^{1/2} - 1.5 \tag{5}$$

3 EXPERIMENTAL CHECK

Let us check the theory, first against the CERN SPS data [10] where the crystal extraction efficiency was measured at 14, 120, and 270 GeV, making use of the same 4-cm long Si(110) crystal, deflecting at 8.5 mrad. The crystal had 3-cm long bent part with two 5-mm straight ends, having in the center pv/R=(0.34 GeV/cm)×(pv/120 GeV). We take θ_c=13.8 μrad×(120 GeV/pv)$^{1/2}$ (as used by the authors of Ref.[10]). The dechanneling length for a straight crystal is taken as 0.569×270=154 mm at 270 GeV, 0.603×120=72.4 mm at 120 GeV, and 0.718×14=10.1 mm at 14 GeV. This length is reduced in a bent crystal by a factor of $(1-p/3R)^2$, Eq.(4). Only the dechanneling over 35 mm is taken into account, as the last 5-mm end is unbent. Table 1 shows good agreement of theory with measurements.

Table 1: Extraction efficiencies (%) from the SPS experiment, Eq.(3), and detailed simulations [11].

pv(GeV)	SPS	Eq.(3)	Monte Carlo
14	0.55±0.30	0.30	0.35±0.07
120	15.1±1.2	13.5	13.9±0.6
270	18.6±2.7	17.6	17.8±0.6

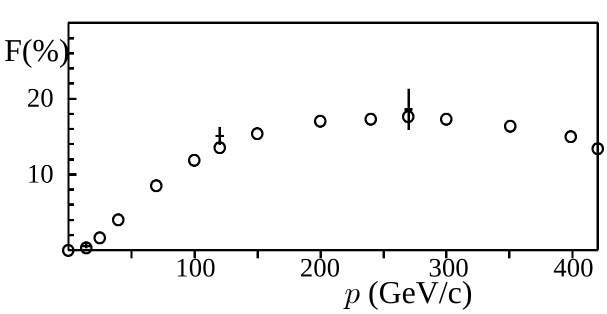

Figure 1: The SPS extraction efficiency as a function of momentum p. The curve (o) is for Eq.(3), the crosses at 14, 120 and 270 GeV/c are for the SPS experiment.

The Tevatron extraction experiment at 900 GeV provides another check at a substantially higher value of efficiency. Here a slight modification of the formulas is needed to account for the non-zero starting divergence, namely σ_0=11.5 μrad (rms). This results in the change in Eq.(2):

$$\Sigma(L/L_N) = \Sigma_{k=1}^{\infty}(k+\sigma_0^2/\sigma_1^2)^{-1/2}\exp(-kL/L_N) \tag{6}$$

Since in this experiment Si(111) planes were used, consisting of narrow (1/4 weight) and wide (3/4 weight) channels, this is to be taken into account in Eq.(3) with respective change in d_p and x_c. The crystal used at the Tevatron was 4 cm long with 8-mm straight ends, having in the center pv/R=0.29 GeV/cm. The theoretical dechanneling length for a straight crystal of Si(111) is 0.646×900 GeV=581 mm; notice that for (111) it is factor of d_p^{111}/d_p^{110}=1.23 higher than for (110). We take into account the dechanneling over 32 mm, as the last 8-mm part is unbent. Eq.(3) then gives an extraction efficiency of 40.8 %. However, a minor correction is discussed below.

Let us note that as the extraction efficiency is getting high, our earlier assumption that the nuclear interactions dominate over the crystal channeling may need correction. To take into account the fact that the circulating particles are efficiently removed from the ring by a crystal extraction as well, one would require a *recurrent* procedure of summation: instead of ΣF_k one has to sum ΣF_k^*, where F_k^*=$F_k(1-F_{k-1}^*)$. This "recurrent" correction doesn't practically affect our earlier SPS calculations; for Tevatron it converts 40.8% into 34.1%, whereas the measured value is on the order of 30% [13], and the Monte Carlo simulation predicted about 35% [3].

4 OPTIMIZATION

For any given energy one can optimize the crystal length L. In optimization we assumed the same proportion between the bent part and the full crystal length, 3 to 4. At 270 GeV the length used at the SPS was close to optimal, 3.0±0.5 cm. At 120 GeV the optimal length is 1.5±0.5 cm resulting in the efficiency of 28%; at 70 GeV it is 0.8±0.2 cm with best efficiency of 38%. The lower energy permits a shorter crystal, and then the multiplicity factor becomes substantial. In Fig.2 one can see that the analytical dependences $F_E(L)$ are very close to those obtained earlier in Monte Carlo simulations [4]. The same maxima at the same optimal lengths are predicted. One obvious conclusion is that the crystal extraction experiments at the SPS and Tevatron have been working rather far from the optimum, so there is a good possibility for improvement. Formula (3) predicts a good efficiency of multipass extraction at a multi-TeV LHC, about 45% for 0.7 mrad deflection, with the optimal length of Si(110) crystal being 6±1 cm.

5 NEW APPLICATIONS

Let us mention here two interesting developments. From Eq.(5) we see that multiplicity factor can be huge if L is very small or L_N big.

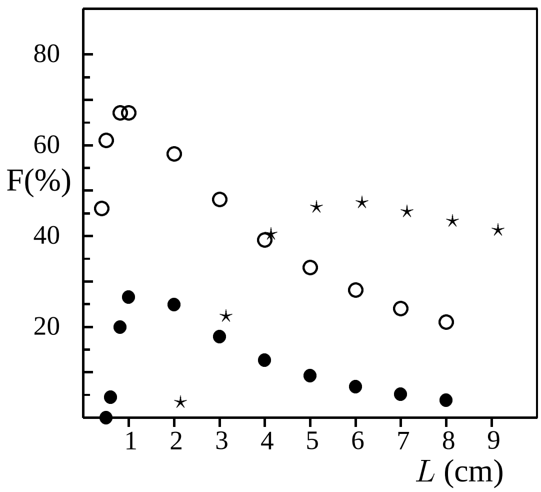

Figure 2: The extraction efficiency, Eq.(3), as a function of the crystal length L; for the SPS (•), Tevatron (o), and Large Hadron Collider (⋆).

MeV extraction. One opportunity (small L) is inspired by the recent successful experiment [14] on bending 3-MeV proton beam by means of graded composition $Si_{1-x}Ge_x$/Si strained layers. These epitaxial layers formed a bent crystal lattice of uniform curvature with a thickness along the beam direction of only L=1 micron (but much bigger across the beam)! This technique allows to grow curved crystals of any size from μm to cm's, though the bending angle achievable is limited to few mrad's [14]. This invention allows to cover the whole spectrum of accelerator energies (from MeV to multi-TeV) by bent crystal channeling technique. In the context of our paper this means that one can consider extraction from accelerators starting with MeV energies, by crystals as short as from 1 μm. Eqs.(1-2) predict that channeling efficiency over 99% can be achieved in sub-GeV (and up to several GeV) range, thus opening a new world for bent channeling crystals applications. With traditional bent crystals, it was common to think that highest efficiencies are achievable at highest (TeV) energies as multiple scattering angles vanish with energy faster than channeling angle does. It's very interesting now to find that channeling efficiency is even more boosted at lower energies due to huge multiplicity factor. One can build a very efficient system to extract beams from accelerators with crystals.

Muon extraction. The other opportunity (big L_N) for high multiplicity factor is muons which have formally $L_N = \infty$. Our theory then says that factor (5) for muons is infinite, and hence efficiency of muon channeling should be 100%. Actually the multiplicity factor for muons is limited by (a) muon lifetime, (b) muon scattering out of accelerator aperture. Quick analysis shows that the first factor dominates. With a muon mean lifetime of 1000 turns in a 2×2 TeV muon collider [15], the typical number of encounters with crystal is ∼300. This is much greater than the corresponding quantity for protons (as limited by L_N).

At muon machine, the backgrounds "have the potential of killing the concept of the muon collider"[15]; one needs a very efficient scraping system to catch muon beam halo. As muons cannot be absorbed, it was proposed[16] to extract 2-TeV muons with electrostatic septum as a primary element. Surely, positive muons can be easily steered away by bent channeling crystals. But can we steer negative muons? Short analysis gives a very encouraging answer. One can channel negative particles in the same bent planes as used for positive ones, e.g. Si(110) [17]. In same crystal Si(110), dechanneling length L_d is shorter by factor of ∼100 for negative particles relative to positive ones. However, at 2 TeV L_d is huge (∼1 meter) for positives and modest (∼1 cm) for negatives. The required deflection angle is only 64 μrad[16] and can be ensured by a Si crystal ∼1 mm long—quite shorter than L_d. Let us say it simply: it is as easy to bend negatives at 64 μrad as it is to bend positives at 6.4 mrad—which is very easy indeed! Now let us recall that multiplicity factor greatly favors muons again, both positive and negative. One can build a very efficient system to handle halos at muon colliders.

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