# ACCURATE AND EFFICIENT COMPUTATION OF SYNCHROTRON RADIATION IN THE NEAR FIELD REGION

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## Abstract

A computer code called Synchrotron Radiation Workshop (SRW) is presented. The code computes the synchrotron radiation from relativistic electrons with high precision and efficiency in the near and far field range. It accepts arbitrary magnetic field description which includes undulators, wigglers, bending magnets (central part and edges), quadrupoles, etc. The polarization, spatial and angular intensity and phase of the radiation, from the millimetre to very hard X-ray range, are accurately computed for a "filament" or "thick" electron beam. For long wavelengths, CPU-efficient SR propagation is implemented using Fourier Optics approach, which handles any number of drift spaces, diffracting apertures, lenses or focusing mirrors. Two examples of the computation illustrating some peculiarities of focusing the bending magnet and undulator radiation are presented.

## 1 INTRODUCTION

The general purpose of the SRW project is to provide users with a collection of computational tools for various applications related to utilization of Synchrotron Radiation (SR). For many applications, it is important to know not only the basic SR characteristics, such as spectrum, flux per unit surface (intensity) or brightness, but also to be able to trace how the SR transforms at propagation through various optical components, and what are the final characteristics of the radiation "on a detector".

The present version of SRW combines in the same code:

- high-accuracy computation of the near-field SR emitted by an electron beam in a magnetic field of nearly arbitrary configuration;
- CPU-efficient propagation of the near-field SR through simple optical elements (thin lenses, focusing mirrors, apertures) and drift spaces.

These two subjects were addressed separately by different authors, at different levels of approximation. The near-field computation of undulator radiation was discussed in [1]; the SR propagation-related topics were treated in very remarkable works [2] and [3].

The computation methods of the near-field SR emission and propagation implemented in the SRW are briefly described in the next section. Section 3 presents two examples of the computation. Finally, the main advantages, problems and limitations of the methods in use are summarized in section 4.

The SRW code is freely available for download from the ESRF Web site [4].

## 2 COMPUTATION METHODS

### 2.1 SR Emission in the Near Field

To compute the near-field SR emission by a single electron in frequency domain, an approach based on retarded potentials is applied [5].

Starting from Fourier transformations of the retarded scalar and vector potentials, one can easily obtain the following expression for the electric field of radiation emitted by a relativistic electron (Gaussian System):

$$\vec{E} = iek \int_{-\infty}^{+\infty} [\vec{\beta} - \vec{n}\,[1 + i\,(kR)^{-1}]]\, R^{-1} \exp[ik(c\tau + R)]\, d\tau, \quad (1)$$

where $k$ is a wave number, $\vec{\beta} = \vec{\beta}(\tau)$ instant relative velocity of electron, $\vec{n} = \vec{n}(\tau)$ unit vector directed from instant electron position to an observation point, $R = R(\tau)$ distance from the electron to the observation point, $c$ speed of light, $e$ charge of electron.

It can be shown that for a non-zero frequency, the method based on the Eq. (1) is analytically equivalent to the more widely used method which treats separately the acceleration and the velocity fields [6]. From a numerical point of view, we believe that the method based on the Eq. (1) is more efficient than the computation made by summing up the acceleration and velocity field contributions.

In the current realization, the code takes into account only transverse components of the electric field of radiation, assuming $kR \gg 1$.

### 2.2 SR Propagation

The SR propagation is implemented in the frame of the Scalar Diffraction theory using the Fourier Optics methods.

Assuming that angles are small and distances are considerably larger than wavelength, the transverse component of the electric field of diffracted synchrotron radiation $\vec{E}_{\perp 2}$ can be computed from the electric field $\vec{E}_{\perp 1}$ at an aperture causing the diffraction by the well-known Huygens-Fresnel principle[1]:

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[1] This can be shown by applying the integral theorem of Helmholtz and Kirchhoff [7] to the synchrotron radiation described, for example, by the expression following from the retarded potentials.

$$\vec{E}_{\perp 2} = -ik(2\pi)^{-1} \iint_\Sigma \vec{E}_{\perp 1} \, S^{-1} \exp(ikS) \, d\Sigma, \qquad (2)$$

where $k$ is a wave number, $\Sigma$ is a surface within the diffracting aperture, $S$ is a distance from a point on this surface to an observation point.

If $\Sigma$ is (a part of) a plane perpendicular to the optical axis, and the observation points belong to another plane which is also perpendicular to the optical axis, then the Eq. (2) is a convolution type integral that can be quickly computed by applying the convolution theorem and 2D Fast Fourier Transforms. This gives a CPU-efficient method of propagation of the SR wave through a drift space.

The propagation of the transverse electric field through a perfect thin lens is described well by a multiplication of the field by a function of transverse coordinates [3]. For more complicated optical components, one can apply physical considerations or analytical methods (for example, the stationary phase method) to derive a proper transformation of the electric field from a transverse plane before the optical component to a plane immediately after it.

## 2.3 Non-zero Emittance of Electron Beam

A non-zero transverse emittance of electron beam can be taken into account in the code by making a convolution over transverse coordinates of the single-electron intensity distribution, with a 2D Gaussian. The rms values of this Gaussian are given by the electron beam sizes propagated through all the optical components to the observation plane, using the rules of the second-order moment propagation.

This method is valid only for the cases when a magnetic field is transversely uniform in the region of the SR emission and the propagated SR distribution is not strongly dominated by diffraction.

## 3 EXAMPLES

### 3.1 Focusing the Bending Magnet Radiation

This example illustrates a case of practical importance for transverse size measurements of electron beam. The visible-range bending magnet radiation is focused by a lens in order to produce an image of the emitting electron beam. The beam energy is 6 GeV, bending magnet field 0.85 T, radiation wavelength 415 nm. The distance from the geometrical source point to the lens is 20 m, and 10 m from the lens to the observation plane (2:1 imaging). A rectangular aperture of 40 mm x 40 mm is placed in front of the lens.

The computed intensity distributions of horizontal and vertical polarization components of the focused SR are presented in Fig. 1. In addition, Fig. 1 shows the vertical profiles of the intensity for a "filament" and a "thick" electron beam. This illustrates the possibility of using the

vertical polarization component of the bending magnet SR for a more precise estimation of a small vertical electron beam size [8].
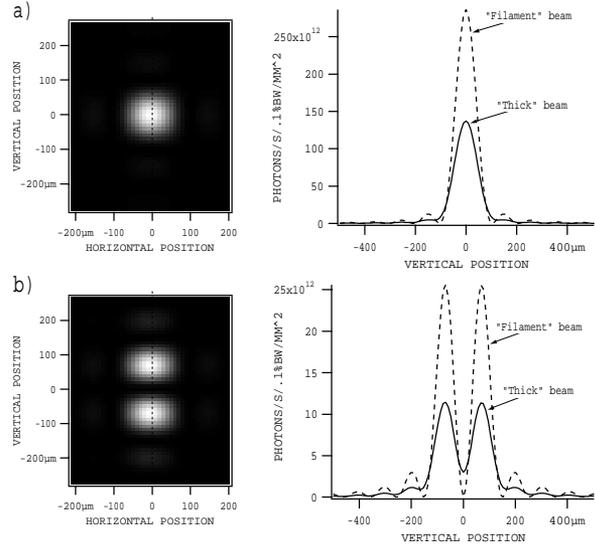


Figure 1. Intensity distributions of the focused bending magnet SR: a) horizontal, b) vertical polarization components. Left: image plots of the distributions for a "filament" electron beam; right: vertical intensity profiles for a "filament" and a "thick" electron beam (in this example, horizontal and vertical rms beam sizes are 115 and 45 μm).

### 3.2 Focusing the Undulator Radiation

This example reveals some peculiarities of focusing the undulator radiation. A 6 GeV electron beam travels in an undulator (46 periods of 35 mm, K = 2.2). The emitted radiation is collected at a distance of 10 m from the source by a thin focusing lens with the focal length of 5 m and observed in a plane located 10 m downstream from the lens (1:1 imaging). Figure 2 presents the spectral flux per unit surface observed in the plane of the lens (a) and in the image plane (b, c) for the photon energies of 8.00, 8.05 and 8.10 keV. The value of 8.10 keV corresponds to the on-axis resonant energy of the third harmonic.

As expected, in the plane of the lens, the highest intensity (i.e., flux per unit surface) is reached around 8.10 keV. However, in the image plane, the highest intensity occurs around 8.05 keV. Clearly, the highest intensity in the image plane is obtained for a photon energy for which the initial spectral angular distribution presents a ring structure rather than a narrow cone. Good focusing of undulator radiation with annular spectral angular distribution was pointed out in [2], where the consideration was done in the Fraunhofer approximation, for an "infinitely long" undulator.

Figure 2c presents the intensity distrubutions in the image plane taking into account the finite electron beam

emittance of the ESRF (horizontal and vertical emittances: 4 and 0.04 nm, beta functions: 0.5 and 2.5 m). The spot size in the image plane is dominated by the electron beam size rather than the single electron emission, and the peak value of the flux per unit surface becomes proportional to the angle integrated flux, which is also more favorable around 8.05 keV rather than 8.10 keV by a factor of ~1.5.
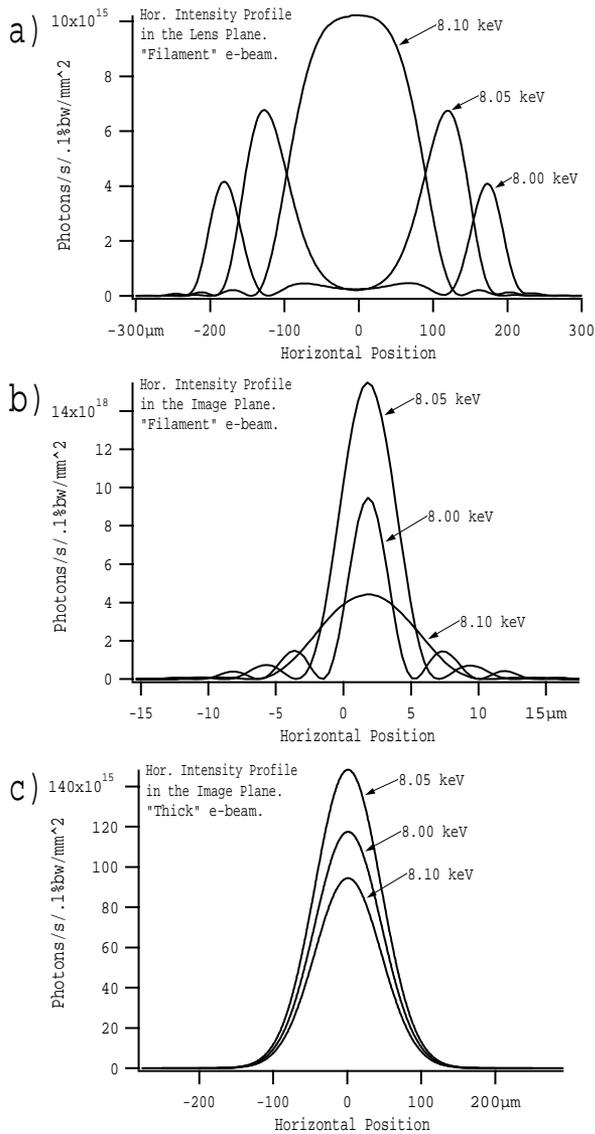


Figure 2: Spectral flux per unit surface vs horizontal position at several photon energies around the third harmonic of emission from an ESRF undulator: (a) at a distance of 10 m from the source for a filament electron beam; (b) in the plane of a 1:1 imaging for a filament electron beam; (c) in the same image plane taking into account the electron beam emittance.

# 4  ADVANTAGES, PROBLEMS & LIMITATIONS

We emphasize the generality and high accuracy of the near-field emission and propagation computation methods used in SRW. Magnetic fields of nearly arbitrary configurations are supported. The radiation can be computed at small distances from the emission region. The theoretical limits of the propagation are essentially those of the Scalar Diffraction theory.

An advantage of the SR propagation method is speed. An SR wave front can be propagated through several optical components and drift spaces for the time from seconds to a minute at 200 MHz CPU clock (which is normally faster than the initial computation of the near-field SR emission). This may allow various optimizations of a beamline.

When computing the SR propagation, one needs to sample the electric field of the wave front over a sufficiently narrow grid in such a way that the phase shift between adjacent points is less than $\pi$. For short wavelengths or long drift spaces this may require thousands of points in both the horizontal and vertical transverse direction, resulting in a very large consumption of memory. For this reason, we believe that the SR propagation, as it is implemented in the current version of the SRW, can be essentially useful for the IR to VUV wavelengths. Nevertheless, a few isolated cases, such as propagation of the central cone of undulator radiation can be computed also in the hard X-ray range.

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