BEAM-BEAM INSTABILITY IN PRESENCE OF BEAM COOLING

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Abstract

Proposed RIKEN Radioactive Isotope Beam Factory [1] is aimed to be used for different kind of collision experiments with unstable ions (see Table 1). Luminosity of collider is proportional to beam-beam parameter, which does not exceed the small value of 0.005 in existing hadron colliders. Implementation of ion beam cooling is expected to be a way to suppress beam-beam instability and to enlarge the maximum achieved value of beam-beam parameter. In this case luminosity can be increased several times as compare with that without cooling. Analytical and numerical treatment of beam-beam interaction in presence of beam cooling is given.

1 LUMINOSITY LIMITATION

Let us consider head-on collisions of strong electron beam against counteracted beam of heavy ion particles with the charge to mass ratio Z/A. Luminosity of an electron-ion collider is given by

$$L = \frac{f\, N_i\, N_e}{4\pi\sigma^2} N_{bunch}\,, \qquad (1)$$

where f is the particle revolution frequency in a ring, N_i and N_e the numbers of ions and electrons per bunch, respectively, N_{bunch} the number of bunches per beam, and σ is the rms (root-mean-square) size of colliding beams.

In existing hadron colliders luminosity is mainly limited by the beam-beam effect, which is expressed as reduction of beam lifetime under particle collisions. Significance of the beam-beam phenomena is characterized by the value of beam-beam parameter ξ, which has a meaning of the linear part of betatron tune shift due to beam-beam collisions:

$$\xi = \frac{r_p\, \beta^* \frac{Z}{A} N_e}{4\pi \gamma_i \sigma^2}\,, \qquad (2)$$

where $r_p = e^2/ 4\pi\varepsilon_o mc^2$ is the value of classical radius of proton, β^* the value of a beta-function of the collider at interaction point (IP), and γ_i the ion beam energy. In existing ion colliders the maximum value of ξ does not exceed the magnitude of ξ=0.005. Limitation in ξ results in constraints of the luminosity L, which follows from Eqs. (1) and (2):

$$L < \frac{f\, N_i\, N_{bunch}\, \gamma_i}{\beta^*\, r_p} \frac{A}{Z} \xi_{max}\,. \qquad (3)$$

Table 1. Parameters of proposed RIKEN Double Storage Ring collider [1].

Circumference	258m
Beam Energy	
proton (GeV)	3.55
ion (Z/A=0.5), Gev/nucleon	1.45
ion (Z/A = 0.387), Gev/nucleon	1.00
Number of ion bunches	50-100
Betatron tune value (Q_x/Q_y)	6.668/5.661
Beta function at Interaction Point (β_x^* / β_y^*, m)	0.1/0.1
Beam sizes at IP ($\sigma_x/\sigma_y/\sigma_z$, mm)	0.4/0.4/100
Luminosity, e^- - ion	10^{26}- 10^{31} $sec^{-1}cm^{-2}$

2 BEAM-BEAM INSTABILITY

Beam-beam instability is usually attributed to excitation of set of nonlinear resonance islands, which, being overlapped, create stochastic particle motion. Another factor in instability of collided particles is the noise in beam-beam interaction. Noise diffusion instability can be easily detected via numerical calculations as well as in analytical study under a fluctuation in the opposite beam size $\sigma_n = \sigma_o (1 \pm u\cdot u_n / 2)$, where u is a noise amplitude and u_n is a uniform random function with unit amplitude. Beam emittance growth after n turns under noise regime is given by [2]

$$\frac{\varepsilon_n}{\varepsilon_o} = \sqrt{1 + D\, n}\,, \qquad (4)$$

where the diffusion coefficient D is a function of the beam-beam parameter, noise amplitude, and ratio of ion beam size, a, to electron beam size, 2σ:

$$D = \pi^2 (\xi u)^2 \left(\frac{a}{2\sigma}\right)^4 . \qquad (5)$$

As it follows from Eq. (5), a noise beam-beam instability can exist under any value of the beam-beam parameter ξ. This can be one of the reasons why a small value of $\xi$was achieved in existing colliders. Two conditions are essential to initiate a noisy beam-beam instability: beam-beam kick has to be a nonlinear function of coordinate and the parameter of the kick (rms beam size σ) has to be subject to noise. In contrast to the stochastic particle motion due to overlapping of nonlinear beam-beam resonance islands, a noise beam-beam instability appears in much more simple conditions without excitation of resonances.

Actual value of the noise amplitude in beam size is not known. It can be estimated from indirect data. In HERA collider [3], beam emittance growth due to particle collisions at the value of beam-beam parameter $\xi = 0.0013$ is

$$\frac{\Delta \varepsilon}{\Delta t} \approx 1 \frac{\pi \text{ mm mrad}}{\text{hour}} . \qquad (6)$$

Initial value of beam emittance is $\varepsilon_o = 18.2\ \pi$ mm mrad. Therefore, after one hour of operation, increment of emittance is $\varepsilon_n / \varepsilon_o = 1.05$. Frequency of particle rotation is $f_o = 4.7 \cdot 10^4$ Hz, therefore, during one hour particles perform $n = 3600 \cdot 4.7 \cdot 10^4 = 1.7 \cdot 10^8$ revolutions. Estimation of diffusion coefficient using Eq. (4) gives

$$D = \frac{(\varepsilon_n/\varepsilon_o)^2 - 1}{n} = 6 \cdot 10^{-10} , \qquad (7)$$

and expected value of noise amplitude from Eq. (5) is:

$$u = \frac{\sqrt{D}}{\pi \xi} = 6 \cdot 10^{-3} . \qquad (8)$$

3 BEAM COOLING

Electron cooling technique is an effective method to decrease the phase space volume of the beam. Cooling time is estimated by the formula [4]:

$$\tau_{cool} = \frac{e \beta^4 \gamma^5}{4 \pi r_e r_i \eta J_e L_c} \left[\frac{\varepsilon_H}{4\beta_H} + \frac{\varepsilon_V}{4\beta_V} + \frac{1}{\gamma^2} \left(\frac{\sigma_p}{p}\right)^2 \right]^{3/2} , \qquad (9)$$

where η is a ratio of cooling section to the ring circumference, J_e is a current density of the electron beam, L_c is the Coulomb logarithm and the expression in square brackets is a temperature of the ion beam.

Let us define the effect of cooling on the beam-beam instability. Decrease in the beam emittance due to cooling at the initial stage can be described by

$$\frac{\varepsilon_n}{\varepsilon_o} = \exp\left(- \frac{n}{N_{damp}}\right) \approx 1 - \frac{n}{N_{damp}} , \qquad (10)$$

where $N_{damp} = \tau_{cool} \cdot f$ is a number of turns required to cool the ion beam. On the other hand, beam emittance growth due to noisy beam-beam instability is described by Eq. (4) as

$$\frac{\varepsilon_n}{\varepsilon_o} \approx 1 + \frac{D\, n}{2} . \qquad (11)$$

Therefore, the required cooling rate to prevent beam-beam instability, expressed in cooling number of turns, is:

$$N_{damp} = \frac{2}{D} . \qquad (12)$$

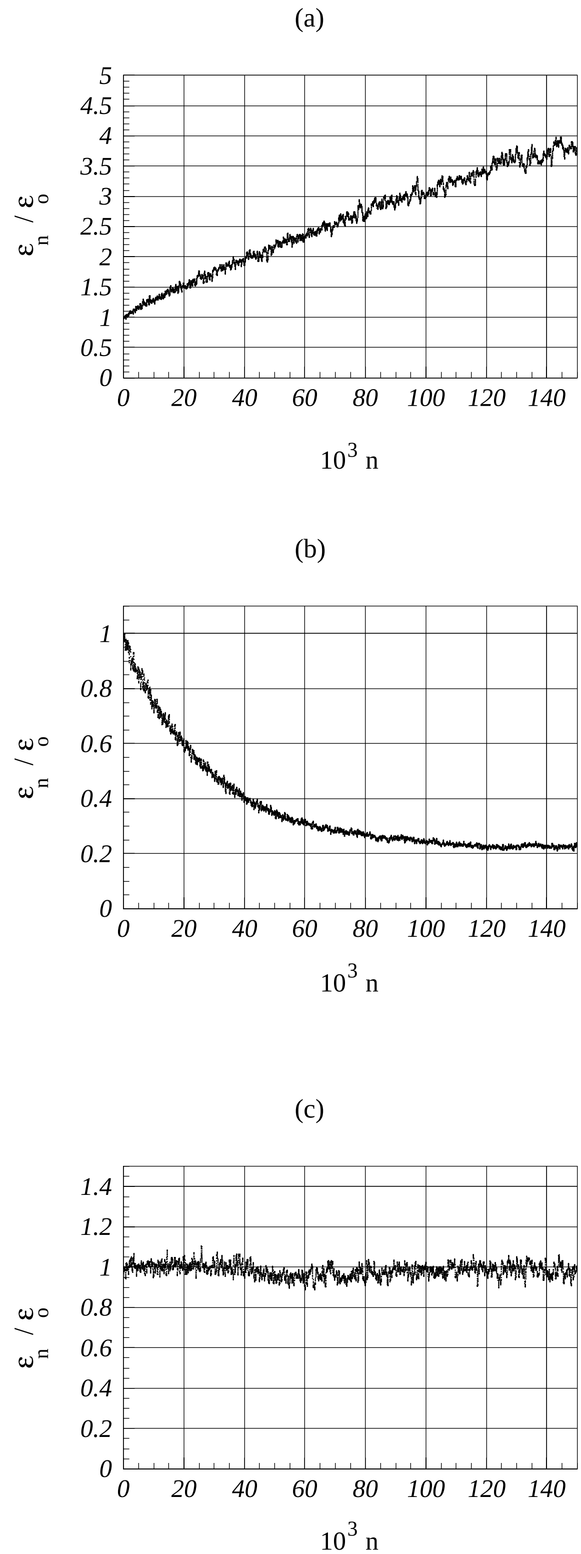

Fig. 1. Beam emittance growth under
(a) noise beam-beam instability,
(b) beam cooling without beam-beam collisions,
(c) combined effect of beam cooling and beam-beam interaction: equilibrium.

Taking the expected number of $N_{damp} = 2\cdot 10^7$, which corresponds to cooling time of 20 sec in the Duoble Storage Ring, and the noise amplitude $u = 6\cdot 10^{-3}$, the maximum value of beam-beam tune shift achieved is

$$\xi_{max} = \frac{1}{\pi u}\sqrt{\frac{2}{N_{damp}}} = 0.017 . \qquad (13)$$

Luminosity is proportional to the maximum value of beam-beam parameter, Eq. 3. Therefore, utilizing the beam cooling can result in increasing of luminosity at least by a factor of 4.

4 NUMERICAL EXAMPLE

Let us consider a numerical model, combining betatron particle motion in the presence of beam-beam collisions with particle cooling:

$$\begin{pmatrix} x_{n+1} \\ p^x_{n+1} \end{pmatrix} = \exp\left(-\frac{1}{N_{damp}}\right) \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x_n \\ p^x_n + \Delta p^x_n \end{pmatrix} + M_x,$$

$$(14)$$

$$\begin{pmatrix} y_{n+1} \\ p^y_{n+1} \end{pmatrix} = \exp\left(-\frac{1}{N_{damp}}\right) \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} y_n \\ p^y_n + \Delta p^y_n \end{pmatrix} + M_y,$$

where θ is a betatron angle, and Δp^x_n a beam-beam kick given by

$$\Delta p^x_n = -4\pi\xi x \frac{1 - \exp(-r^2/2\sigma^2_n)}{(r^2/2\sigma^2_n)} , \qquad (15)$$

analogously for Δp^y_n. Matrices M_x, and M_y describe the ion beam excitation due to scattering [5]

$$x \to x + \lambda_1 \hat{r}_1 , \qquad p_x \to p_x + \lambda_2 \hat{r}_2 ,$$
$$(16)$$
$$y \to y + \lambda_3 \hat{r}_3 , \qquad p_y \to p_y + \lambda_4 \hat{r}_4 ,$$

where λ_i (i = 1,..., 4) are parameters defining the final equilibrium phase space volume of the beam, $\hat{r}_i$ the Gaussian random numbers with average value <r> = 0 and rms value $<\hat{r}^2> =1$.

Fig. 1 presents results of the numerical simulation of combined effect of beam-beam interaction and beam cooling. The value of noise amplitude, u = 2.5%, as well as number of dumping turns, $N_{damp} = 5\cdot 10^4$, and other parameters were taken arbitrary to demonstrate the main features of the process. As can be seen, without cooling, the beam emittance monotonously increases due to beam-beam collisions, while with a cooling, an equilibrium in the beam size is achieved if cooling rate is equal to the increment of beam-beam instability.

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