BEAM LINES DESIGN CODES: DYNAMIC MODELING APPROACH

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Abstract

This paper presents the new codes for design of beam lines based on the dynamic modeling approach. This approach uses three levels of modeling process which operate by the following objects: physical, mathematical (in symbolic form) and computer ones. The human man–machine interface is described.

1 THE BASIC DEFINITIONS AND CONCEPTS

Constructing of effective software in the field of beam physics is actual today. Such codes are used for exploring and developing beamlines of different types. There is a set of complex approaches for the process of model creation, so a researcher (designer) has to know different fields of science (not only physics, but numerical analysis, control theory and etc.). In this way it is obviously that corresponding computer codes should have a natural interface (friendly for users) for manipulation by the calculation and graphics computer resources. Here we are presented one of the possible ways for such kind of software creation.

1.1 Mathematical Tools

The necessary mathematical tools are based on the Lie algebraic methods, in the first place on the Lie transformation (map) ideology in the matrix representation [1].

The Lie Transformations It is known that time evolution in dynamic systems may be represented by one–parameter groups of maps acting on the initial values of phase space variables $\mathcal{M} : X_0 \to X = \mathcal{M} \circ X_0$. In the case of Hamiltonian systems such maps form symplectic groups of symplectic maps – the so called Lie maps. In this way one have to compute the action of this group for given dynamic systems [2, 3].

The Matrix Formalism Let

$$\frac{dX}{dt} = F(X, U, t)$$

be a motion equation for particles in a beam line and there is an expansion $F(X,t) = \sum_{k=0}^{\infty} \mathbf{F}_k(t) X^{[k]}$. Here X is a phase vector in a local coordinate system, U is a control vector describing external control fields (generated, for example, by dipoles, quadrupoles and so on) and corresponding geometrical parameters. $X^{[k]} = \underbrace{X \otimes \ldots \otimes X}_{ktimes}$ is the Kronecker power of a phase vector X of k-th order, $\mathbf{P}^{1k}(U;t)$ is a $(n \times \binom{n+k-1}{k})$ –dimensional matrix. This representation is similar the Hamiltonian expansion which well known in beam physics because of to Alex Dragt's works (see, for example, [2]). For nonautonomous systems we can use the so called Magnus's representation [1]. This approach allows us to pass from the time–ordered exponent operator to a routine exponential operator. The expansion of the function F generates an expansion of the function $G(X;t|t_0) = \sum_{k=0}^{\infty} \mathbf{G}_k(t|t_0) X^{[k]}$ which appears in the Magnus's representation and so we can write

$$\mathcal{M}(t|t_0) = \exp\left\{\sum_{k=0}^{\infty} \mathcal{L}_{G_k(X;t|t_0)}\right\},$$

$$G_k(X;t|t_0) = \mathbf{G}_k\, X^{[k]}.$$

The similar to the Dragt–Finn factorization for the Lie transformations allows to rewrite the exponential operator as an infinite product of exponential operators of Lie operators

$$\mathcal{M} = \ldots \cdot \exp\{\mathcal{L}_{H_2}\} \cdot \exp\{\mathcal{L}_{H_1}\} =$$
$$= \exp\{\mathcal{L}_{V_1}\} \cdot \exp\{\mathcal{L}_{V_2}\} \cdot \ldots,$$

where $H_k = \mathbf{H}_k X^{[k]}$, $V_k = \mathbf{V}_k X^{[k]}$ are homogeneous polynomials of k-th order. The matrices $\mathbf{H}_k$ or $\mathbf{V}_k$ can be calculated with the help of the continuous analogue of the CBH– and Zassenhauss formulae and by using the Kronecker product and Kronecker sum technique for matrices. Moreover using the matrix representation for the Lie operators one can write a matrix representation for the Lie map generated by these Lie operators

$$\mathcal{M} \cdot X = \mathbf{M}\, X^{\infty} = (\mathbf{M}^{10}\mathbf{M}^{11}\mathbf{M}^{12} \ldots \mathbf{M}^{1k} \ldots) X^{\infty} =$$

$$= \sum_{k=0}^{\infty} \mathbf{M}^{1k}\, X^{[k]},$$

$$X^{\infty} = (1\ X\ X^{[2]} \ldots X^{[k]} \ldots)^{*},$$

where the matrices $\mathbf{M}^{1k}$ (**solution matrices**) can be calculated according to the recurrent sequence of formulae of the following types:

$$\mathcal{M}_k \cdot X^{[l]} = \exp\{\mathcal{L}_{G_k}\} \cdot X^{[l]} =$$

$$X^{[l]} + \sum_{m=1}^{\infty} \frac{1}{m!} \prod_{j=1}^{m} \mathbf{G}_m^{\oplus((j-1)(k-1)+l)}\ X^{[m(k-1)+l]},$$

where $\mathbf{G}^{\oplus l} = \mathbf{G}^{\oplus(l-1)} \otimes \mathbf{E} + \mathbf{E}^{[l-1]} \otimes \mathbf{G}$ –the Kronecker sum of l-th order. Using the generalized Gauss's algorithm we can write the matrix representation for the inverse map $\mathcal{M}^{-1}$:$X \to X_0 = \mathcal{M}^{-1} \cdot X$.

1.2 The Dynamic Modeling Approach

Success or fail in the beamline creation is closely connected with model selection (or selection a whole family of models), in [4] is described the paradigm of dynamic modeling approach. This term we consider as a possibility of flexible manipulating with elementary objects (from which our beamline consists). There are some problems connected with optimal and correct selection of elementary object set: what is it the elementary physical object, what kind of properties must be provided by these objects, what kind of mathematical object can be associated to these physical objects and etc. But at the moment exist a wide set of object–oriented methods for the abstract system model creation (also its known as object–oriented modeling). These methods help us to formalize our modeling process.

1.3 Computer Implementations

In this report we describe a prototype of the designing system for construction of beam lines with desired characteristics. This designer's tools has to admit including different kind of mathematical methods used for solution searching too [5, 6]. Instruments collection should guarantee the convenience and effectiveness all manipulation for designing process.

2 BASIC CONCEPTS OF DYNAMIC MODELING

Here we consider the idea and concept of dynamic modeling paradigm for beamlines design. This approach is based on two aspects: matrix formalism for Lie algebraic methods and object-oriented programming approach. Using of these approaches permits to create a base of an expert system for studying of beamlines. For creation of corresponding databases and knowledge bases the computer algebra codes are used.

Usual approaches in beam physics deal with some set of different on their nature (mathematical one first of all) elements. This is one of the principal difficulty which interferes in building **knowledge bases** – the first part of any expert system. Indeed, these elements have different methodological description, operations and representations. In this case the knowledge engineering (i.e. the process of building expert systems) is hindered because we must use different kind of elements, define necessary connections between them (in the form of semantic nets, for instance), construct human interface for manipulation elements of the expert system. The analysis of the present status in the accelerator physics allow to make the following conclusions:

$\diamond$ the usage mathematical methods have to be adequate both to physical models and modern programming methods, to put it in another way the mathematical methods have to permit using of artificial intelligence methods and facilities;

$\diamond$ the manipulated objects must maintain a natural extension of the set of processes and effects being investigated;

$\diamond$ the manipulated objects must admit the most simple (from the a computational point of view) operations necessary for calculations.

From this point of view selection of the Lie algebraic methods is justified. According to the paradigm dynamic modeling we can go by the following way:

$\star$ to define an initial set of decisions (preliminary judgements of design problem);

$\star$ to apply one or more mathematical models of various properties of the dynamic system (in our case of the beam line system). On this step in the frame of the matrix formalism we must select some of set of matrices from databases and include them to a calculation module. If there are no necessary matrices one can calculate them using computer algebra codes (in this work we usually use the $REDUCE$ and $MAPLE$ codes;

$\star$ to analyze the solution and then accept or reject corresponding model, and in the later case, define a new set of decisions and return to the second step.

The running of calculation experiments allows the filling our knowledge bases, for example in the form of recommendations and exclusions which can help for selections of models in future.

In the frame of this approach the mathematical model are regarded as objects, which have such very important characteristics as the inheritance and polymorphism.

From point of view of the matrix formalism the **bricks** of the corresponding databases are matrices of low dimensions calculated in the symbolic form with the help of computer algebra codes.

These matrices are objects from which we build our models of a dynamic system - beam line system [7]. The extension of usual matrix algebra is realized on account of an introduction of the Kronecker sum and product. We have to note the possibility of using of parallel processing both in the symbolic and numeric modes of calculations.

3 CONCLUSION

Modern trends in soft– and hardware development give us a chance for more effective computing process realization. For this purpose it is more comfortable to use homogeneous objects and operations. In this case we have more low requests to machine resources. The symbolic representation of necessary information gives us a way for flexible and efficient manipulation by our mathematical objects for designer requirements. It is obviously, that for complex physical–mathematical models using the designer has need of a man-machine interface, which gives a possibility for making modifications of package's settings (for requirements in selected fields of beam physics). In this paper computer codes for design process of beam line is presented. This package has the properties of opening and enlarging environment (the designer has tools to include and manipulate by mathematical models which connected with such physical objects as dipole, quadropole, etc.), mathematical methods to solve of corresponding equations [6] and optimization methods [7] too.

A graphics interface makes to construct a starting beam lines for investigation more simple and comfortable. The using mathematical methods allow us to realize advantages of parallel and distributed calculations.

4 REFERENCES

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